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ABSTRACT. An expanded concept of state is given in relation to the observation system. The exact solution of the nonstationary Schrödinger Equation made it possible to obtain very important mathematical models that adequately describe the behavior of elementary particles at the microlevel of matter. To obtain an exact solution to the nonstationary Schrödinger equation, it was necessary to introduce the concept of an algorithmically predictable generalized function. An imaginary Riccati-type leveling is obtained. The solution to this equation created the prerequisites for creating models at the microlevel of matter. As a result, a model of an antiparticle was obtained, which is in the wave function together with the particle. The electric field gradient at which real particles are formed from the physical vacuum is determined. The corpuscular-wave dualism of particles at the microlevel of matter is substantiated. A gravitational wave model has been created. The important role played by photons – quanta of electromagnetic radiation – is noted. Using the example of a gravity wave, it is shown that gamma radiation certainly accompanies the identification of a gravitational wave.

Key words: Schrödinger equation, microcosm, elementary particles, Riccati equation, dispersion, diffusion.

Introduction

It is widely known that the solution of the stationary Schrödinger Equation made it possible to obtain significant results in relation to processes occurring at the mesolevel of matter. Some scientists came to the conclusion that Schrödinger Equation was created to serve de Broglie waves: <historically and logically, the Schrödinger Equation originated as an equation for de Broglie waves.> [1].

The author of this work believes that the Schrödinger Equation has its value. The exact solution of the nonstationary Schrödinger Equation, which was postponed indefinitely, conceals very important models of the physical microcosm. This paper shows that the exact solution of a non-stationary equation makes it possible to obtain a new class of mathematical models that adequately describe the behavior of elementary particles at the microlevel of matter. This proves that the exact solution of the nonstationary Schrödinger Equation has its own value.

Some data on optimization problems of Euler-Lagrange and Hamilton

E. Schrödinger was the first to express the idea about the existence of the optimal properties of the elementary particles, when he was writing his equation for the particles of the physical microcosm using Hamiltonian function. After establishing the adequacy of modeling by means of the equation of stationary processes of the microcosm it became clear that the microcosm is organization the basis of optimal principles. However, existed solution of the Schrödinger

Equation did not allow to model the wave function of the elementary particles on the microlevel. The present work solved that problem: the results obtained allow to model the wave function of the elementary particles existing in the microlevel.

Let us now define the essence of the optimization principle prevailing in the physical microcosm. According to that principle, under the action of conservative forces, any dynamic system moves in such a way as to minimize the time average value of the difference between kinetic and potential energies, i.e.

$$\delta\int_{t_1}^{t_2}(T-V)\,dt=0 \qquad (0.1^*)$$

or taking into account the equation (0.1^*), we can write

$$\int_{t_1}^{t_2}\delta L dt=0, \qquad (0.2^*)$$

where $T(q,p)$ – kinetic energy, $V(q)$ – potential energy, $L(q,p)$ – Lagrange function, q – generalized coordinate, $p=\dot{q}$ generalized impulse.

The variation of the Lagrange function in the integrand (0.2^*) is

$$\int_{t_1}^{t_2}\delta L dt=\int_{t_1}^{t_2}\frac{\partial L}{\partial p}\delta p dt+\int_{t_1}^{t_2}\frac{\partial L}{\partial q}\delta q dt=\int_{t_1}^{t_2}\frac{\partial L}{\partial q}\delta q dt+\frac{\partial L}{\partial p}\delta q\Big|_{t_2}^{t_1}-\int_{t_1}^{t_2}\frac{d}{dt}\left(\frac{\partial L}{\partial p}\right)\delta q dt=$$

$$=\int_{t_1}^{t_2}\left[\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial p}\right)\right]\delta q dt=0.$$

In the last expression it is assumed that $\delta q = 0$ for $t_1 = t$ and $t_2 = t$.

Since the number of generalized coordinates q is equal to the number of degrees of freedom and as δq does not depend on time, the latter equality is satisfied if the expression in square brackets is equal to zero, i.e.

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} \equiv \frac{dp}{dt} + \frac{\partial H}{\partial q} = 0 \Rightarrow \dot{p} = -\frac{\partial H}{\partial q}, \quad (0.1)$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{p}} - \frac{\partial L}{\partial p} \equiv 0 - \dot{q} + \frac{\partial H}{\partial p} = 0 \Rightarrow \dot{q} = \frac{\partial H}{\partial p}, \quad (0.2)$$

where $H = T + V$ is the Hamiltonian function (Hamiltonian). Expressions (0.1) and (0.2) show that the Euler-Lagrange equations are equivalent to the Hamilton equations, representing the right-hand side (with respect to the equivalence signs <<≡>>) of expressions (0.1) and (0.2). *Schrödinger used the Hamilton function* H *as a basis for the synthesis of his equation.*

The solution of the Euler-Lagrange equation is a functional. This fact determines the existence of an important Euler-Lagrange equation property of invariance to arbitrary transformation of coordinates.

The optimization principles of this equation imply not only the property of invariance, but also the possibility of continuum and discrete aspects of the system modeling.

It should be noted that the Euler-Lagrange equation belongs not only to physics, as noted in [3], but is the property of any national economic or scientific field for solving applied optimization problems.

§1. Extension of the concept of state in relation to the observation system

From the point of view of optimality, the concept of the integrity of a dynamic system, i.e. its indivisibility into separate subsystems is very important. It is convenient to interpret the integrity property in terms of observations (measurements).

Let the observation system be given by scalar equations:

$$\dot{x} = -\alpha x + \xi(t), \tag{1}$$

$$y = x + \varsigma(t) \tag{2}$$

of the object (1) and observation channel (2). In expressions (1) and (2) $\xi(t)$ and $\varsigma(t)$ are scalar random processes of the white noise type with the following stochastic characteristics:

$$E[\xi(t)] = 0, \quad E[\xi(t)\xi(t')] = \rho\delta(t-t'),$$
$$E[\varsigma(t)\varsigma(t')] = r\delta(t-t'), \quad E[\varsigma(t)] = 0,$$

where E is the mathematical expectation operator, δ – Dirac function, parameters α, ρ, r are constant; the processes ξ and ς are not correlated.

And the following designations: $E\left[x(0)\right] = 0,\ E\left[x^2(0)\right] = v_0,\ v = E\left[\left(\hat{x} - x\right)^2\right],$ where $\hat{x}$ denotes the conditional estimation of variable x, obtained by the least squares method, and v is dispersion of the variable x. In such a case, the equation for dispersion v will be given by [2]:

$$\dot{v} = -2\alpha v - \left(1/r\right)v^2 + \rho, \quad v\left(0\right) = v_0.^{1} \tag{3}$$

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[1] Equation (3), where the constant term is equal to zero, i.e. $\rho = 0$, we refer to as the Riccati equation.

Expression (3) is scalar form of the Riccati equation. The right-hand side of equation (3) can be written as a soliton [3]

$$\frac{dv}{dt} = -A\,\text{sech}^2(\beta^* t - \phi). \tag{6}$$

Soliton solutions of the integrity dynamical systems have an important property. The property lies in the optimality of the soliton solution of the Riccati equation (3). The general solution of equation (3) has the following form [2]:

$$v = v_1 + \frac{v_1 + v_2}{\left[(v_0 + v_2)/(v_0 - v_1)\right]e^{2\beta^* t} - 1}, \tag{3a}$$

where

$$\beta^* = \sqrt{\alpha^2 + \rho / r}, \tag{7}$$

$$v_1 = r\left(\beta^* - \alpha\right), \tag{8}$$

$$v_2 = r\left(\beta^* + \alpha\right), \tag{9}$$

$$\phi = \ln\left(\sqrt{c}\right)^{-1}, \quad c = \frac{v_2 + v_0}{v_1 - v_0}, \quad v_1 > v_0, \quad A = D\beta^*, \quad D = \frac{v_1 + v_2}{2\sqrt{c}}$$

and v_0 is the dispersion value v at the initial moment of time $t_0 = 0$, i.e. $v_0 = v(0)$.

Finally, solution of equation (4) allows us to determine the dispersion

$$v = -A\int_{t_o}^{t} \text{sech}^2\left[\beta^*\left(t' - t_0\right) - \phi\right]dt'. \tag{10}$$

Representation of the observation (measurement) system in the form of object (1) and observation channel (2) is formal. In natural conditions the observation system is an integrity formation; it cannot be divided into an object (1) and an observation channel (2). The observation channel (2) is an integral part of the observation object (1). Representation of a real observation system in the form of expressions (1) and (2) is appropriate for mathematical processing of the results of indirect observations. The class of integrity dynamic systems includes the systems modelled simulated by the following Riccati equations:

$$\dot{z} = mz(n-z), \quad z(t_0) = \mathrm{z}_0, \tag{11a}$$

$$\dot{z} = -mz(n-z), \quad z(t_0) = z_0. \tag{11b}$$

The solution of equations (11a) and (11b) is given by:

$$z = \frac{1}{4}n^2 m \int_{t_0}^{t} \operatorname{sech}^2\left[\frac{1}{2}mn(t-t_0)\right]dt, \tag{12a}$$

$$z = -\frac{1}{4}n^2 m \int_{t_0}^{t} \operatorname{sech}^2\left[\frac{1}{2}mn(t'-t'_0)\right]dt'. \tag{12b}$$

From the parity property of the soliton it follows that the parameter n can have both positive and negative signs in solutions (12a) and (12b). It should be noted that equations (11a) and (11b) are the particular forms of equation (3).

Solutions of integrity dynamical systems (10), (12a), (12b) have the dissipative property. Dissipative functions are not invertible with respect to the corresponding argument. Conservative functions are invertible; their second derivative with respect to the argument does not reverse the sign.

The ι time derivative of both sides of solution (3a) is the soliton differential equation (4), whose solution (10) sati*sfies the Euler-Lagrange optimization equations.* It is easy to verify that the functionals $L = L(v)$ and $L = L(z)$ (see (10), (12a), (12b)) satisfy the Euler-Lagrange equations (0.1) and (0.2).

The fact that the functional L satisfies the Euler-Lagrange equation means that the variance is zero $v = E\left[(\hat{x} - x)^2\right] = 0$, where $\hat{x} = E[x/y]$, i.e. the object (1) and the observation channel (2) represent one whole: the system (1), (2) is integrity. *Thus, soliton solutions of integrity dynamical systems have the following important properties:*

1. They satisfy the Euler-Lagrange optimization equations.

2. They are dissipative in time functions, i.e. these functions are time irreversible.

3. They do not allow to represent the equations of the object and the observation channel separately.

Further, the solution to the problem of mathematical modeling of the dispersion of the elementary particle is given at the microlevel of the matter having those properties.

* Denotations p and r given above are independent from those given below.

§2. The Analysis of Schrödinger and Stochastic Mechanics Equations

In the middle of the twenties of the last century, Austrian physicist Erwin Schrödinger using de Broglie's hypothesis of optico-mechanical analogy for the behavior of the micro particles and basing on the Hamilton optimization principle, synthesized the key equation of quantum mechanics named after him:

$$j\varepsilon \frac{\partial \Psi}{\partial t} = -\frac{1}{2}\varepsilon^2 \frac{\partial^2 \Psi}{\partial x^2} + \left(\frac{U(x)}{m}\right)\Psi\,, \tag{5}$$

where $j = \sqrt{-1}$, $\varepsilon = \hbar / m$, $\hbar = 1,05459 \cdot 10^{-34}\, J \cdot s$ is the Planck constant divided by 2π, Ψ, the wave function of the particle to be found, $U(x)$ the potential energy of the particle with mass m and coordinate x.

Schrödinger Equation is extraordinary. The extraordinary nature of the equation lies in the fact that it simultaneously belongs to two levels of the matter, partly to the micro-level (the left-hand side of the equal sign "=") and to the meso-level (the right-hand side); the meso-level of the matter is between the micro-level and the macro-level.

The solution to the Schrödinger Equation can be found in three ways.

The first way is used to solve the nonstationary equation (5). The second one is used to solve the stationary equation, i.e. for $\dot{\Psi} = 0$. This method was used by Schrödinger himself. Finally, the third way of solution uses the function close to a generalized function.[2] The latter method, applied by the author for solution of equation (5), allows to obtain the wave function of an elementary particle at the micro-level. In such a case, the Schrödinger Equation entirely belongs to the left-hand side of the plane with respect to the equal sign "=".

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[2]The function close to a generalized function will hereafter referred to as a normalized algorithmically realizable generalized function (ARGF).

Consider the solutions to the Schrödinger Equation at three levels of the matter separately.

1) Introduce denotations $\Psi(x,t)=\psi(x)\varphi(t)$[3]. In such a case, equation (5) can be written as follows

$$-j\varepsilon\frac{1}{\varphi}\frac{\partial\varphi}{\partial t}=\frac{1}{2}\varepsilon^2\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2}-\frac{U(x)}{m}. \qquad (6)$$

Since the left-hand side of equation (6) is the function of time, and the right-hand side is the function of coordinates, equation (6) is satisfied if and only if both parts are equal to a constant value [1]. We denote this constant value by W/m, where W is the total energy of the particle. When the above condition is satisfied, equation (6) is divided into two equations

$$j\varepsilon\frac{1}{\varphi}\frac{\partial\varphi}{\partial t}=\frac{W}{m},$$

$$\frac{\partial^2\psi}{\partial x^2}+2\frac{1}{\varepsilon^2 m}(W-U)\psi=0. \qquad (13)$$

Thus, the solution of the non-stationary equation (5) has no practical value;

2) Schrödinger solved the stationary equation (13) $(\dot{\Psi}=0)$ applied to the hydrogen atom (using a spherical coordinate system) and obtained a spectrum for the energy eigenvalues that coincides with the well-known experimental data. That showed that the stationary equation (13) correctly describes the motion of the electron in the potential electric field. Therefore,

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[3]Such an approach is valid if the potential energy of the particle does not depend on time.

equation (13) was taken as the basic equation of stationary states of quantum mechanics;

3) The entire Schrödinger Equation transferred to the micro-level was not obtained neither by Schrödinger nor other scientists. In this work, the solution of the Schrödinger Equation is transferred to the micro-level of matter.

Such an approach to the solution of the Schrödinger Equation will make it possible to solve number of problems up to now known from heuristic considerations.

To pass from the solution of the continuous equation (5) to the solution of the equation transferred to the micro-level, it is necessary to use the system of equations of stochastic mechanics transferred to the microlevel with the help of ARGF [4]. The ARGF is considered to transfer dispersions and diffusions from real areas in to imaginary areas.

The system of equations of stochastic mechanics has the following form:

$$\frac{\partial P}{\partial t}+\Delta\cdot\left(Pv\right)=0, \tag{14}$$

$$\frac{\partial v}{\partial t}+\left(v\cdot\Delta\right)v=\frac{F}{m}+\left(u\cdot\Delta\right)u-\frac{1}{2}\varepsilon\,\Delta^2 u, \tag{15}$$

$$Pu=-\frac{1}{2}\varepsilon\Delta P, \tag{16}$$

where v and u are unidimensional vectors of real dispersion and diffusion of the elementary particle; $\Delta=\frac{\partial}{\partial x}$, unidimensional vector-operator; $F=\frac{dU}{dx}$, the gradient of the field U, i.e., $F=\mathrm{grad}U$; of the point “.” denotes scalar product.

Since in such a case the angle between the vectors equals to 0 degrees, the point in equations (14) and (15) can be omitted, i.e. the scalar product can be replaced by the ordinary product.

Equation (16) can be written as follows:

$$u = -\frac{1}{2}\varepsilon\left(\ln P\right)'_x = -\frac{1}{2}\varepsilon\frac{\Delta P}{P}, \tag{16a}$$

where $\left(\ln P\right)'_x$, is the rate of change of continuous probability density P of the real random process of the diffusion with coefficient $-\frac{1}{2}\varepsilon$.

Further, equations (14)-(16) already transferred to the microlevel of the matter are used. And formula (16a) is used to transfer diffusion u to the microlevel of the matter.

§3. General Considerations for Transferring the Solution to the Schrödinger Equation Mapped to the Micro-level of the Matter

It is well known that the wave function of an elementary particle satisfying the Schrödinger Equation (5) can be given by:

$$\Psi = \sqrt{P}\exp\left\{\frac{j}{\varepsilon}\int v dx\right\}. \tag{17}$$

Substituting the wave function (17) in equation (5), taking into consideration equation (16), we will have the differential relation

$$-j\varepsilon\Delta\Psi = \left(v + ju\right)\Psi.$$

From that relation we can pass to the wave function which is solution of non-stationary Schrödinger Equation

$$\Psi(x,t) = e^{\frac{j}{\varepsilon}\int_{\tau_0}^{\tau} v(x,t)dx - \frac{1}{\varepsilon}\int_{\tau_0}^{\tau} u(x,t)dx}, \qquad (18)$$

where τ_0 is the very little time but other than zero, i.e. $\tau_0 \neq 0$.

The role of ARGF is, together with the equations of stochastic mechanics, to transfer the real dispersion function v and diffusion function u into the class of imaginary functions. Such a transfer allows to transform the real wave function Ψ into an imaginary wave function and, consequently, to obtain a solution to the Schrödinger Equation at the micro-level of the matter [6,7].

The Laplace transform from ARGF is given by $\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)$, where $s = \sigma + j\omega$, $\tau_0 = const.$

If we use the symbol $<< \xrightarrow[\square]{\square} >>$ of correspondence between the Laplace transform and its original, then it will be possible to determine ARGF in the time domain:

$$\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right) \xrightarrow[\square]{\square} (-1)^{n-1}, \quad n-1 < \frac{t}{\tau_0} < n, \qquad (19)$$

where s is the current time, $n = 1, 2, ...$

Apart from formula (19) ARGF can also be defined by the use of the inverse Laplace transform operator L^{-1}:

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = (-1)^{n-1}, \quad n-1 < \frac{t}{\tau_0} < n. \qquad (20)$$

If in formula (20) we take into consideration the equality $-1 = e^{\pi j(2k+1)}$, $k = 0,1,2,...,$ then the last expression will be written as follows:

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = e^{\pi j(2k+1)(n-1)\tau_0}, \ (n-1)\tau_0 \le t < n\tau_0 \text{ for.} n \text{»} 1. \quad (21)$$

Introduce designation:

$$\tau = n\tau_0 = x. \quad (22)$$

Clearly, $t = 0$ if $n = 1$. If $(n-1)\tau_0 = t$, according to (21) for $n = 2,3,...$ we will have

$$L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\} = e^{\pi j(2k+1)t}. \quad (23)$$

Let us introduce the distribution function of an imaginary random diffusion process defining the function by the right-hand side of expression (23). Then the density function $P(j,x,t,k)$ of the probability distribution of an imaginary random diffusion process will be found according to the expression

$$\left(L^{-1}\left\{\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)\right\}\right)'_t = \pi j(2k+1)e^{\pi j(2k+1)t} \equiv P(j,x,t,k). \quad (24)$$

Further, we use the wave function of just an imaginary random diffusion process, i.e., solution $\Psi(j,x,t,k)$, of non-stationary Schrödinger Equation, which is the mapping of solution (18) to the micro-level of the matter:

$$\Psi(j,x,t,k)=e^{\frac{j}{\varepsilon}\int\limits_{\tau_0}^{n\tau_0} v(j,x,t,k)dx-\frac{1}{\varepsilon}\int\limits_{\tau_0}^{n\tau_0} u(j,k)dx}, \tag{18a}$$

where $v(j,x,t,k)$, dispersion of an imaginary random diffusion process mapped to the microlevel. According to formula (18a), it is necessary to substitute diffusion $u(j,k)$ in it. The imaginary diffusion of an elementary particle is determined by the formula (16a) transferred to the micro-level of the matter. To that end, we use the density function $P(j,x,t,k)$ of distribution of the probability of an imaginary random diffusion process (24) taking into consideration the notations (22), (23):

$$u(j,k)=-\frac{\varepsilon}{2}\pi j(2k+1). \tag{25}$$

It can be seen from (25) that the diffusion of an imaginary random process for a concrete k is constant.

§4. Obtaining of the Dispersion Equation for the Elementary Particles Mapped onto the Micro-level of the Matter

In what follows, it is assumed everywhere that the system of equations of stochastic mechanics (14)-(16) consists of the functions $P(j,x,t,k)$, $u(j,k)$ and $v(j,x,t,k)$ mapped[4] to the

[4]Further, instead of the words "transfer to", their synonym "mapping to" will be used.

micro-level. It should be noted that $P(j,x,t,k)$ and $u(j,k)$ are already known, they are defined by the formulae (24) and (25). The mapped diffusion function $u(j,k)$ and the dispersion $v(j,x,t,k)$ function are used to substitute them in the mapped wave function formula (18a).

Below given sequence of mathematical operations allows to determine the equation satisfying the mapped dispersion $v(j,x,t,k)$.

To find the equation of the dispersion v mapped to the microlevel, we substitute the density $P(j,x,t,k)$ determined according to (24) into equation (14)

$$-\pi^2(2k+1)^2 e^{\pi j(2k+1)t} - v\pi^2(2k+1)^2 e^{\pi j(2k+1)t}$$
$$=-\pi j(2k+1)e^{\pi j(2k+1)t}\frac{\partial v}{\partial x}|:\pi j(2k+1)e^{\pi j(2k+1)t}.$$

The latter gives the differential equation

$$\frac{\partial v}{\partial x} = -\pi j(2k+1)(v+1). \qquad (26)$$

If we put the diffusion value (25) into the Nelson equation (15) mapped to the micro-level, we will have

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{F(x)}{m}. \qquad (27)$$

The joint solution of equations (26) and (27) leads to $2k+1$ $(k=0,1,2,...)$ number of Riccati-type equations mapped to the micro-level

$$\frac{\partial v}{\partial t} = \pi j(2k+1)v + \pi j(2k+1)v^2 + F(x)/m. \qquad (28)$$

For a certain k the expression (28) is the mapped scalar Riccati equation with constant coefficients.

Consider equation (28) as equation (3) mapped to the micro-level of the matter. In this reflection, the parameters $-2\alpha,\ -\frac{1}{r},$ and ρ of equations (3) are mapped to the parameter of equation (28), respectively. The mapping process can be schematically represented as follows:

$$-2\alpha \text{ is mapped to } \pi j\left(2k+1\right), \tag{29}$$

$$-\frac{1}{r} \text{ is mapped to } \pi j\left(2k+1\right), \tag{30}$$

$$\rho \text{ is mapped to } F(x)/m. \tag{31}$$

Thus, the joint solution of the equations of stochastic mechanics mapped to the micro-level (14)-(16) allows to obtain the Riccati equation (28) mapped to the micro-level, satisfying the dispersion $v\left(j,x,t,k\right)$ of elementary particle at the micro-level of the matter.

§5. Solution of the Equation Determining the Dispersion of Elementary Particle at the Microlevel in a Steady State

If instead of parameters $-2\alpha,\ -\frac{1}{r},$ and ρ we take into consideration their mapped values (29)-(31), then the structure of the solution to equation (28) will be the same (see (3a)) as it was in solution of equation (3). In such a case, the parameters $\beta,\ v_1,\ v_2$ are determined by taking into consideration the mappings (29)-(31), in accordance to formulae (7)-(9):

$$\beta(x) \equiv \beta(j,x,k) = \sqrt{-\frac{\pi^2}{4}(2k+1)^2 - j\pi(2k+1)\frac{F(x)}{m}}, \quad (32)$$

$$v_1(x) \equiv v_1(j,x,k) = \left[j\pi(2k+1)\right]^{-1}\left[\beta(x) - j\frac{\pi}{2}(2k+1)\right], \quad (33)$$

$$v_2(x) \equiv v_2(j,x,k) = \left[j\pi(2k+1)\right]^{-1}\left[\beta(x) + j\frac{\pi}{2}(2k+1)\right]. \quad (34)$$

Taking the above parameters into consideration, the solution of the mapped dispersion equation (28) for a certain k will be written as follows:

$$v(j,x,t) = v_1(j,x) + \frac{v_1(x)+v_2(x)}{\frac{v_0(x)+v_2(x)}{v_0(x)-v_1(x)}e^{2\beta(x)t} - 1}, \quad (35)$$

where $v_0(x) \equiv v(j,x,0)$ is the imaginary dispersion for $t = 0$.

Without losing generality, in solution of equation (28), we can assume that $v_0 = 0$. In such a case, the soliton solution of equation (28) will be given by

$$v(x,t) = -D(x)\beta(x)\int_0^\infty \operatorname{sech}^2\left[\beta(x)t - \phi(x)\right]dt, \quad (36)$$

where $D(x) = \frac{v_1(x)+v_2(x)}{2\sqrt{c}}$, $c = \frac{v_2(x)}{v_1(x)}$, $\phi(x) = \ln\left(\sqrt{c}\right)^{-1}$.

The functional determining the dispersion $v(x,t)$ in the time interval $t \in (0,\infty)$ depends on the coordinates of the particle in a complex way; therefore, the substitution of the mapped dispersion $v(x,t)$ in formula of wave function (18a) greatly complicates the calculation of the function, making calculation practically impossible. However, to determine the dispersion $v(x,t)$ in the stationary case, i.e. when $t=\infty$ and at the initial moment when $t=0$, the calculation of the dispersion is possible.

Indeed, for $t=\infty$, according to formula (35), we have $v(x)=v_1(x)$, and for $t=0$, then from (35) we receive $v_0(x)=0$. Consequently, calculation of integral (36) in the stationary state and at the initial moment will be given by

$$v\left(x,t=_0^\infty\right)\equiv v(x)=v_1(x)-0=v_1(x). \tag{37}$$

According to formula (33), expression (37) will be written as

$$v(x)=v_1(x)=\left[j\pi(2k+1)\right]^{-1}\beta(x)-\frac{1}{2}. \tag{38}$$

If in formula (32) we take out the term $-\frac{\pi^2}{4}(2k+1)^2$ for the radical sign, then we will have

$$\beta(x)=j\frac{\pi}{2}(2k+1)\sqrt{1+j\frac{4}{\pi m(2k+1)}F(x)}. \tag{39}$$

If in expression (38) we take into consideration (39), then we get[5]

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[5]If both sides of the formula (40) are multiplied by the expressions conjugate to that formula, respectively, i.e. to the $v^*=\frac{1}{2}\left[\sqrt{1+j\chi(x)}-1\right]$, we get the square of the dispersion of an elementary particle at the microlevel $v^2=\frac{1}{4}\left[1+j\chi(x)-1\right]=\frac{j}{\pi m(2k+1)}F(x)$. Hence, $\chi(x)=4v^2$ and, consequently, $r=\sqrt{1+\chi^2}=\sqrt{1+16v^4}$.

$$v(x)=\frac{1}{2}\sqrt{1+j\aleph F(x)}-\frac{1}{2}=\frac{1}{2}\sqrt{1+j\chi(x)}-\frac{1}{2}, \qquad (40)$$

where $\aleph=\dfrac{4}{\pi m(2k+1)}$, $\chi(x)=\aleph F(x)$.

With account of the formula

$$\sqrt{1+j\chi}=\pm\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right],\ r=\sqrt{1+\chi^2}, \qquad (41)$$

expression (39) will be written as

$$\beta(x)=\pm\left[j\eta(x)-\gamma(x)\right],$$

where $\eta(x)=\dfrac{\pi}{2}(2k+1)\sqrt{\dfrac{r+1}{2}}$, $\gamma(x)=\dfrac{\pi}{2}(2k+1)\sqrt{\dfrac{r-1}{2}}$.

§6. Obtaining a Mathematical Model of an Antiparticle

In 1930, guided by physical considerations, P. Dirac predicted the existence of antiparticles: each elementary particle corresponds to its antiparticle; the positron is the antiparticle for the electron. All predictions on the existence of antiparticles were confirmed experimentally and the antiprotons, antineutrons, etc. were discovered.

The mathematical substantiation of this phenomenon is of interest. For mathematical confirmation of this phenomenon, it is reasonable to consider the wave function of the particle and antiparticle at the microlevel of the matter. According to formula (18a), mathematical operations are performed in exponential order; the exponential order will be obtained, if we take into consideration the expressions (25) and (40) in formula (18a):

$$\frac{j}{2\varepsilon}\int_{\tau_0}^{n\tau_0}\sqrt{1+j\chi(x)}dx-\frac{j}{2\varepsilon}\int_{\tau_0}^{n\tau_0}dx+\frac{1}{2}j\pi(2k+1)\int_{\tau_0}^{n\tau_0}dx. \tag{42}$$

Represent the expression $\sqrt{1+j\chi(x)}$ separately according to formula (41): the terms with the positive sign and with the negative sign will be considered separately.

First, we take into consideration the plus sign (+) and then the minus sign (-). In such a case, expression (42) will be written in two variants:

$$\frac{jt}{2\varepsilon}\left\{\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]-1\right\}+\frac{jt}{2}\pi(2k+1), \tag{43}$$

$$\frac{jt}{2\varepsilon}\left\{-\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]-1\right\}+\frac{jt}{2}\pi(2k+1). \tag{44}$$

Further, it is assumed that the particle and the antiparticle are in a constant potential energy, i.e. $U=const$ and, consequently, $r=1$.

According to expressions (43) and (44), the wave function can be written as follows:

$$\Psi_0=\left[e^{j\frac{\pi}{2}(2k+1)}\right]^t,\ \ k=1,2,3,... \tag{45}$$

$$\Psi_{1,2}=\left\{e^{-j\left[\frac{1}{\varepsilon}-\frac{\pi}{2}(2k+1)\right]}\right\}^t. \tag{46}$$

It should be noted that the wave function (45) does not contain the mass of the elementary particle, therefore, further, it will not be taken into consideration. The wave function (46) contains both the wave function of the particle and the wave function of the antiparticle. If we take into consideration Euler's formula $\left(e^{jg}=\cos g+j\sin g\right)$, then for obtaining the wave function of the antiparticle it is necessary to open the brackets of expression $\frac{\pi}{2}\left(2k+1\right)$. In such a case, the arguments of the sinusoidal and cosinusoidal functions will consist of three summands ε^{-1}, πk, $\frac{\pi}{2}$:

$$\Psi_1=\left[e^{-j\left(\frac{1}{\varepsilon}-k\pi-\frac{\pi}{2}\right)}\right]^t=\left[\cos\left(A+B+C\right)+j\sin\left(A+B+C\right)\right]^t. \tag{47}$$

To obtain the wave function of the particle it is not necessary to open the brackets of expression $\frac{\pi}{2}\left(2k+1\right)$;

in such a case, the arguments of the sinusoidal and cosinusoidal functions will consist of two summands ε^{-1}, $\frac{\pi}{2}\left(2k+1\right)$:

$$\Psi_2=\left\{e^{-j\left[\frac{1}{\varepsilon}-\frac{\pi}{2}(2k+1)\right]}\right\}^t=\left[\cos\left(a+b\right)-j\sin\left(a+b\right)\right]^t, \tag{48}$$

where

$$A = a = \frac{1}{\varepsilon},\ B =-\pi k,\ C = -\frac{\pi}{2},\ b = -\frac{\pi}{2}\left(2k+1\right),\ k = 1,3,5,...,$$

for particle and $k = 0,2,4,...,$ for antiparticle.

For the antiparticle we will have

$$\cos\left(A+B+C\right) = \cos A\cos B\cos C - \sin A\sin B\cos C - sinA\cos B\sin C -$$
$$-\cos A\sin B\sin C = -\sin\left(\frac{1}{\varepsilon}\right),$$

(49)

$$\sin\left(A+B+C\right) = \sin A\cos B\cos C + \cos A\sin B\cos C + \cos A\cos B\sin C -$$
$$-\sin A\sin B\sin C = \cos\left(\frac{1}{\varepsilon}\right).$$

(50)

For the particle we will have

$$\cos\left(a+b\right) = \cos a\ \cos b - \sin a\ \sin b = \sin\left(\frac{1}{\varepsilon}\right), \qquad (51)$$

$$\sin\left(a+b\right) = \sin a\ \cos b + \cos a\ \sin b = -\cos\left(\frac{1}{\varepsilon}\right). \qquad (52)$$

Consequently, the difference between the particle and the antiparticle is reduced to the group or the absence of group of summands πk and $\frac{\pi}{2}$.

If we take the results (51) and (52) into consideration in formula (48), taking de Moivre formula $\left(\cos g + j\sin g\right)^{t} = \cos tg + j\sin tg$ into consideration, then we get the wave function of the particle

$$\Psi_2 = \sin\left(\frac{t}{\varepsilon}\right) - j\cos\left(\frac{t}{\varepsilon}\right). \qquad (51a)$$

If we take the results (49) and (50) into consideration in formula (47) there we get the wave function of the antiparticle

$$\Psi_1 = -\sin\left(\frac{t}{\varepsilon}\right) + j\cos\left(\frac{t}{\varepsilon}\right). \qquad (49a)$$

Taking into account the expression (45) for the photon, as well as the formulas (49a), (51a) and exponent (42), the solution to the non-stationary Schrödinger equation takes the form

$$\Psi = \Psi_1 + \Psi_2 + \Psi_0.$$

Consequently, we have the annihilation

$$\Psi_1 + \Psi_2 = 0.$$

Taking into account annihilation, the solution to the nonstationary Schrödinger equation (5) will be written as follows:

$$\Psi(t) = \Psi_0 \equiv e^{j\frac{\pi}{2}(2k+1)t}. \qquad (18b)$$

The resulting solution (18b) makes no sense. On the left side of the sign <=> there is a wave function Ψ (t) that changes over time. On the right side of the sign <=> there is a photon Ψ_0 – quantum of electromagnetic radiation, depending on the imaginary unit j. To get rid of the imaginary unit j it is necessary to use the inverse Laplace L^{-1} transform operator (see the formula (20) in relation to the formula $\frac{1}{s}th\left(\frac{\tau_0 s}{2}\right)$. For this purpose, let us imagine the photon (45) in the following form:

$$\Psi_0 \equiv e^{j\frac{\pi}{2}(2k+1)t} = (-1)^{\frac{t}{2}} = (-1)^{\frac{\tau_0}{2}(n-1)}, \quad n-1 < \frac{t}{\tau_0} < n,$$

where $t = \tau_0 (n-1), \quad n = 1, 2, \ldots$

Let's introduce the following notation

$$\gamma(t) = L^{-1}\left\{\frac{1}{s} th\left(\frac{\tau_0 s}{4}\right)\right\}.$$

Using this notation we finally obtain a solution to the nonstationary equation (5) in the form

$$\Psi(t) = \gamma(t).$$

$\gamma(t)$ function is graphically depicted in Fig.1.

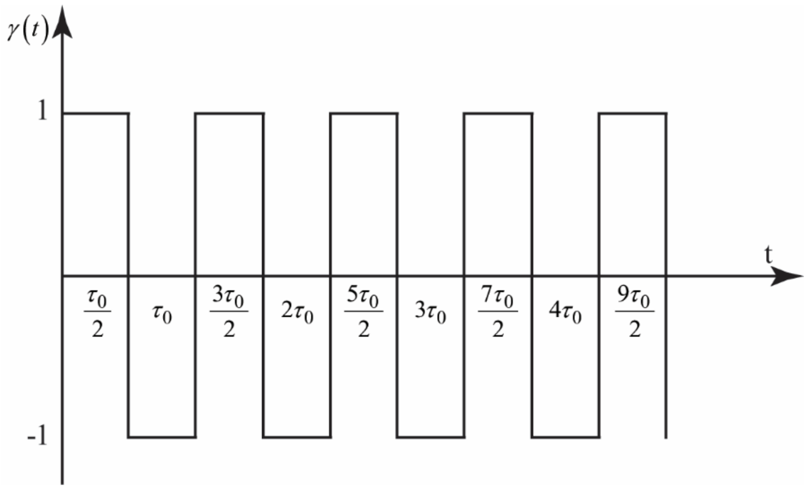

Fig.1. Discrete linear spectrum of γ radiation.

Gamma radiation is a very short-wave electromagnetic radiation with a wavelength not exceeding 10^{-2} nm. The discrete line spectrum of $\gamma(t)$ radiation is confirmation of the existence of discrete energy levels of nuclei.

§7. Model of Emergence of Elementary Particles from the Vacuum

The physical vacuum is teeming with virtual particles; it contains various kinds of virtual elementary particles. Physical vacuum exerts comprehensive pressure on any elementary particle both on the scale of the Universe and in laboratory conditions. It is considered that for any type of vacuum, there is a resonant wavelength. The data presented in show how sharply the concentration of elementary particles decreases with the change of the vacuum type. The present work shows how significantly the free path of neutrinos changes depending on the resonance wave of a certain type of vacuum. Below, a model is constructed based on the results of this work, which shows how elementary particles emerge from "nothing" in a strong electric field.

To determine the gradient of the electric field, where the real particles emerge from virtual particles, we use formula (41) instead of the first term of the expression (42); and without taking into consideration the coefficient $\frac{1}{2}$, we obtain

$$j\int_{\tau_0}^{n\tau_0}\left\{\pm\frac{1}{\varepsilon}\left[\sqrt{\frac{r+1}{2}}+j\sqrt{\frac{r-1}{2}}\right]-\frac{1}{\varepsilon}+\pi(2k+1)\right\}dx. \tag{53}$$

After multiplying (53) by j, in formula (53), we will have imaginary and non-imaginary parts separately

$$\int_{\tau_0}^{n\tau_0}\left\{-\frac{1}{\varepsilon}\left[\pm\sqrt{\frac{r-1}{2}}\right]+\left[\pm\frac{j}{\varepsilon}\sqrt{\frac{r+1}{2}}\right]-\frac{j}{\varepsilon}+j\pi(2k+1)\right\}dx. \tag{54}$$

If virtual particles are absent, in expression (54) the sum of the coefficients for imaginary terms will be equal to zero,

$$\frac{1}{\varepsilon}\left[\pm\sqrt{\frac{r+1}{2}}\right]-\frac{1}{\varepsilon}+\pi\left(2k+1\right)=0. \tag{55}$$

When the virtual particles are absent, we have only real particles. As $r=\sqrt{1+\aleph^2F^2}$, from equality (55) it follows that

$$\sqrt{1+\aleph^2F^2}=G\left(k\right), \tag{56}$$

where $G\left(k\right)=1-4\pi\varepsilon\left(2k+1\right)+2\pi^2\varepsilon^2\left(2k+1\right)^2$.

From equality (56) we find the gradient of the electric field, where the real particles emerge

$$F\left(k\right)=\left|\pm\frac{1}{\aleph\left(k\right)}\sqrt{G\left(k\right)^2-1}\right|. \tag{57}$$

The sign of the absolute value in formula (57) follows from the fact that the expression $\chi=\dfrac{4F}{\pi m\left(2k+1\right)}$ is positive and, consequently, $F>0$.

If we to take into consideration (55) in expression (54) and also consider the formulae (18a), (25), (40) and the denotation $\tau_0\left(n-1\right)=t$ we obtain the final result of emergence of real particles from "nothing"

$$\Psi\left(k,t\right)=e^{\frac{t}{2\varepsilon}\sqrt{\frac{r(k)-1}{2}}},\ t=\text{const}.$$

The parameter k, where the elementary particles begin to emerge from the vacuum, is determined from the following inequality

$$\frac{\tau_0\sqrt{\dfrac{r\left(k\right)-1}{2}}}{2\varepsilon}>1.$$

§8. Model of Corpuscular-Wave Dualism

After Planck's postulate about the discrete nature of energy radiation by atoms-oscillators (1900), the idea of quantization was developed by A. Einstein (1905). He suggested that quantum properties are inherent in light in general. It follows from Einstein's hypothesis that light must be considered not as a wave, but as a stream of quanta (photons) with the energy $E_0 = h\nu_0$[6] and impulse $p = \hbar\omega_0/c$[7] each. In terms of cognition, this hypothesis did not accept the position in classical physics about the essential difference between the matter and radiation; it affirmed the fundamental principle of the physics of the microcosm of wave – a model of corpuscular-wave dualism is given. To obtain a mathematical model of corpuscular-wave dualism. The hypothesis of the great thinker was of considerable theoretical value.

In this section, one of the main problems of this work is solved – a model of corpuscular-wave dualism is given. To obtain a mathematical model of corpuscular-wave dualism, consider the wave function of an elementary particle taking into consideration expression (53) and coefficient $\frac{1}{2}$ writing it as

$$\Psi(k,t) = e^{\frac{1}{2}\int\limits_{\tau_0}^{n\tau_0}\left\{\pm\left[-\frac{1}{\varepsilon}\sqrt{\frac{r-1}{2}}\right]\pm\left[\frac{j}{\varepsilon}\sqrt{\frac{r+1}{2}}\right]-\frac{j}{\varepsilon}+j\pi(2k+1)\right\}dx}, \qquad (58)$$

where $k = 1, 3, 5, \ldots$

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[6] h is the Planck constant, v_0 – the frequency of electromagnetic radiation.

[7] c – the speed of light in emptiness, ω_0 – the angular frequency of electromagnetic radiation.

The potential energy in which the elementary particle is located is constant, i.e. $U = \text{const}$. This means that $r = 1$ and the content in the first square brackets goes to zero.

If we put a plus sign (+) in front of the second bracket in formula (58), we get a wave function without a mass particle, i.e., a photon

$$\Psi_0 = e^{j\frac{\pi}{2}(2k+1)t}. \tag{45'}$$

If we put a minus sign (-) in front of the second bracket in formula (58), we obtain the wave function of an elementary particle with mass m

$$\Psi_1(k,t) = e^{-j\left[\frac{t}{\varepsilon}-\frac{\pi}{2}t(2k+1)\right]}. \tag{59}$$

We introduce the notation

$$a = \frac{1}{\varepsilon},\ b = -\frac{\pi}{2}(2k+1). \tag{60}$$

Taking these designations into consideration, the wave function (59) will be written as

$$\Psi_1(k,t) = \left[e^{-j(a-b)}\right]^t. \tag{61}$$

Using the Euler formula in the square brackets, the wave function (61) will be given by

$$\Psi_1(k,t) = \left[\cos(a+b) - j\sin(a+b)\right]^t. \tag{62}$$

Transformation of trigonometric functions and taking into consideration notation (60) gives

$$\cos(a+b) = \cos a\cos b - \sin a\sin b = \sin a, \tag{63}$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b = -\cos a. \tag{64}$$

Taking into consideration expressions (63) and (64) in formula (62), we obtain

$$\Psi_1(m,t) = [\sin a + j\cos a]^t. \tag{65}$$

Since time is discrete $t = (n-1)\tau_0$, the Moivre formula can be used; as a result, formula (65) will be written as

$$\Psi_1(m,t) = \sin(at) + j\cos(at). \tag{66}$$

Taking into consideration the fact that only the real part of the complex function has physical meaning, with account of designation (60) formula (66) will be given by

$$\Psi_1(m,t) = \sin\left(\frac{1}{\varepsilon}t\right). \tag{67}$$

To pass from formula (67) to the wave-particle duality, it is necessary to use the de Broglie formula

$$\lambda = \frac{h}{p} = \frac{h}{mV},^{8} \tag{68}$$

where λ is wave-length.

Using formulas (68) and (67), the mass of an elementary particle can be eliminated. Replacing it with the speed of that particle and determining the mass $m = \frac{h}{\lambda V}$ from (68) and substituting it in formula (67) we will have

$$\Psi_1(V,t) = \sin\left(\frac{2\pi}{\lambda V}t\right) = \sin\left(\frac{\omega_0}{V}t\right), \tag{69}$$

[8] λ is the wavelength of the elementary particle; V – the velocity of an elementary particle; p – the impulse of an elementary particle.

where $\omega_0 = \dfrac{2\pi}{\lambda}$ is the angular frequency.

As for small values ℓ we have $\sin \ell \approx \ell$, formula (69) can be given in two expressions

$$\Psi_1(V,t) = \sin\left(\frac{\omega_0}{V}t\right), \tag{69a}$$

$$\Psi_1(V,t) = \frac{\omega_0}{V}t. \tag{70}$$

If in formulas (69a) and (70), we take into account the formula for photon (45′), then we get

$$\Psi(V,t) = \sin\left(\frac{\omega_0}{V}t\right) + \Psi_0, \tag{69b}$$

$$\Psi(V,t) = \frac{\omega_0}{V}t + \Psi_0. \tag{70a}$$

Since formula (45′) for the photon Ψ_0 can be represented in unfolded in time form, i.e. in the form $\gamma'(t) = \mathrm{L}^{-1}\left\{\frac{1}{s}\,\mathrm{th}\left(\frac{\tau' s}{2}\right)\right\}$, then the wave (69 a) and corpuscular (70) functions of the non-stationary Schrödinger equation (5) will be written as follows

$$\Psi(V,t) = \sin\left(\frac{\omega_0}{V}t\right) + \gamma'(t), \tag{69c}$$

$$\Psi(V,t) = \frac{\omega_0}{V}t + \gamma'(t). \tag{70b}$$

Formula (69c) corresponds to the wave motion of the particle, and formula (70b) to corpuscular motion. So, the motion of an elementary particle can be viewed from two positions: waves and corpuscles.

§9. Gravitational Wave Model

Although the existence of gravitational waves was predicted by the general theory of relativity, their detection was possible only after a hundred years.

In the mid-seventies of the last century, the problem of indirect detection of gravitational waves was solved in [5]. In the article, the probabilistic (correlation) relationship between the earthquakes and flares passing through the solar chromosphere was proved. The studies presented in this work allows:

1) To ascertain that gravitational waves as such exist;

2) To ascertain that the gravitational waves consist of neutrinos as neutrinos freely pass through the Earth;

3) To ascertain that the flares in the solar chromosphere show that gravitational waves are composed of the matter, i.e. from neutrinos that have rest mass;

4) To determine the velocity of gravitational wave υ; since the distance from the earth to the sun is well known ($l = 15 \cdot 10^7$ km), the time t of flight of the neutrino cluster from the moment of the earthquake to the moment of the flares in the chromosphere, consequently, $\upsilon = \frac{l}{t}$.

Thus, it becomes clear that the gravitational wave is associated with the transfer of the matter in the form of a large aggregate of elementary particles of neutrino of the same type.

The necessity of introduction of neutrinos was determined by the law of conservation of energy in the process of the β^{+} – decay of the atomic nuclei. W. Pauli suggested a hypothesis (in 1931-1932) about the existence of neutrinos. E. Fermi gave the name "neutrino" to the particle due to lack of charge and its very small dimensions. He also expressed the idea that the neutrino is not in a "ready-made form" in the nucleus of an atom, but in some way, it is instantly formed from the energy of the nucleus.

Researchers of the composition of the cosmic rays reached the conclusion that all the ordinary matters in the Universe consists of two lightest leptons, an electron e and an electron neutrino ν_e [9].

For 30 years after the discovery of neutrino, it was believed that this particle had zero rest mass. The papers published at that time considered that gravitational waves carried energy and impulse, but they had nothing to do with the transfer of the matter [6].

It became known to cosmology that the total mass of neutrinos in the cosmos many times prevails the total mass of luminous objects and therefore the neutrino makes the main contribution to cosmic gravity [6].

Such an abundance of neutrinos in the Universe has created the prerequisites for creation of a new science – neutrino

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[9] Further, the index e by ν_e will be omitted. In addition to the electron neutrino ν, there exist τ – neutrinos and μ – neutrinos, but they are rare. We consider all neutrinos as three states of one particle. This is possible in the case when the laws of conservation of the lepton charges are violated.

astronomy. Consequently, the detection of gravitational waves belongs to that science.

Finally, the time came for direct detection of the gravitational wave: it occurred on February 11, 2016, when two highly sensitive detectors of the LIGO gravitational observatory, located in Washington and state of Louisiana, simultaneously recorded the signal GW150914, lasting about 0.2 seconds.

The position is accepted that the structure of the gravitational wave exponent coincides with the structure of the particle-wave exponent, that is, with the structure of the wave function (58). The value of the constant potential energy $U = \text{const}$ is the same as in the previous paragraph. However, the parameter ε has a different meaning defined as $\varepsilon^* = \dfrac{\hbar}{M}$.

The gravitational wave function has the form

$$\Psi(k,t) = e^{\frac{1}{2}\int\limits_{\tau 0}^{n\tau 0}\left\{\pm\left[-\frac{1}{\varepsilon^*}\sqrt{\frac{r^*-1}{2}}\right]\pm\left[\frac{j}{\varepsilon^*}\sqrt{\frac{r^*+1}{2}}\right]-\frac{j}{\varepsilon^*}+j\pi(2k+1)\right\}dx}, \qquad (71)$$

where $k = 1, 2, 3, \ldots,$ $r^* = \sqrt{1+\chi^{*2}}$, $\chi^* = \dfrac{4}{\pi M(2k+1)} \times \dfrac{dU}{dx}$, M – mass of neutrino beam is determined below.

This means that $r^* = r = 1$ and the content in the first square brackets goes to zero.

If we put a plus sign (+) before the second bracket in formula (71), then we get a wave function without a mass particle, i.e., a photon

$$\Psi_0 = e^{j\frac{\pi}{2}(2k+1)}. \qquad (45'')$$

If we put a minus sign (-) in front of the second bracket, we get the wave function of the gravitational wave

$$\Psi_1\left(k,t\right)=e^{-j\left[\frac{t}{\varepsilon^*}-\frac{\pi}{2}t\left(2k+1\right)\right]}, \tag{72}$$

where $t=\tau''\left(n-1\right)$.

We introduce the notation

$$a^*=\frac{1}{\varepsilon^*},\ b=\frac{\pi}{2}\left(2k+1\right). \tag{73}$$

Taking these designations into consideration, the wave function (72) will be written as

$$\Psi_1\left(k,t\right)=\left[e^{-j\left(a^*-b\right)}\right]^t. \tag{74}$$

Using the Euler formula in the square brackets, the wave function (74) will be given by

$$\Psi_1\left(k,t\right)=\left[\cos\left(a^*-b\right)-j\sin\left(a^*-b\right)\right]^t. \tag{75}$$

Transformation of trigonometric functions and taking into consideration notation (73) gives

$$\cos\left(a^*-b\right)=\cos a^*\cos b+\sin a^*\sin b=\sin a^*, \tag{76}$$

$$\sin\left(a^*-b\right)=\sin a^*\cos b-\cos a^*\sin b=-\cos a^*. \tag{77}$$

Taking into consideration expressions (76) and (77) in formula (75), we obtain

$$\Psi_1\left(M,t\right)=\left[\sin a^*+j\cos a^*\right]^t. \tag{78}$$

Since time is discrete $t=\left(n-1\right)\tau_0$, the Moivre's formula can be used; as a result, formula (78) will be written as

$$\Psi_1\left(M,t\right)=\sin\left(a^*t\right)+j\cos\left(a^*t\right). \tag{79}$$

Taking into consideration the fact that only the real part of the complex function has physical meaning, with account of designation (73) formula (79) will be given by

$$\Psi_1(M,t) = \sin\left(\frac{1}{\varepsilon^*}t\right). \quad (80)$$

The above model of the wave function of an elementary particle located at the micro-level, can be used (formula (80)) for modeling the gravitational wave, since neutrino belongs to the class of elementary particles. Therefore, the gravitational wave model is given by the formula

$$\Psi_1(M,t) = \sin\left(\frac{M}{\hbar}t\right). \quad (81)$$

If we take into account the wave function of the photon (45″), then the wave function of the gravitational wave will be written as follows

$$\Psi(M,t) = \Psi_1(M,t) + \Psi_0. \quad (82)$$

Formula (82) can be represented as

$$\Psi(M,t) = \sin\left(\frac{M}{\hbar}t\right) + \gamma''(t), \quad (83)$$

where $\gamma''(t)$ – the gamma function having a discrete spectrum; M is the relativistic mass of the neutrino beam. The neutrino beam consists of N – number of neutrinos of the same type; therefore, the relativistic mass of the neutrino beam is determined as follows

$$M = \sum_{i=1}^{N} m_i = Nm, \quad (84)$$

where m_i is the relativistic mass of one neutrino particle. The relativistic mass of one neutrino particle is determined according to the Lorentz transformation

$$m = \frac{m_0}{\sqrt{1-\left(\frac{\upsilon}{c}\right)^2}} = \mu m_0 , \tag{85}$$

where $\frac{1}{\sqrt{1-\left(\frac{\upsilon}{c}\right)^2}} = \mu$ is a constant parameter depending on the neutrino velocity, υ is the velocity of neutrinos; m_0, the rest mass of the neutrino.

Thus, the relativistic mass M of the neutrino beam is determined according to (84), taking into consideration the relativistic mass of the i- th particle (85):

$$M = N\mu m_0.$$

Since for a small value ℓ we have $\sin\ell \approx \ell$, then for the detected signal, according to formula (83), we obtain

$$\Psi(M,t) = \frac{M}{\hbar}t + \gamma''(t). \tag{86}$$

What Fermi Gamma-ray Space Telescope recorded in 11.02. 2016 should not be a <puzzle> [7], since the gravitational wave (86) model we obtained provides for the existence of gamma-ray $\gamma''(t)$ appearing when identifying a gravitational wave [7].

A feature of gamma-ray $\gamma(t)$, $\gamma'(t)$, $\gamma''(t)$ may be a different interval of discreteness $\tau(\tau_0, \tau', \tau'')$, i.e. quantization interval of these radiations.

* * *

Interaction carriers

Among the four types of fundamental interactions (strong, gravitational, weak and electromagnetic), only electrically charged particles and photons – quanta of electromagnetic radiation – participate in electromagnetic interaction. Photon is a typical representative of a new, important class of micro-objects – interaction carriers. The $\gamma(t)$ radiation is a particular type of electromagnetic radiation. The γ radiation is emitted by excited atomic nuclei during radioactive transformations, nuclear reactions, thermonuclear fusion, as well as other processes.

Results

An essential achievement of §1 was the expansion of the concept of the integrity of the system, that is, the quantumness of the entire system in relation to the observation system. The reason for expanding the concept of quantumness was the soliton solution of the Riccati equation, which satisfies the conditional dispersion v of the observational system. The dubious solution of the Riccati equation (10) satisfies the Euler-Lagrange equation (0.1). Therefore, the found solution is optimal for the problem posed, in which the observation channel belongs entirely to the observation system. The fact of quantization concept extension was known, but the reason for the extended concept of quantization was not understandable.

The main result of §2 is the author's statement, according to which he owns the method of using a generalized function to obtain a wave function in solving the Schrödinger Equation, which belongs to the microlevel of matter.

There are two main results in §3. The first result is the derivation of the structure of the solution of the non-stationary Schrödinger Equation (5). The second result is a mathematical model for the practical implementation of the generalized function algorithm (20). Subsequently, these results, together with the equations of stochastic mechanics, are used to transfer dispersion v and diffusion u to the class of imaginary functions, in which we have a model for solving the nonstationary Schrödinger Equation.

An essential result of §4 can be considered the derivation of an equation of the Riccati type (28) for determining the dispersion of an elementary particle located at the microlevel of matter. The solution of the imaginary Riccati equation (28), given in §5, became a harbinger of achieving the main goal of this monograph: obtaining new models of the physical microcosm and showing their optimality. Solution (18a) of the non-stationary Schrödinger Equation (5) allows creating a mathematical model of the antiparticle (47). This question is solved in §6. In the same paragraph, the process of annihilation that occurs when an antiparticle collides with a particle is shown. In addition to the antiparticle and particle, when solving the non-stationary Schrödinger Equation, one more particle is formed, whose wave function does not contain rest mass (45). This particle in elementary particle physics is called a photon: it does not participate in the process of annihilation.

It is well known that elementary particles are formed from the physical vacuum in a strong electric field. The solution (18a) of the non-stationary Schrödinger Equation (5) makes it possible to obtain a mathematical model of the process of formation of elementary particles from "nothing". In §7, the electric field gradient is determined, at which elementary particles with a certain mass are born (from vacuum). This model uses the parameter k, which has no dimension. Although the process of formation of elementary particles from the physical vacuum is widely known, many details of this phenomenon remain unknown. Here is what is said in the book of Paul Davies "Superforce" – "The phenomenon of birth from "nothing" occurs in a sufficiently strong electric field". This proposal does not say how the mass of the formed elementary particle affects the gradient of a "sufficiently strong electric field". So the model of the gradient (57) obtained in §7, of a strong electric field, in which elementary particles are formed from the physical vacuum, will shed light on many questions.

The use of the solution (18a) of the non-stationary Schrödinger Equation (5) together with the de Broglie formula (68) made it possible to obtain a mathematical model of wave-particle duality in §8. According to this model, the speed of movement of an elementary particle determines the choice between a wave and a particle.

An important problem of describing a gravitational wave is discussed in §9. The structure of the gravitational wave proposed in this paragraph is the same as in the wave model given in §8. However, this time the mass of the elementary particle was replaced by a beam consisting of neutrinos. To date, it is believed that the neutrino has a rest mass other than zero. The mass of a beam consisting of individual neutrinos is used instead of the mass of an individual elementary particle in the gravitational wave model (86).

Conclusion

The fact that the exact method of solving the problem is preferable to the approximate method is an indisputable fact. In this case, the significance of the exact method is enhanced by the fact that the solution of the non-stationary Schrödinger Equation gives physical science four new directions in the knowledge of the physical microcosm (see Fig. 2).

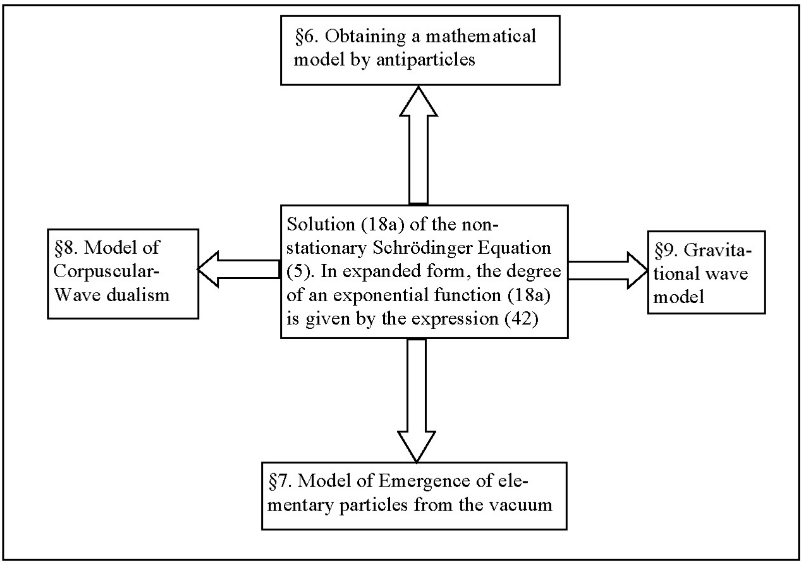

Fig. 5. Block diagram of new directions obtained thranks to solution (18a) of Schrödinger non-stationary equation (5).

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