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Editorial Office:
Georgian Academy of Sciences
52, Rustaveli Avenue,
Tbilisi, 380008,
Republic of Georgia

Telephone : **+995 32 99.75.93**
Fax : **+995 32 99.88.23**
E-mail : **BULLETIN@PRESID.ACNET.GE**

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A. Tavkhe

Academician Albert N. Tavkhelidze

Editor in Chief

The President of the Georgian Academy of Sciences

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MATHEMATICS

S.Saneblidze

Lusternik-Schnirelmann Category of a Weak Formal Space

Presented by Corr. Member of the Academy N.Berikashvili, December 26, 1994

ABSTRACT. **The paper presents the Lusternik-Schnirelmann category computed for a class of spaces by the integral cohomology ring.**

The Lusternik-Schnirelmann category of a topological space X, $cat(X)$, is the least integer n such that X has an open cover $n+1$ sets, each of which is contractable in X. Lusternik and Schnirelmann [1] proved that if a smooth real - valued function $M \to R$ on a manifold M is bounded below, then it has at least $cat(M)+1$ critical points. A simple approximation of $cat(X)$ is the *cup - length*, $c(X)$, of the integral cohomology $H(X)$. It is the least integer k such that $a_1 a_2 \dots a_{k+1} = 0$ for all $a_i \in \tilde{H}(X)$. In general, $cat(X) \geq c(X)$.

Here we describe a class of spaces for which the equality just holds above. Namely for a space X with $c(X) = n$ consider the following conditions:

(A) The diagonal map $X \to \Pi^{n+1} X = X \times \dots \times X$ is weak formal [2];

(B) The following short sequence is exact

$$0 \to H^i(X;\, \pi_{i-1}(F_X)) \xrightarrow{u^*} H^i(X;\, H_{i-1}(F_X))$$

where F_X denotes $n+1$ - fold join of the loop space ΩX of X and u^* is induced by the Hurewicz homomorphism in coefficients.

(C) For any sequence of homomorphisms preserving $H^*(X;\, R\pi_*(F_X))$

$$f_h^j : H^i(X;\, R_q H_i(F_X)) \to \oplus_{k=1}^{j} H^{i+k}(X;\, R_{q+j-k} H_{i+j-1}(F_X)),\ j \geq 2,$$

of the form $f_h^j = h \cup_-$, some $h \in H^*(X;\, Hom(RH_*(F_X),\, RH_*(F_X)))$ (the cup product is defined by the evaluation map in coefficients), $RH_*(F_X)$ is a free group resolution of $H_*(F_X)$, and for elements $a^{(n)} = (a^1, \dots, a^n)$, $a^i \in H^i(X;\, R_0 H_i(F_X)) \oplus H^{i+1}(X;\, R_1 H_i(F_X))$ and $c^{(n-1)} = (c^1, \dots c^{n-1})$, $c^i \in H^i(X;\, R_0 \pi_i(F_X)) \oplus H^{i+1}(X;\, R_1 \pi_i(F_X))$ with $[f_h(a^{(i-1)} - c^{(i-1)})]^{i+1} = 0$ and $[f_h(a^{(n)})]^{n+2} \in H^{n+2}(X;\, \pi_{n+1}(F_X))$, $f_h = \left\{ f_h^j \right\}$ there is an element

$$c^n \in H^n(X;\, R_0 \pi_n(F_X)) \oplus H^{n+1}(X;\, R_1 \pi_n(F_X)) \text{ such that } [f_h(a^{(n)} - c^{(n)})]^{n+2} = 0.$$

We have the following

Theorem 1. *If a simply connected space X having the homotopy type of a CW - complex satisfies conditions (A), (B) and (C), then*

$$cat(X) = c(X).$$

It is well - known that if $cat(X) = 1$, *then X is a co-H-space* [3].

Thus, we obtain the following

Corollary 2. *Let* $H^*(X)$ *have the trivial multiplication. If X satisfies (A) and (B), then X is a co-H-space.*

For example, the hypotheses of the theorem are satisfied for spheres, finite projective spaces, etc.

Now let for a map g: $X \to Y$, $c(g)$ denote the cup - length of the ring $Im(g^*)$, g^*: $H(Y) \to H(X)$ [3].

Then we have the following

Theorem 3. *Let $g:X \to Y$ be a weak formal map with $c(g) = n$. If X, Y satisfy conditions (A), (B') and (C'), where (B') and (C') are obtained respectively from (B) and (C) by replacing F_X by F_Y, then*

$$cat(g) = c(g).$$

The proofs of the theorems are based on an obstruction theory to the section problem in a fibration developed in [2].

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A.Razmadze Mathematical Institute
Georgian Academy of Sciences

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MATHEMATICS

M.Ashordia

On the Correctness of the Multipoint Boundary Value Problem for the System of Generalized Ordinary Differential Equations

Presented by Academician I.Kiguradze, April 10, 1995

ABSTRACT. **The sufficient conditions are given to guarantee the correctness of the problem**

$$dx(t)=dA(t)\cdot f(t,x(t)),$$
$$x_i(t_i)=\varphi_i(x)\ (i=1,\dots,n),$$

where $t_1,\dots,t_n\in[a,b]$, $A:[a,b]\to R^{n\times n}$ is a matrix-function with bounded variation components, $f:[a,b]\times R^n\to R^n$ is a vector-function belonging to the Caratheodory class corresponding of A, and $\varphi_i(i=1,\dots,n)$ are the continuous functionals from the space of vector-functions of bounded variation into R.

Let $t_1,\dots,t_n\in[a,b]$; $A_0=(a_{ik0})_{i,k=1}^{n}:[a,b]\to R^{n\times n}$ be a matrix-function with bounded variation components; $a_{ik0}(t)\equiv a_{1ik}(t)-a_{2ik}(t)$, where a_{jik} is a function nondecreasing on the every intervals $[a,t_i[$ and $]t_i,b]$ for $j\in\{1,2\}$ and $i,k\in\{1,\dots,n\}$;

$$A^{(j)}(t)=(a_{jik}(t))_{i,k=1}^{n}\ (j=1,2);\ f_0=(f_{i0})_{i=1}^{n}:[a,b]\times R^n\to R^n$$

be a vector-function belonging to the Caratheodory class corresponding to matrix-function A_0 and $\varphi_i:BV_s([a,b],R^n)\to R$ $(i=1,\dots,n)$ be a continuous functionals in general nonlinearity.

For the system of generalized ordinary differential equations

$$dx(t)=dA_0(t)\cdot f_0(t,x(t)) \tag{1}$$

consider the multipoint boundary value problem

$$x_i(t_i)=\varphi_i(x)\quad (i=1,\dots,n). \tag{2}$$

Consider a sequence of matrix-functions of bounded variation $A_m:[a,b]\to R^{n\times n}$ $(m=1,2\dots)$, a sequence of vector-functions $f_m=(f_{im})_{i=1}^{n}:[a,b]\times R^n\to R^n$ $(m=1,2,\dots)$ belonging to the Caratheodory class corresponding the matrix-function A_m, a sequence of points $t_{1m},\dots,t_{nm}\in[a,b]$ $(m=1,2,\dots)$ and a sequence of continuos functionals $\varphi_{1m},\dots,\varphi_{nm}:BV_s([a,b],R^n)\to R$ $(m=1,2,\dots)$.

In this paper sufficient conditions are given that guarantee both the solvability of the problem

$$dx(t)=dA_m(t)\cdot f_m(t,x(t)), \tag{1_m}$$
$$x_i(t_{im})=\varphi_{im}(x)\quad (i=1,\dots,n) \tag{2_m}$$

for any sufficiently large m and convergence of its solutions as $m\to+\infty$ to the solution of the problem (1), (2) if this problem is solvable.

The results of analogous character are contained in [1] for the Cauchy-Nicolletti's boundary value problem ($\varphi_i(x)=c_i$, c_i=const) and in [2,3] for the multipoint boundary value problems for the systems of the ordinary differential equations. The theory of generalized ordinary differential equations enables to investigate the ordinary differential and the different equations from the common standpoint [1, 4-8].

Throughout the paper the following notations and definitions will be used.

$R=]-\infty,+\infty[$, $R_+=[0,+\infty[$; $[a,b]$ $(a,b\in R)$ is a closed segment. $R^{n\times m}$ is a space of all real $n\times m$ - matrices $X=(x_{ij})_{i,j=1}^{n,m}$ with the norm $\|X\|=\max\limits_{j=1,\dots,m}\sum\limits_{i=1}^{n}|x_{i,j}|$; $R_+^{n\times m}=\left\{(x_{ij})_{i,j=1}^{n,m}: x_{ij}\ge 0\ (i=1,\dots,n;\ j=1,\dots,m)\right\}$.

If $X=(x_{ij})_{i,j=1}^{n,m}\in R^{n\times m}$, then $|X|=\left(|x_{ij}|\right)_{i,j=1}^{n,m}$, $[X]_+=\left(\dfrac{|x_{ij}|+x_{ij}}{2}\right)_{i,j=1}^{n,m}$. $R=R^{n\times 1}$ is a space of all real column n-vectors $x=(x_i)_{i=1}^{n}$; $R_+^n=R_+^{n\times 1}$. If $X\in R^{n\times n}$, then X^{-1} and $det(X)$ are respectively the matrix inverse to X and the determinant of X; I is the identity $n\times n$-matrix.

$\overset{b}{\underset{a}{V}}(X)$ is the sum of total variations of components $x_{ij}(i=1,\dots,n;\ j=1,\dots,m)$ of the matrix-function $X:[a,b]\to R^{n\times m}$; $V(X)(t)=(v(x_{ij})(t))_{i,j=1}^{n,m}$, where $v(x_{ij})(a)=0$ and $v(x_{ij})(t)=\overset{t}{\underset{a}{V}}(x_{ij})$ for $a<t\le b$; $X(t-)$ and $X(t+)$ are the left and the right limit of the matrix-function $X:[a,b]\to R^{n\times m}$ at the point $t(X(a-)=X(a),\ X(b+)=X(b))$. $d_1X(t)=X(t)-X(t-)$, $d_2X(t)=X(t+)-X(t)$; $\|X\|_s=\sup\left\{\|X(t)\|: t\in[a,b]\right\}$.

$BV_s([a,b],R^{n\times m})$ is a normed space of all matrix-functions of the bounded variation $X:[a,b]\to R^{n\times m}$ (i.e., such that $\overset{b}{\underset{a}{V}}(X)<+\infty$) with the norm $\|X\|_s$; $BV_s([a,b],R_+^n)=\{x\in BV_s([a,b],R^n): x(t)\in R_+^n\ (t\in[a,b])\}$. If $y\in BV_s([a,b],R^n)$ and $r\in\,]0,+\infty[$, then $U(y;r)=\{x\in BV_s([a,b],R^n): \|x-y\|_s<r\}$; $D(y;r)$ is a set of all $x\in R^n$ such that $\inf\left\{\|x-y(\tau)\|: \tau\in[a,b]\right\}<r$.

If $D\subset R$ is an interval, then $C(D,R^n)$ is a set of all continuous vector-functions $x:D\to R^n$; $C(D,R_+^n)=\{x\in C(D,R^n): x(t)\in R_+^n$ for $t\in[a,b]\}$.

If $g:[a,b]\to R$ is nondecreasing function, $x:[a,b]\to R$ and $a\le s<t\le b$, then $\int_s^t x(\tau)dg(\tau)=\int_{]s,t[} x(\tau)dg(\tau)+x(t)d_1g(t)+x(s)d_2g(s)$, where $\int_{]s,t[} x(\tau)dg(\tau)$ is the Lebesgue-Stieltjes integral over the open interval $]s,t[$ with respect to the measure μ_g corresponding to the function g (if $s=t$, then $\int_s^t x(\tau)dg(\tau)=0$).

A matrix-function is said to be nondecreasing if every its components are the same.

If $G=(g_{ij})_{i,j=1}^{l,n}:[a,b]\to R^{l\times n}$ is a nondecreasing matrix-function and $D\subset R^{n\times m}$, then $L([a,b], D; G)$ is a set of all matrix-functions $X=(x_{jk})_{j,k=1}^{n,m}:[a,b]\to D$ so that

$$\int_a^b |x_{jk}(t)|\, dg_{ij}(t)<+\infty \quad (i=1,\dots,l;\ j=1,\dots,n;\ k=1,\dots,m);$$

$$\int_s^t d\,G(\tau)\cdot X(\tau)=\left(\sum_{j=1}^{n}\int_s^t x_{jk}(\tau)dg_{ij}(\tau)\right)_{i,k=1}^{l,m} \quad \text{for } a\le s\le t\le b.$$

If $D_1\subset R^n$ and $D_2\subset R^{n\times m}$, then $K([a,b]\times D_1,D_2;\ G)$ is the Caratheodory class, i.e., the set of all mappings $F=(f_{jk})_{j,k=1}^{n,m}:[a,b]\times D_1\to D_2$ such that for each $i\in\{1,\dots,l\}$, $j\in\{1,\dots,n\}$ and $k\in\{1,\dots,m\}$: (a) the function $f_{jk}(\cdot,x):[a,b]\to D_2$ is $\mu_{g_{ij}}$-measurable for every $x\in D_1$; (b) the function $f_{jk}(t,\cdot):D_1\to D_2$ is continuous for $\mu_{g_{ij}}$-almost every where $t\in[a,b]$ and $\sup\{\|f_{jk}(\cdot,x)\|:x\in D_0\}\in L([a,b], R_+;\ g_{ij})$ for every compactum $D_0\subset D_1$.

If $G_j:[a,b]\to R^{l\times n}$ $(j=1,2)$ are a nondecreasing matrix-functions, $G=G_1-G_2$ and $X:[a,b]\to R^{n\times m}$, then $\int_s^t dG(\tau)\cdot X(\tau)=\int_s^t dG_1(\tau)\cdot X(\tau)-\int_s^t dG_2(\tau)\cdot X(\tau)$ for $a\le s\le t\le b$; $K([a,b]\times D_1,D_2;G)=\bigcap_{j=1}^{2}K([a,b]\times D_1,D_2;G_j)$.

If $B\in BV([a,b], R^n)$, then $M([a,b]\times R_+, R_+^n;\ B)$ is the set of all vector-functions $\omega\in K([a,b]\times R_+, R_+^n;\ B)$ such that $\omega(t,\cdot)$ is nondecreasing and $\omega(t,0)=0$ for $t\in[a,b]$.

Inequalities between both the vectors and matrices are understood as componentwise.

If B_1 and B_2 are the normed spaces, then an operator $\varphi:B_1\to B_2$ is called positive homogeneous if $\varphi(\lambda x)=\lambda\varphi(x)$ for every $\lambda\in R_+$ and $x\in B_1$. An operator $\varphi:BV_s([a,b], R^n)\to R^n$ is called nondecreasing if for every $x, y\in BV_s([a,b], R^n)$ such that $x(t)\le y(t)$ for $t\in[a,b]$ the inequality $\varphi(x)(t)\le\varphi(y)(t)$ is fulfilled for $t\in[a,b]$.

A vector-function $x\in BV_s([a,b], R^n)$ is said to be a solution of the system (1) if

$$x(t)=x(s)+\int_s^t d\,A_0(\tau)\cdot f_0(\tau,x(\tau)) \text{ for } a\le s\le t\le b.$$

Under a solution of the sysem of the generalized ordinary differential inequalities $dx(t)-dA_0(t)\cdot f_0(t,x(t))\le 0$ (≥ 0) we understand a vector-function $x\in BV_s([a,b], R^n)$ such that $x(t)-x(s)-\int_s^t d\,A_0(\tau)\cdot f_0(\tau,x(\tau))\le 0$ (≥ 0) for $a\le s\le t\le b$.

Let $l:BV_s([a,b], R^n)\to R^n$ be a linear continuous operator and let $l_0:BV_s([a,b], R^n)\to R_+^n$ be a positive homogeneous continuous operator. We shall say that a matrix-function $P:[a,b]\times R^n\to R^{n\times n}$ satisfies the Opial condition with respect to the triplet $(l, l_0; A_0)$ if : (a) $P\in K([a,b]\times R^n, R^{n\times n}; A_0)$ and there exists a matrix-function $\Phi\in L([a,b], R_+^{n\times n}; A_0)$ such that $|P(t,x)|\le\Phi(t)$ on $[a,b]\times R^n$; (b) $det(I+(-1)^j d_jB(t))\neq 0$ for $t\in[a,b]$ $(j=1,2)$ and the problem $dx(t)=dB(t)\cdot x(t)$, $|l(x)|\le l_0(x)$ has only trivial solution for every $B\in BV_s([a,b], R^{n\times n})$ provided, there exists a sequence $y_k\in BV_s([a,b], R^n)$ $(k=1,2,\ldots)$ such that $\lim_{k\to+\infty}\int_a^t dA_0(\tau)\cdot P(\tau,y_k(\tau))=B(t)$ uniformly on [a,b].

Let x^0 be a solution of the problem (1), (2) and let r be a positive number.

The solution x^0 is said to be a strongly isolated in the radius r if there exist $P\in K([a,b]\times R^n, R^{n\times n}; A_0)$, $q\in K([a,b]\times R^n, R^n; A_0)$, linear continuous operator $l:BV_s([a,b], R^n)\to R^n$, positive homogeneous continuous operator $l_0:BV_s([a,b], R^n)\to R_+^n$ and continuous operator $\widetilde{l}:BV_s([a,b], R^n)\to R^n$ such that: (a) $f_0(t,x)=P(t,x)x+q(t,x)$ for $t\in[a,b]$, $\|x-x^0(t)\|<r$ and the equality $h(x)=l(x)+\widetilde{l}(x)$ is fulfilled on $U(x^0;r)$; (b) vector-functions $\alpha(t,\rho)=max\,\{|q(t,x)|:\|x\|\le\rho\}$ and $\beta(\rho)=sup\{[\,|\widetilde{l}(x)|-l_0(x)]_+:\|x\|_s\le\rho\}$ satisfy conditions $\lim_{\rho\to+\infty}\frac{1}{\rho}\int_a^b d(A^{(1)}(t)+A^{(2)}(t))\cdot\alpha(t,\rho)=0$, $\lim_{\rho\to+\infty}\frac{\beta(\rho)}{\rho}=0$; (c) the problem $dx(t)=dA_0(t)\cdot[P(t,x(t))x(t)+q(t,x(t))]$, $l(x)+\widetilde{l}(x)=0$ has no solution differing from x^0; (d) the matrix-function P satisfies the Opial condition with respect to the triplet (l, l_0, A_0).

Let $h(x)=(h_i(x))_{i=1}^n$, $h_i(x)=x_i(t_i)-\varphi_i(x)$ $(i=1,\ldots,n)$ and $h_m(x)=(h_{im}(x))_{i=1}^n$, $h_{im}(x)=x_i(t_{im})-\varphi_{im}(x)$ $(i=1,\ldots,n;\ m=1,2,\ldots)$ for $x=(x_i)_{i=1}^n\in BV_s([a,b], R^n)$. By $W_r(A_0, f_0, h; x_0)$ we denote a set of all sequences (A_m, f_m, h_m) $(m=1,2,\ldots)$ such that: (a) $\lim_{m\to+\infty}\int_a^t dA_m(\tau)\cdot f_m(\tau,x)=\int_a^t dA_0(\tau)\cdot f_0(\tau,x)$ uniformly on $[a,b]$ for every $x\in D(x^0;r)$; (b) $\lim_{m\to+\infty}h_m(x)=h(x)$ uniformly on $U(x^0;r)$; (c) there exists a sequence $\omega_m\in M([a,b]\times R_+, R_+^n; A_m)$ $(m=1,2,\ldots)$ such that
$$sup\left\{\left\|\int_a^b dV(A_m)(t)\cdot\omega_m(t,r)\right\|: m=1,2,\ldots\right\}<+\infty,$$
$$\lim_{s\to 0+} sup\left\{\left\|\int_a^b dV(A_m)(t)\cdot\omega_m(t,s)\right\|: m=1,2,\ldots\right\}=0$$
and $|f_m(t,x)-f_m(t,y)|\le\omega_m(t,\|x-y\|)$ on $[a,b]\times D(x^0;r)$ $(m=1,2,\ldots)$.

The problem (1), (2) is said to be $(x^0;r)$-correct if for every $\varepsilon\in\,]0,r[$ and $((A_m, f_m, h_m))_{m=1}^{+\infty}\in W_r(A_0, f_0, h, x^0)$ there exists a natural m_0 such that the problem (1_m), (2_m) has at least one solution containing in $U(x^0;r)$ and every such solution belongs to the ball $U(x^0;\varepsilon)$ for any $m\ge m_0$.

The problem (1), (2) is said to be correct if it has a unique solution x^0 and for every $r>0$ it is $(x^0;r)$-correct.

We shall say that a pair $((c_{il})_{i,l=1}^{n};(\varphi_{0i})_{i=1}^{n})$ consisting of the matrix-function $(c_{il})_{i,l=1}^{n}\in BV_s([a,b],\ R^{n\times n})$ and of the positive homogeneous nondecreasing operator $(\varphi_{0i})_{i=1}^{n}:BV_s([a,b],\ R_+^n)\to R_+^n$ belongs to the set $U(t_1,\dots,t_n)$ if functions $c_{il}(i\neq l;\ i,\ l=1,\dots,n)$ are nondecreasing on $[a,b]$ and continuous at the point t_i, $d_jc_{ii}(t)\geq 0$ for $t\in[a,b]$ $(j=1,2;\ i=1,\dots,n)$ and the problem

$$[dx_i(t)\text{-}sign(t-t_i)\sum_{l=1}^{n}x_l(t)dc_{il}(t)]sign(t-t_i)\leq 0\ (i=1,\dots,n),$$

$$(-1)^j d_jx_i(t_i)\leq x_i(t_i)d_jc_{ii}(t_i)\ (j=1,2;\ i=1,\dots,n);$$

$$x_i(t_i)\leq\varphi_{0i}(|x_1|,\dots,|x_n|)\ (i=1,\dots,n)$$

has no nontrivial, non-negative solution.

Theorem 1. *Let conditions* $(-1)^{\sigma+1}f_{k0}(t,x_1,\dots,x_n)sign[(t-t_i)x_i]\leq\sum_{l=1}^{n}p_{\sigma ikl}(t)|x_l|+q_k(t,\|x\|)$

for $\mu_{a_{\sigma ik}}$ *- almost everywhere* $t\in[a,b]\setminus\{t_i\}$ $(i,k=1,\dots,n)$,

$$[(-1)^{\sigma+j+1}f_{k0}(t_i,\ x_1,\dots,x_n)sign x_i\text{-}\sum_{l=1}^{n}\alpha_{\sigma ikjl}|x_l|\text{-}q_k(t_i,\|x\|)]d_ja_{\sigma ik}(t_i)\leq 0\ (j=1,2;\ i,k=1,\dots,n)$$

be fulfilled on R^n *for every* $\sigma\in\{1,2\}$ *and let inequalities*

$$|\varphi_i(x_1,\dots,x_n)|\leq\varphi_{0i}(|x_1|,\dots,|x_n|)+\gamma\left(\sum_{l=1}^{n}\|x_l\|_s\right)\ (i=1,\dots,n)$$

be fulfilled on $BV_s([a,b],\ R^n)$, *where* $\alpha_{\sigma ikjl}\in R$ $(j,\sigma=1,2;\ i,\ k,\ l=1,\dots,n)$, $(p_{\sigma ikl})_{k,l=1}^{n}\in L([a,b],\ R^{n\times n};\ A^{(\sigma)})$ $(\sigma=1,2;\ i=1,\dots,n)$, $q=(q_k)_{k=1}^{n}\in K([a,b]\times R_+,\ R_+^n;\ A^{(\sigma)})$ $(\sigma=1,2)$ *is a vector-function nondecreasing with respect to the second variable,* $\gamma\in C(R_+,R_+)$ *and*

$$\lim_{\rho\to+\infty}\frac{1}{\rho}\int_a^b d(A^{(1)}(t)+A^{(2)}(t))\cdot q(t,\rho)=0,\quad \lim_{\rho\to+\infty}\frac{\gamma(\rho)}{\rho}=0.$$

Moreover, let there exist a matrix-function $(c_{il})_{i,l=1}^{n}\in BV_s([a,b],\ R^{n\times n})$ *such that*

$$(c_{il})_{i,l=1}^{n};\ (\varphi_{0i})_{i=1}^{n}\in U(t_1,\dots,t_n),\tag{3}$$

$$\sum_{\sigma=1}^{2}\sum_{k=1}^{n}\int_s^t p_{\sigma ikl}(\tau)da_{\sigma ik}(\tau)\leq c_{il}(t)\text{ - }c_{il}(s)$$

$$\text{for } a\leq s\leq t<t_i \text{ and } t_i<s<t\leq b\ (i,l=1,\dots,n),\tag{4}$$

$$\sum_{\sigma=1}^{2}\sum_{k=1}^{n}\alpha_{\sigma ikjl}\,d_ja_{\sigma ik}(t_i)\leq d_jc_{il}(t_i)\ (j=1,2;\ i,l=1,\dots,n).\tag{5}$$

If the problem (1), (2) *has no more one solution, then it is correct.*

Theorem 2. *Let conditions* $(-1)^{\sigma+1}[f_{k0}(t,x_1,\dots,x_n)\text{-}f_{k0}(t,y_1,\dots,y_n)]sign[(t-t_i)(x_i-y_i)]\leq\sum_{l=1}^{n}p_{\sigma ikl}(t)|x_l-y_l|$

for $\mu_{a_{\sigma ik}}$*-almost everywhere* $t\in[a,b]\setminus\{t_i\}$ $(i,k=1,\ldots,n)$,

$$\{(-1)^{\sigma+j+1}[f_{k0}(t_i,x_1,\ldots,x_n)-f_{k0}(t_i,y_1,\ldots,y_n)]sign(x_i-y_i)-\sum_{l=1}^{n}\alpha_{\sigma ik\,jl}|x_l-y_l|\}d_j a_{\sigma ik}(t_i)\le 0$$

$$(j=1,2;\ i,k=1,\ldots,n)$$

be fulfilled on R^n *for every* $\sigma\in\{1,2\}$, *and let the inequalities* $|\varphi_i(x_1,\ldots,x_n)-\varphi_i(y_1,\ldots,y_n)|\le\varphi_{0i}(|x_1-y_1|,\ldots,|x_n-y_n|)$ $(i=1,\ldots,n)$ *be fulfilled on* $BV_s([a,b],R^n)$, *where* $\alpha_{\sigma ikjl}\in R$ $(j,\sigma=1,2;\ i,k,l=1,\ldots,n)$, $(p_{\sigma ikl})_{k,l=1}^{n}\in L([a,b],R^{n\times n};A^{(\sigma)})$ $(\sigma=1,2;\ i=1,\ldots,n)$. *Moreover, let there exist a matrix-function* $(c_{il})_{i,l=1}^{n}\in BV_s([a,b],R^{n\times n})$ *such that conditions* (3)-(5) *hold. Then the problem* (1), (2) *is correct.*

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MATHEMATICS

M.Bakuridze

Some Properties of Fourier Trigonometric Series of Even and Odd Functions

Presented by Academician L.Zhizhiashvili, June 20, 1995

ABSTRACT. **Problems of continuity of sums of definite type "cosine" and "sine" series are considered. The criterion of the sum of "sine" series from the class H^{ω} is established.**

1. Let $f \in L(T)$ be an even function and $g \in L(T)$ be an odd function. Let Fourier series of these functions be, correspondingly

$$\sigma[f] = \sum_{K=1}^{\infty} a_K \cos Kx \quad \text{and} \quad \sigma[g] = \sum_{K=1}^{\infty} b_K \sin Kx.$$

Together with these series we consider the following series [2-4]

$$\sum_{K=1}^{\infty} U_K \cos Kx, \tag{1}$$

$$\sum_{K=1}^{\infty} U_K \sin Kx, \tag{2}$$

$$\sum_{K=1}^{\infty} U_K^* \cos Kx, \tag{3}$$

$$\sum_{K=1}^{\infty} U_K^* \sin Kx, \tag{4}$$

where $U_K = \frac{1}{K} \sum_{j=1}^{K} a_j$ and $U_K^* = \sum_{j=K}^{\infty} \frac{a_j}{j}$.

Also, we consider the series

$$\sum_{K=1}^{\infty} M_K \cos Kx, \tag{5}$$

$$\sum_{K=1}^{\infty} M_K \sin Kx, \tag{6}$$

$$\sum_{K=1}^{\infty} M_K^* \cos Kx, \tag{7}$$

$$\sum_{K=1}^{\infty} M_K^* \sin Kx, \tag{8}$$

where

$$M_K = \frac{1}{K}\sum_{j=1}^{K} b_j \quad \text{and} \quad M_K^* = \sum_{j=K}^{\infty} \frac{b_j}{j}.$$

Let ω be the modulus of continuity [4]. It is said, that ω satisfies Bari-Stechkin's condition [1], if

$$\int_0^{\sigma} \frac{\omega(t)}{t} dt + \sigma \int_{\sigma}^{\pi} \frac{\omega(t)}{t^2} dt = O(\omega(\sigma)), \qquad \sigma \to t$$

In the sequel, if $f \in C(t)$ then by $\omega(\sigma, t)_C$ we denote the modulus of continuity of the function f, and by H^{ω} - the following class of functions:

$$H^{\omega} = \{f: f \in C(t), \quad \omega(\sigma, t)_C = O(\omega(\sigma))\}$$

The theorems stated in the paper concern the behaviour of (1) - (8) series.

Theorem 1. *Let* $f \in C(T)$ *be an even function and* $a_k \geq 0$ $(k = 1, 2, \ldots)$. *Then the sums of the series* (3) *and* (4) *are continuous and the sums of the series* (1) *and* (2) *are not generally continuous.*

Theorem 2. *Let* $g \in C(T)$ *be an odd function and* $b_k \geq 0$. *Then the sum of the series* (8) *is continuous function and the sums of the series* (5), (6) *and* (7), *are not generally continuous.*

Theorem 3. *Let the modulus of continuity* ω *satisfies Bari - Stechkin condition* (9), $g \in C(T)$ *be an odd function and* $b_k \geq 0$ $(k = 1, 2, \ldots)$. *Then, the necessary and sufficient condition for the sum of the series* (8) *to be of* H^{ω} *class is*

$$k \cdot M_k^* \leq A \cdot \omega\left(\frac{1}{k}\right), \ k \geq 1.$$

I.Javakhishvili Tbilisi State University

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MATHEMATICS

T.Akhobadze

On Some Analogies of Riemann-Lebesgue Theorem for Functions of Multiple Variables - I

Presented by Academician L.Zhizhiashvili, June 21, 1995

ABSTRACT. **In the present paper the limits in Pringsheim's sense of the type**

$$\lim_{\lambda_1,\lambda_2\to+\infty}\iint_I f(u,v)\alpha(\lambda_1 u,\ \lambda_2 v)\,du\,dv$$

for functions f and α from various classes are investigated.

The Riemann-Lebesgue theorem asserts that Fourier coefficients C_ν of integrable functions concerning the trigonometric system tend to zero as $|\nu|\to\infty$. There exist some generalizations of this theorem. Kahane [2] continued investigations in this direction. In particular he considered the problem of evaluating limits of the type

$$\lim_{\lambda\to+\infty}\int_I f(t)\beta(\lambda t)\,dt \tag{1}$$

under various assumptions regarding the functions f and β over the interval I. On the other hand there exist analogies of the Riemann-Lebesgue theorem for multiple cases (i.e. for functions of multiple variables) and some its generalizations.

The purpose of the present paper is to investigate the behaviour of analogy of the limit (1) for multiple case, where the convergence is definite in Pringsheim's sense. From the obtained results in particular, we can get Kahane's [2] corresponding statements and some already known theorems. For the presentation simplicity of our results we shall formulate them for two-dimensional case, although analogies of these statements are valid for arbitrary finite-dimensional cases.

As usual we denote by $L_p\{I\}$ ($1\le p\le+\infty$) the class of Lebesgue integrable to the p-th power functions on the two-dimensional interval, moreover if $p=+\infty$ we designate by $L_\infty\{I\}$ the class of essentially bounded (in Lebesgue's sense) functions on I.

Theorem 1. *Let $\alpha\in L_\infty\{[0,+\infty)^2\}$. Then the necessary and sufficient condition for*

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_0^{+\infty}\int_0^{+\infty} f(u,v)\alpha(\lambda_1 u,\ \lambda_2 v)\,du\,dv$$

to exist in Pringsheim's sense for every function $f\in L_1\{[0,+\infty)^2\}$ is existence of the limit

$$M(\alpha)\equiv\lim_{T_1,T_2\to+\infty}\int_0^{T_1}\int_0^{T_2}\alpha(u,v)\,du\,dv \tag{2}$$

also in Pringsheim's sense; moreover this being the case

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_0^{+\infty}\int_0^{+\infty} f(u,v)\alpha(\lambda_1 u,\ \lambda_2 v)\,du\,dv=\left(\int_0^{+\infty}\int_0^{+\infty} f(u,v)\,du\,dv\right)M(\alpha).$$

Let $\alpha_{11}(u,v)$, $\alpha_{12}(u,v)$, $\alpha_{21}(u,v)$ and $\alpha_{22}(u,v)$ coincide respectively with $\alpha(u,v)$, $\alpha(u,-v)$, $\alpha(-u,v)$ and $\alpha(-u,-v)$ for $u\geq 0$ and $v\geq 0$.

Corollary 1. *Let $\alpha\in L_\infty\{(-\infty,+\infty)^2\}$ then the necessary and sufficient condition for*

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_0^{+\infty}\int_0^{+\infty} f(u,v)\alpha(\lambda_1 u,\ \lambda_2 v)dudv$$

to exist in Pringsheim's sense for every function $f\in L_1\{(-\infty,+\infty)^2\}$ is that for each function $\alpha_{ij}\in L_\infty\{[0,+\infty)^2\}$, $i,j=1,2$; to exist finite limit

$$\mathrm{M}(\alpha_{ij})=\lim_{T_1,T_2\to+\infty}\frac{1}{T_1T_2}\int_0^{T_1}\int_0^{T_2}\alpha_{ij}(u,v)dudv$$

in Pringsheim's sense. When these mean values exist the following representation is valid.

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(u,v)\alpha(\lambda_1 u,\lambda_2 v)dudv=\left(\int_0^{+\infty}\int_0^{+\infty} f(u,v)dudv\right)\mathrm{M}(\alpha_{11})+$$

$$+\left(\int_0^{+\infty}\int_{-\infty}^{0} f(u,v)dudv\right)\mathrm{M}(\alpha_{12})+\left(\int_{-\infty}^{0}\int_0^{+\infty} f(u,v)dudv\right)\mathrm{M}(\alpha_{21})+$$

$$+\left(\int_{-\infty}^{0}\int_{-\infty}^{0} f(u,v)dudv\right)\mathrm{M}(\alpha_{22}).$$

It is clear that Theorem 1 and Corollary 1 contain analogy of Riemann-Lebesgue statement for two-dimensional case. In particular from this theorem concerning functions of two variables we can get various analogies for already known statements of Fejer [3], Zygmund [1] and other authors which can be found for example in Polya-Szegö [4].

Let γ be a positive, locally-integrable function on $[0,+\infty)^2$. We say that a function γ belongs to the class $M(\gamma\in M)$ if for every positive constant C there exists a positive constant $M(C)$ such that for all positive T

$$\frac{1}{T}\int_0^C\int_0^T\gamma(u,v)dudv\leq M(C),$$

$$\frac{1}{T}\int_0^T\int_0^C\gamma(u,v)dudv\leq M(C). \qquad (3)$$

Theorem 2. *Let α be a locally-integrable to the p-th $(1<p<+\infty)$ power function on the interval $[0,+\infty)^2$, moreover suppose that $|\alpha|\in M$. Then, in order that the limit*

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_a^b\int_c^d f(u,v)\alpha(\lambda_1 u,\lambda_2 v)dudv\quad(0\leq a<b<+\infty,\ 0\leq c<d<+\infty) \qquad (4)$$

to exist in Pringsheim's sense for every function $f\in L_q\{[a,b]\times[c,d]\}$, where $\frac{1}{p}+\frac{1}{q}=1$, it is necessary and sufficient that

1) the averages

$$\frac{1}{T_1T_2}\int_{\frac{a}{b}T_1}^{T_1}\int_{\frac{c}{d}T_2}^{T_2}|\alpha(u,v)|^p\,dudv$$

be bounded as $T_1,T_2\to+\infty$, *and*

2) *to exist limit* (2) *in Pringsheim's sense. The conditions* 1) *and* 2) *holding then*

$$\lim_{\lambda_1,\lambda_2\to+\infty}\int_a^b\int_c^d f(u,v)\alpha(\lambda_1u,\lambda_2v)dudv=\left(\int_a^b\int_c^d f(u,v)dudv\right)M(\alpha). \quad (5)$$

Remark 1. If in the Theorem 2 a=0 and c=0, then the condition $|\alpha|\in M$ is unnecessary. If a or c are positive then the condition $|\alpha|\in M$ in the definite sense can't be weakened.

Theorem 3. *Let* α *be locally-integrable function on* $[0,+\infty)^2$. *Suppose that the function* $\gamma(u,v)=|\alpha(u,v)|$ *satisfies the condition* (3). *Then the necessary and sufficient condition for finite limit* (4) *to exist in Pringsheim's sense for every essentially bounded function f is that*

1)
$$\frac{1}{T_1T_2}\int_0^{T_1}\int_0^{T_2}|\alpha(u,v)|\,dudv$$

be bounded as $T_1,T_2\to+\infty$, *and* 2) *to exist limit* (2) *in Pringsheim's sense. If the conditions* 1) *and* 2) *are fulfilled then* (5) *is valid.*

Remark 2. If in the Theorem 3 a=0 and c=0 then the condition (3) for $\gamma(u,v)=|\alpha(u,v)|$ function are unnecessary too. If a or c are positive then the condition $|\alpha|\in M$ can't be weakened.

Remark 3. The Theorem 3 is new in one-dimensional case too, moreover in this case condition (3) are unnecessary.

I.Javakhishvili Tbilisi State University

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MATHEMATICS

I.Khasaia

Problems of Convergence Multiple Expansions in Complex Sphere for Polynomial Differential Bundle with Constant Coefficients

Presented by Academician L.Zhizhiashvili, December 20, 1995

ABSTRACT. **In case of constant coefficiental operator of the third order the problem of convergence spectral expansion has been studied in complex domain. The analogue of Abeli's well-known theorem about convergence power series has been obtained.**

Let's discuss differential bundle made by the differential equation

$$y^{(3)} - a\lambda y^{(2)} + a\lambda^2 y^{(1)} - \lambda^3 y = 0 \qquad (1)$$

and with expanded boundary conditions

$$y(0) = y'(0) = y(1) = 0 \quad (a = 1 - \sqrt{2}). \qquad (2)$$

Roots of characteristic equation are indicated with $\alpha_1, \alpha_2, \alpha_3$.

Suppose they are numbered so that $\alpha_1 = 1$, $\alpha_2 = e^{i\theta}$, $\alpha_3 = e^{-i\theta}$, where $\theta = 3/4\pi$.

The fundamental system of solution of equation (1) has the following form

$$y_j = e^{\lambda \alpha_j x}, \ 1 \le j \le 3 \ . \qquad (3)$$

λ-plane is divided into such S_j $(0 \le j \le 5)$ sectors that

$$\mathrm{Re}\lambda\alpha_1 \ge \mathrm{Re}\lambda\alpha_2 \ge \mathrm{Re}\lambda\alpha_3, \ \lambda \in S_0$$
$$\mathrm{Re}\lambda\alpha_1 \ge \mathrm{Re}\lambda\alpha_3 \ge \mathrm{Re}\lambda\alpha_2, \ \lambda \in S_1$$
$$\mathrm{Re}\lambda\alpha_3 \ge \mathrm{Re}\lambda\alpha_1 \ge \mathrm{Re}\lambda\alpha_2, \ \lambda \in S_2$$
$$\mathrm{Re}\lambda\alpha_3 \ge \mathrm{Re}\lambda\alpha_2 \ge \mathrm{Re}\lambda\alpha_1, \ \lambda \in S_3$$
$$\mathrm{Re}\lambda\alpha_2 \ge \mathrm{Re}\lambda\alpha_3 \ge \mathrm{Re}\lambda\alpha_1, \ \lambda \in S_4$$
$$\mathrm{Re}\lambda\alpha_2 \ge \mathrm{Re}\lambda\alpha_1 \ge \mathrm{Re}\lambda\alpha_3, \ \lambda \in S_5.$$

The following lemma takes place.

Lemma 1. For any $\lambda \in S_j$ $(0 \le j \le 5)$ the following unequalities take place

$$\frac{\theta}{2}(j - 1) \le \arg\lambda \le \frac{\theta}{2}j, \ j = 0, 1$$

$$\frac{\pi}{2}(j - 2 - 2\delta_{j,5}) + \frac{\theta}{2} \le \arg\lambda \le \pi(j - 1 - 2\delta_{j,5}) - \frac{\theta}{2}, \ j = 2, 5$$

$$\pi - \frac{\theta}{2}(j - 2 - 2\delta_{j,4}) \le \arg\lambda \le \pi + \frac{\theta}{2}(j - 3), \ j = 3, 4,$$

where $\delta_{j,k}$-is the symbol of Cronecker.

It isn't difficult to notice that the equation has the form for its eigen numbers [1]

$$\Delta(\lambda) = 0,$$

where

$$\Delta(\lambda) = \lambda \sum_{j=1}^{3} e^{\lambda \alpha_j} \Omega_j$$

and

$$\Omega_1 = \begin{vmatrix} 1 & 1 \\ \alpha_2 & \alpha_3 \end{vmatrix}, \quad \Omega_2 = \begin{vmatrix} 1 & 1 \\ \alpha_3 & \alpha_1 \end{vmatrix}, \quad \Omega_3 = \begin{vmatrix} 1 & 1 \\ \alpha_1 & \alpha_2 \end{vmatrix}$$

Theorem 1. (1)-(2) boundary problem has three infinite sequence of eigen numbers, that satisfy the following asymptotic

$$\lambda_{k,1} = \frac{\pi + 2\pi k - i \ln b_1}{2 \sin \frac{\theta}{2}} e^{-\frac{\theta}{2} i} + O(\frac{1}{k}) = \lambda^\circ_{k,1} + O(\frac{1}{k}),$$

$$\lambda_{k,2} = \frac{-\pi + 2\pi k - i \ln b_2}{2 \sin \frac{\theta}{2}} e^{\frac{\theta}{2} i} + O(\frac{1}{k}) = \lambda^\circ_{k,2} + O(\frac{1}{k}),$$

$$\lambda_{k,3} = \frac{-\pi + 2\pi k - i \ln b_1}{2 \sin \theta} e^{\pi i} + O(\frac{1}{k}) = \lambda^\circ_{k,3} + O(\frac{1}{k}),$$

where b_j $(1 \le j \le 3)$ are to λ independent numbers.

Let's take function

$$\psi(x, \lambda) = \lambda^{-1} \varphi(x, \lambda),$$

where

$$\varphi(x, \lambda) = \begin{vmatrix} y_1(0) & y_2(0) & y_3(0) \\ y'_1(0) & y'_2(0) & y'_3(0) \\ y_1(x, \lambda) & y_2(x, \lambda) & y_3(x, \lambda) \end{vmatrix}$$

$\psi(x, \lambda)$ is eigen function that is corresponding $\lambda = \lambda_k$ to eigen number (λ_k, $k \in N$ eigen numbers are counted according the growth of the model).

From here and (3) we get

$$\psi(x, \lambda) = \sum_{j=1}^{3} e^{\lambda \alpha_j x} \Omega_j .$$

The following lemmas are proved using corresponding questions from [2,3].

Lemma 2. If $\gamma_p^{(i)} \le \arg z \le \delta_p^{(i)}$ $(1 \le i \le 3,\ 1 \le p \le 3)$,

where $\gamma_1^{(i)} = 0,\ \gamma_{p+1}^{(i)} = \delta_p^{(i)} = \dfrac{(5+\delta_{i,3})p - i + 2}{8} \pi,\ 1 \le p \le 2,\ \delta_3^{(i)} = 2\pi,\ 1 \le i \le 3,$

then

$$\left|\psi(z, \lambda_{k,i})\right| \le \begin{cases} C\left|e^{\lambda_{k,i} \alpha_1 z}\right|, & i = p,\ 1 \le p \le 3 \\ C\left|e^{\lambda_{k,i} \alpha_{6-p-i} z}\right|. & 3-i \le p \le 3,\ 3-i+1 \le p \le 2 \end{cases}$$

Constant C does not depend upon k and p.

Lemma 3. *Suppose* $z_0 = |z_0| e^{\theta_0 i}$,

where $\theta_0 \in (\gamma_q^{(i)}, \delta_q^{(i)})$, $1 \le q \le 3$, $\gamma_1^{(i)} = 0$, $\gamma_{q+1}^{(i)} = \delta_q^{(i)} = \dfrac{(5+\delta_{i,3})q - i + 2}{8}\pi$

$\delta_3^{(i)} = 2\pi$, $(1 \le i \le 3)$.

Then

$$\left|\psi(z_0, \lambda_{k,i})\right| \ge \begin{cases} C\left|e^{\lambda_{k,i}\alpha_i z_0}\right|, & i = q, (1 \le p \le 3) \\ C\left|e^{\lambda_{k,i}\alpha_{6-q-i} z_0}\right|, & \text{the rest for } q. \end{cases}$$

Lemma 4. *Suppose that* $[c, d]$ *is an arbitrary segment*

$$c = |c| e^{\theta_0 i},\ d = |d| e^{\theta_0 i},\ 0 < |c| < |d| < 1.$$

Then K_0 *is dependent upon* $[c, d]$ *that* $\forall k > k_0$, $\exists z_k \in [c, d]$.

Then

1) $\lambda_k = \lambda_{k,1}$

$$\left|\psi(z_k, \lambda_{k,1})\right| \ge \begin{cases} C\left|e^{\lambda^0_{k,1}\alpha_1 z_k}\right|, & \theta_0 = 0, \frac{3}{4}\pi \\ C\left|e^{\lambda^0_{k,1}\alpha_3 z_k}\right|, & \theta_0 = \frac{11}{8}\pi \end{cases}$$

2) $\lambda_k = \lambda_{k,2}$

$$\left|\psi(z_k, \lambda_{k,2})\right| \ge \begin{cases} C\left|e^{\lambda^0_{k,2}\alpha_1 z_k}\right|, & \theta_0 = 0 \\ C\left|e^{\lambda^0_{k,2}\alpha_2 z_k}\right|, & \theta_0 = \frac{5}{8}\pi, \frac{5}{4}\pi \end{cases}$$

3) $\lambda_k = \lambda_{k,3}$

$$\left|\psi(z_k, \lambda_{k,3})\right| \ge \begin{cases} C\left|e^{\lambda^0_{k,3}\alpha_2 z_k}\right|, & \theta_0 = 0, \frac{5}{8}\pi \\ C\left|e^{\lambda^0_{k,3}\alpha_1 z_k}\right|, & \theta_0 = \frac{11}{8}\pi \end{cases}$$

Let's T_{z_0} $(z_0 = |z_0| e^{\theta_0 i})$ be polygon with vertex:

1) if $\theta_0 \in (0, \frac{5}{8}\pi)$

$|z_0|\cos(\theta_0 - \frac{\pi}{4})\cos^{-1}\frac{\pi}{4}$, $|z_0| e^{\theta_0 i}$, $|z_0|\cos(\theta_0 - \frac{9}{8}\pi)\cos^{-1}\frac{\pi}{4} e^{\frac{5}{8}\pi i}$, $|z_0| e^{\left(\frac{5}{4}\pi - \theta_0\right)i}$,

$|z_0| e^{\left(\frac{5}{4}\pi + \theta_0\right)i}$, $|z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{4} e^{\frac{11}{8}\pi i}$, $|z_0| e^{-\theta_0 i}$;

2) if $\theta_0 \in (\frac{5}{8}\pi, \frac{3}{4}\pi)$

$|z_0|\cos(\theta_0 + \pi)\cos^{-1}\frac{\pi}{4}$, $|z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{5}{8}\pi) e^{\left(\theta_0 - \frac{3}{8}\pi\right)i}$,

$$|z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{\pi}{2})e^{(\theta_0 - \frac{\pi}{8})i},\ |z_0|e^{\theta_0 i},\ |z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{8}e^{\left(\frac{\pi}{8}\right)i},$$

$$|z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{4}e^{\frac{11}{8}\pi i},\ |z_0|\cos(\theta_0 - \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{5}{8}\pi)e^{-\left(\theta_0 - \frac{3}{8}\pi\right)i};$$

3) if $\theta_0 \in (\frac{3}{4}\pi, \frac{5}{4}\pi)$

$$|z_0|\cos(\theta_0 + \pi)\cos^{-1}\frac{\pi}{4},\ |z_0|e^{\left(\frac{5}{4}\pi - \theta_0\right)i},\ |z_0|\cos^{-1}\frac{\pi}{4}\cos(\theta_0 + \frac{9}{8}\pi)e^{\frac{5}{8}\pi i}$$

$$|z_0|e^{\theta_0 i},\ |z_0|\cos^{-1}\frac{\pi}{4}\cos(\theta_0 - \pi)\cos(\theta_0 - \frac{9}{8}\pi)e^{\frac{11}{8}\pi i},\ |z_0|e^{\left(\theta_0 - \frac{5}{4}\pi\right)i};$$

4) if $\theta_0 \in (\frac{5}{4}\pi, \frac{11}{8}\pi)$

$$|z_0|\cos(\theta_0 + \pi)\cos^{-1}\frac{\pi}{4},\ |z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{5}{4}\pi)e^{(\theta_0 - \pi)i},$$

$$|z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{4}e^{\frac{5}{8}\pi i},\ |z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{3}{4}\pi)e^{\left(\theta_0 - \frac{3}{8}\pi\right)i},$$

$$|z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{3}{4}\pi)e^{\left(\theta_0 - \frac{\pi}{4}\right)i},\ |z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{4}e^{\frac{11}{8}\pi i},$$

$$|z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}(\theta_0 - \frac{5}{4}\pi)e^{(\pi - \theta_0)i};$$

5) if $\theta_0 \in (\frac{11}{8}\pi, 2\pi)$

$$|z_0|\cos(\theta_0 + \frac{\pi}{4})\cos^{-1}\frac{\pi}{4},\ |z_0|e^{(2\pi - \theta_0)i},\ |z_0|\cos^{-1}\frac{\pi}{4}\cos(\theta_0 + \frac{3}{8}\pi)e^{\frac{5}{8}\pi i},$$

$$|z_0|e^{\left(\theta_0 - \frac{3}{4}\pi\right)i},\ |z_0|e^{-\left(\theta_0 + \frac{\pi}{4}\right)i},\ |z_0|\cos(\theta_0 + \frac{3}{8}\pi)\cos^{-1}\frac{\pi}{4}e^{\frac{11}{8}\pi i},\ |z_0|e^{\theta_0 i};$$

6) if $\theta_0 = 0$

$$|z_0|,\ |z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4}e^{\frac{5}{8}\pi i},\ |z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{8}e^{\pi i},\ |z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4}e^{\frac{11}{8}\pi i};$$

7) if $\theta_0 = \frac{5}{8}\pi$

$$|z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4},\ |z_0|e^{\frac{5}{8}\pi i},\ |z_0|e^{\frac{11}{8}\pi i};$$

8) if $\theta_0 = \frac{3}{4}\pi$

$|z_0|,\ |z_0|e^{\frac{3}{4}\pi i},\ |z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4}e^{\frac{11}{8}\pi i}$;

9) if $\theta_0 = \frac{5}{4}\pi$

$|z_0|,\ |z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4}e^{\frac{5}{8}\pi i},\ |z_0|e^{\frac{5}{4}\pi i}$;

10) if $\theta_0 = \frac{11}{8}\pi$

$|z_0|\cos\frac{3}{8}\pi\cos^{-1}\frac{\pi}{4},\ |z_0|e^{\frac{5}{8}\pi i},\ |z_0|e^{\frac{11}{8}\pi i}$

By the given lemmas and the results from the work [4] the following theorem is proved

Theorem 2. *If*

$$\sum_{k=1}^{\infty} a_k \lambda_k^s \psi(x, \lambda_k) \tag{4}$$

$$(s = 3p + \nu,\ p = 0, 1, 2, \ldots,\ \nu = 0, 1, 2)$$

from the series one of them is uniformly convergent in $z_0 = |z_0|e^{\theta_0 i}$ *point complex plane, where*

a) $\theta_0 \in (\gamma_j, \delta_j),\ 1 \le j \le 5,\ \gamma_1 = 0,$

$\gamma_{j+1} = \delta_j = \dfrac{j+4+3(\delta_{j,3}+\delta_{j,4})}{8}\pi,\ \delta_5 = 2\pi$

b) $\theta_0 = 0,\ \gamma_{j+1},\ 2\pi \quad (1 \le j \le 4,\ |z_0| < 1)$.

Then

1) (4) series are absolutely and uniformly convergent in any closed domain that wholly belongs to T_{z_0} and consequently in domain T_{z_0} it represents analytical function;

2) if f_ν ($\nu = 0, 1, 2$) are the sums of the first three series then the sum result of every following three series is defined by the formula

$$f_s = f_{s-3}^{(3)} - a f_{s-2}^{(2)} + a f_{s-1}^{(1)} \quad (s = 3p + \nu,\ p = 1, 2, \ldots)$$

3) f_s functions satisfy boundary conditions to zero.

Georgian Subtropical Institute

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MATHEMATICS

N.Inasaridze

Non-Abelian Tensor Products of Finite Groups with Non-Compatible Actions

Presented by Corr. Member of the Academy H.Inasaridze, February 20, 1996.

ABSTRACT. **Some sufficient conditions for finiteness of a generalized non-abelian tensor product of groups are established.**

The non-abelian tensor product of groups was introduced by Brown and Loday [1-3].

It was defined for any A and B groups which act on themselves by conjugation and each of them acts on the other, so that the following compatibility conditions hold

$$^{(ab)}a'={}^{aba^{-1}}a',\quad {}^{(ba)}b'={}^{bab^{-1}}b', \tag{1}$$

for all $a,a'\in A$ and $b,b'\in B$ and $aba^{-1}, bab^{-1}\in A*B$ (free product of groups).Conditions (1) are very important for the investigation of the tensor product of Brown and Loday. It is known from G.Ellis's paper [4] the finiteness of this tensor product and (1) is important to prove this fact too.

The non-abelian tensor product of groups introduced by Brown and Loday was not available for constructing of the non-abelian homology of groups [5]. Therefore a new tensor product was introduced [5] without conditions (1) and with extra condititions. Just this tensor product will be considered in this paper.

Definition 1. *Let A and B be arbitrary groups which act on themselves by conjugation and each of them acts on the other.The non-abelian tensor product $A\otimes B$ of the groups A and B is the group generated by the symbols $a\otimes b$ and defined by the relations*

$$aa'\otimes b=({}^{a}a'\otimes {}^{a}b)(a\otimes b)$$

$$a\otimes bb'=(a\otimes b)\,({}^{b}a\otimes {}^{b}b')$$

$$(a\otimes b)\,(a'\otimes b')=({}^{[a,b]}a'\otimes {}^{[a,b]}b')(a\otimes b)$$
$$(a'\otimes b')\,(a\otimes b)=(a\otimes b)\,({}^{[b,a]}a'\otimes {}^{[b,a]}b')$$

for all $a,a'\in A$ and $b,b'\in B$, where $[a,b]=aba^{-1}b^{-1}\in A*B$.

Remark 2. *If the actions of the pair A,B of groups satisfy the compatibility conditions (1) then we obtain the tensor product of Brown and Loday.*

In this paper we concern the finiteness of this tensor product of groups.In general it is an open problem, but we give some sufficent conditions of finiteness.

Definition 3 . *Let A and B be arbitrary groups which act on themselves by conjugation and each of them acts on the other. Then Com-subgroup of A with B denoted by Com A(B) is the normal subgroup of A generated by the elements* ${}^{(ab)}a'\,{}^{aba^{-1}}a'^{-1}$ *where* $a, a'\in A$, $b'\in B$.

Note that if A and B act on each other compatibly then $Com\ A(B)$ and $ComB(A)$ are trivial groups.

Definition 4. *Let A and B be groups which act on each other. Under these actions A acts on B perfectly if the action of A on B induces the action of A on Com. B(A) and*

A,B are groups with perfect actions if under these actions A acts on B perfectly and B acts on A perfectly.

Definition 5. *Let (A,B) be an arbitrary pair of groups which acts on each other. Then Com-pairs of (A,B) are the pairs (ComA(B),B) and (A, Com B(A)) of groups.*

Let (A,B) be an arbitrary pair of groups. On the zero stage take the pair (A,B) of groups. If (A,B) are with perfect actions then we can go over to the first stage and take its *Com*-pairs with induced actions. If the obtained pairs are with perfect actions then we can go over to the second stage and take their *Com*-pairs with induced actions if not,this process stop at the first stage and so on.

Definition 6. *The family of these obtained pairs will be called the compatibility resolution of the pair (A,B).*

Definition 7. *Let A and B be groups acting on each other. It should be said that A and B act on each other half compatibly if for every pair of groups (C,D) of the compatibility resolution of (A,B) the actions are perfect and the following conditions hold:* ${}^{h}dd^{-1} \in ComD(C)$, ${}^{g}cc^{-1} \in ComC(D)$ *for each* $c\in C$, $d\in D$, $h\in ComC(D)$, $g\in ComD(C)$ *and at some n-th stage,* $n \geq 0$, *of the compatibility resolution of (A, B) every pair* (A_n, B_n) *is a pair of groups with compatible actions.*

Clearly if A and B are groups which act on each other compatibly (i.e. conditions (1) hold) then they act on each other half compatibly.

Theorem 8. *Let A and B be finite groups acting on each other half compatibly, then* $A \otimes B$ *is finite.*

Theorem 9. *Let A and B be finite groups. Let A acts on B and B acts on A trivially.If* $B^{(n)}$ *is abelian for some* $n\geq 1$, *then* $A \otimes B$ *is finite,where* $B^{(n)} = [B^{(n-1)}, B^{(n-1)}]$.

Example 10. *Let A and B be finite groups. Let* $[A,A]$ *be abelian and* $[A,A]\not\subset Z(A)$. *Then the actions of* $A\times B$ *on A by conjugation and of A on* $A\times B$ *trivially do not satisfy the compatibility conditions (1).*

From Theorem 9 $(A\times B) \otimes A$ *is finite.*

Now it will be shown the existence of such finite group A. Suppose $M= Z_p^{+}$ *(additive group) and* $N = Z_p^{\times}\backslash\{0\}$ *(multiplicate group) for any simple* $p > 2$. *Assume that N acts on M by multiplication of* Z_p *i.e.* ${}^{[n]}[m]=[nm]$ *for all* $[n] \neq 0$, $[m]\in Z_p$. *Let consider* $M\rhd\!\!\lhd N$ *(semi-direct product of M and N). Then the commutant* $[M\rhd\!\!\lhd N, M\rhd\!\!\lhd N]=M$ *and therefore it is abelian. It can be shown that* $[M\rhd\!\!\lhd N, M\rhd\!\!\lhd N]\not\subset Z(M\rhd\!\!\lhd N)$.

Thus $M\rhd\!\!\lhd N$ *is an example of the above mentioned finite group A and therefore* $\{(M\rhd\!\!\lhd N)\times B\} \otimes (M\rhd\!\!\lhd N)$ *is finite for any finite group B.*

Definition 11. *Let A and B be groups and A acts on B. Then [A,B] is a normal subgroup of B generated by elements* ${}^{a}bb^{-1}$ *for all* $a\in A$, $b\in B$, *and we can define*

$$[A,B]^n = [[A,B]^{n-1}], \; n > 1,$$

since the action of A on B induces the action of A on [A,B].

Theorem 12. *Let A and B be finite groups. A acts on B and B acts on A trivially. If* $[A,B]^n$ *is abelian for some* $n \geq 1$,*then* $A \otimes B$ *is finite.*

Note that Example 10 is available as example for Theorem 12.

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A.Razmadze Mathematical Institute
Georgian Academy of Sciences

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MATHEMATICS

M.Tchumburidze

About Matrices of Singular and Fundamental Solutions for Equations of the Generalised Couple - Stress Thermoelasticity Plane Theory

Presented by Academician T.Burchuladze, March 26, 1996

ABSTRACT. **The matrices of fundamental and other singular solutions are constructed explicit for the system of stationary equations of two - dimensional generalised couple - stress thermoelasticity theory and some of their properties are indicated.**

Basic dynamic homogeneous system of partial differential equations of generalized Green -- Lindsay couple - stress thermoelasticity on the plane for the homogeneous, izotropyc elastic media with the centre of symmetry has the form [1, 4, 5]:

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) v(x, t) = 0, \tag{1}$$

where,

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) = \left\| L_{j,k}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial t}\right) \right\|_{4\times 4},$$

$$L_{jk} = \delta_{jk}\left[(m+\alpha)\Delta - \rho\frac{\partial^2}{\partial t^2}\right], + (\lambda + \mu - \alpha)\frac{\partial^2}{\partial x_j\,\partial x_k}, j, k = 1,2$$

$$L_{jk} = \delta_{jk}\left[(m+\alpha)\Delta - 4\alpha - I\frac{\partial^2}{\partial t^2}\right], j = 3,\ k = 3,4$$

$$L_{jk} = -2\alpha\sum_{p=1}^{2}\varepsilon_{jkp}\frac{\partial}{\partial x_p},\quad \begin{matrix} j = 1,2,\ k = 3 \\ k = 1,2,\ j = 3 \end{matrix}$$

$$L_{j4} = -\gamma_\tau\frac{\partial}{\partial x_j},\quad j = 1,2$$

$$L_{4k} = -\eta\frac{\partial}{\partial t}\frac{\partial}{\partial x_k},\quad k = 1,2$$

$$L_{43} = L_{34} = 0,$$

$$L_{44} = \Delta - \frac{1}{\chi_\tau}\frac{\partial}{\partial t}, \gamma_\tau = \gamma\left(1 + \tau_1\frac{\partial}{\partial t}\right), \frac{1}{\chi_\tau} = \frac{1}{\chi}\left(1 + \tau_0\frac{\partial}{\partial t}\right),$$

where $V = (v_1, v_2, v_3, v_4)^T = (v, v_3, v_4)^T$, $v = (v_1, v_2)$ is the displacement vector, v_3 - characteristic of the rotation and v_4 is the temperature variation, symbol T - transpose operator, $x = (x_1, x_2)$ is the point of the twodimensional Euclidean space R^2 and t is the time, Δ is the two dimensional Laplacian operator, δ_{kj} - symbol of Kronikery, ε_{jkp} - symbol of Levi-Chivita, $\gamma, \chi, \eta, \rho, \lambda, \mu, \alpha, \nu, \beta, I$ - constant which satisfies the following conditions [2]: $\mu > 0,\ I > 0,\ \rho > 0,\ \alpha > 0,\ \nu > 0,\ \beta > 0,\ \chi > 0,\ \lambda + \mu > 0,\ \frac{\gamma}{\nu} > 0,\ \tau_0,\ \tau_1$ - the constants of relaxations [2]: $\tau_1 \geq \tau_0 \geq 0$.

In case of harmonic oscillations and pseudooscillations the (1) system to the $V(x, \omega) = (u_1, u_2, u_3, u_4)^T = (U, u_4)^T$, $U = (u_1, u_2, u_3)$ has come to the following form [2, 4]:

$$L\left(\frac{\partial}{\partial x}, -i\omega\right) v(x, \omega) = 0, \tag{2}$$

where ω is real or complex parameter respectively.

Let us search the matrix of fundamental solutions:

$\Phi(x, \omega) = \|\Phi_{jk}\|_{4\times4}$

with the following form:

$$\Phi(x, \omega) = \hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right) \varphi(x, \omega), \tag{3}$$

where $\hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right)$ - is the connecting matrix of $L\left(\frac{\partial}{\partial x}, -i\omega\right)$:

$$\hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right) L\left(\frac{\partial}{\partial x}, -i\omega\right) = L\left(\frac{\partial}{\partial x}, -i\omega\right) \hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right) = I \det L\left(\frac{\partial}{\partial x}, -i\omega\right) \tag{4}$$

I - is the unit matrix of dimension 4×4, $\varphi(x, \omega)$ - is researching scalar function.

As it is known $\hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right) = \left\|\hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right)\right\|_{4\times4}$, $\hat{L}\left(\frac{\partial}{\partial x}, -i\omega\right)$ - algebrical addition of $L_{jk}\left(\frac{\partial}{\partial x}, -i\omega\right)$ element on the $L\left(\frac{\partial}{\partial x}, -i\omega\right)$ matrix. With (3) and (4) force we get differential equations of the eighth *row* according to the $\varphi(x, \omega)$:

$$\det L\left(\frac{\partial}{\partial x}, -i\omega\right) \varphi(x, \omega) = (\lambda + 2\mu)(\mu + \alpha)(\nu + \beta) \prod_{k=1}^{4} \left(\Delta + \sigma_k^2\right) \varphi(x, \omega) = 0, \tag{5}$$

where σ_k^2, $k = 1,4$ characteristic parametres, which are explained from the following equations:

$$\begin{aligned} \sigma_1^2 + \sigma_2^2 &= \rho_0 \omega^2 (1 + \tau_2) + \frac{i\omega}{æ}(1 + \varepsilon) \\ \sigma_1^2 \cdot \sigma_2^2 &= \rho_0 \frac{i\omega}{æ} \omega^2 + \omega^4 \tau_3 \\ \sigma_3^2 + \sigma_4^2 &= \frac{\rho\omega^2}{\mu + \alpha} + \frac{I\omega^2 - 4\alpha}{\nu + \beta} + \frac{4\alpha^2}{(\mu + \alpha)(\nu + \beta)} \\ \sigma_3^2 \cdot \sigma_4^2 &= \frac{\rho\omega^2}{\mu + \alpha} \cdot \frac{I\omega^2 - 4\alpha}{\nu + \beta} \end{aligned} \tag{6}$$

Here, $\rho_0 = \dfrac{\rho}{\lambda+2\mu}$, $\tau_2 = \dfrac{\gamma\eta\tau_1}{\rho} + \dfrac{\lambda+2\mu}{\rho\chi}\tau_0 = \dfrac{1}{\rho_0\chi}(\varepsilon\tau_1+\tau_0)$, $\tau_3 = \dfrac{\rho_0\tau_0}{\chi}$, $\varepsilon = \dfrac{\chi\gamma\eta}{\lambda+2\mu}$.

According to (5) we have:

$$\varphi(\chi,\omega) = \sum_{k=1}^{4} a_k \cdot H_0^{(1)}(\sigma_k|x|), \tag{7}$$

where $H_0^{(1)}(\sigma_k|x|)$ - are Hunkel functions (the first kind the zero row), $|x| = \sqrt{x_1^2+x_2^2}$, a_k are constants. As it is known in the area close to zero we have Hunkel function with the following form [1]:

$$\frac{\Pi}{2i} H_0^{(1)}(\sigma_k|x|) = \ln|x| - \frac{\sigma_k^2}{2^2}|x|^2\ln|x| + \frac{\sigma_k^4}{2^6}|x|^4\ln|x| - \frac{\sigma_k^6}{2^8\cdot 3^2}|x|^6\ln|x| + $$
$$+\ \mathrm{const} + O|x|^8\ln|x|.$$

Let us, chose a_k, $k=\overline{1,4}$ constants so that 6th rows $\varphi(x,\omega)$ partial production close to zero area have speciality of $\ln|x|$ form. For this consider

$$a_k = \frac{i}{2\Pi(\lambda+2\mu)(\mu+\alpha)(\nu+\beta)(\sigma_{k+1}^2-\sigma_k^2)(\sigma_{k+2}^2-\sigma_k^2)(\sigma_{k+3}^2-\sigma_k^2)}, \quad k=\overline{1,4} \tag{8}$$

where, $\sigma_5=\sigma_1$, $\sigma_6=\sigma_2$, $\sigma_7=\sigma_3$.

If we check - up we get

$$\sum_{k=1}^{4} a_k = \sum_{k=1}^{4} a_k\,\sigma_k^2 = \sum_{k=1}^{4} a_k\,\sigma_k^4 = 0,\ \sum_{k=1}^{4} a_k\,\sigma_k^6 = \frac{i}{2\Pi(\lambda+2\mu)(\mu+\alpha)(\nu+\beta)},$$

$$\Phi_{pj}(x,\omega) = \sum_{k=1}^{4}\left\{\delta_{p\gamma}A_k + (-1)^{p+j}B_k\frac{\partial^2}{\partial x_{p+1}\partial x_{j+1}} + 4\alpha^2 C_k\frac{\partial^2}{\partial x_p\partial x_j}\right\}H_0^{(1)}(\sigma_k|x|),$$
$$x_3=x_1, \quad p,j=\overline{1,2}$$

$$\Phi_{pj}(x,\omega) = \sum_{k=3}^{4}\left\{N_K\sum_{n=1}^{2}\varepsilon_{pjn}\frac{\partial}{\partial x_n}\right\}H_0^{(1)}(\sigma_k|x|), \quad \begin{matrix} p=\overline{1,2}, & j=3 \\ j=\overline{1,2}, & p=3 \end{matrix}$$

$$\Phi_{uj}(x,\omega) = -i\omega\eta\frac{\partial}{\partial x_j}\sum_{k=1}^{2}E_k H_0^{(1)}(\sigma_k|x|), \quad j=\overline{1,2}$$

$$\Phi_{p4}(x,\omega) = \gamma(1-i\omega\tau_1)\frac{\partial}{\partial x_p}\sum_{k=1}^{2}E_k H_0^{(1)}(\sigma_k|x|), \quad p=\overline{1,2},$$

$$\Phi_{33}(x,\omega) = \sum_{k=3}^{4}D_k H_0^{(1)}(\sigma_k|x|),$$

$$\Phi_{44}(x,\omega) = \sum_{k=1}^{2}F_k H_0^{(1)}(\sigma_k|x|) + 8\alpha^2\frac{\partial^4}{\partial x_1^2\partial x_2^2}\sum_{k=1}^{4}a_k H_0^{(1)}(\sigma_k|x|),$$

where

$$A_k=\frac{a_k(\lambda_3^2-\sigma_k^2)(\lambda_4^2-\sigma_k^2)(\lambda_5^2-\sigma_k^2)}{(\mu+\alpha)(\nu+\beta)},\quad k=\overline{1,4},$$

$$B_k=\frac{a_k(\lambda_4^2-\sigma_k^2)(\lambda_6^2-\sigma_k^2)}{(\lambda+\mu-\alpha)(\nu+\beta)},\quad k=\overline{1,4}$$

$$C_k=a_k(\lambda_5^2-\sigma_k^2),\quad k=\overline{1,4},\quad N_k=\frac{a_k(\sigma_1^2-\sigma_k^2)(\sigma_2^2-\sigma_k^2)}{2\alpha(\lambda+2\mu)},\quad k=\overline{3,4}$$

$$D_k=\frac{a_k(\sigma_1^2-\sigma_k^2)(\sigma_2^2-\sigma_k^2)(\lambda_3^2-\sigma_k^2)}{(\mu+\alpha)(\lambda+2\mu)},\quad k=\overline{3,4}$$

$$E_k=\frac{a_k(\sigma_3^2-\sigma_k^2)(\sigma_4^2-\sigma_k^2)}{(\nu+\beta)(\mu+\alpha)},\quad k=\overline{1,2}$$

$$F_k=\frac{a_k(\sigma_3^2-\sigma_k^2)(\sigma_4^2-\sigma_k^2)(\lambda_7^2-\sigma_k^2)}{(\mu+\alpha)(\nu+\beta)(\lambda+2\mu)},\quad k=\overline{1,2}$$

It is easy to check - up, that

$$\sum_{k=1}^{4}B_k=\sum_{k=1}^{4}C_k=\sum_{k=3}^{4}N_k=\sum_{k=1}^{2}E_k=0,$$

where

$$\lambda_3^2=\frac{\rho\omega^2}{\mu+\alpha},\quad \lambda_4^2=\frac{I\omega^2-4\alpha}{\nu+\beta},\quad \lambda_6^2=\frac{\omega^2}{\text{æ}}(\tau_0+\tau_1\varepsilon_1)+\frac{i\omega}{\text{æ}}(1+\varepsilon_1),\quad \lambda_7^2=\rho_0\omega^2,$$

$$\varepsilon_1=\frac{\gamma\eta\text{æ}}{(\lambda+\mu-\alpha)}.$$

In the same way we can construct $\Phi(x,0)=\Phi(x)$ the matrix of fundamental solutions for static equations ($\omega=0$). In this case from (6) we have $\sigma_1=\sigma_2=\sigma_3=0$, $\sigma_4^2=-\dfrac{4\alpha\mu}{(\mu+\alpha)(\nu+\beta)}$.

The following notation is introduced: $\delta_4=\left[\dfrac{4\alpha\mu}{(\mu+\alpha)(\nu+\beta)}\right]^{1/2}$.

Taking this into account and carrying out some simple calculations, we find:

$$\varphi(x)=-\frac{\pi i}{2\delta_4^6}H_0^{(1)}(i\delta_4|x|)-\frac{1}{\delta_4^6}\ln|x|-\frac{1}{2^2\delta_4^4}|x|^2\ln|x|-$$

$$-\frac{1}{2^6\delta_4^2}|x|^4\ln|x|\equiv\frac{|x|^6\ln|x|}{2^8 3^2\pi(\lambda+2\mu)(\mu+\alpha)(\nu+\beta)}+O(|x|^8\ln|x|).$$

The elements of the matrix $\Phi(x)$ are equation:

$$\Phi_{kj}(x)=\frac{i\delta_{kj}\pi}{2\mu}(\mu+\alpha)^2(\nu+\beta)\left\{\frac{1}{\mu+\alpha}H_0^{(1)}(i\delta_4|x|)-\frac{2i}{\pi}\ln|x|\right\}+$$

$$+\frac{(-1)^{k+j+1}\pi i}{8\mu^2}(\lambda+\mu-\alpha)(\nu+\beta)^2(\mu+\alpha)\times$$

$$\times\frac{\partial^2}{\partial x_{k+1}\partial x_{j+1}}\left\{-H_0^{(1)}(i\delta_4(x))+\frac{2i}{\pi}\ln|x|+\frac{2i\mu}{\pi(\nu+\beta)}|x|^2\ln|x|\right\}-\frac{i(\mu+\alpha)^2(\nu+\beta)^2}{8\alpha\mu^2}\times$$

$$\times\frac{\partial}{\partial x_k\partial x_j}\left\{H_0^{(1)}(i\delta_4|x|-\frac{2i}{\pi}\ln|x|-\frac{i\alpha\mu}{\pi(\mu+\alpha)(\nu+\beta)}|x|^2\ln|x|\right\}+$$

$$+\delta_{kj}\frac{(13\mu+9\alpha+\lambda)(\nu+\beta)(\mu+\alpha)}{8\mu},\quad k,j=\overline{1,2},\ x_3=x_1$$

$$\Phi_{kj}(x)=-\frac{\pi i}{4\mu}(\lambda+2\mu)(\mu+\alpha)(\nu+\beta)\sum\varepsilon_{kjl}\,\partial\left\{H_0^{(1)}(i\delta_4|x|-\frac{2i}{\pi}\ln|x|\right\},\quad \begin{matrix}k=\overline{1,2},\ j=3\\ j=\overline{1,2},\ k=3\end{matrix}$$

$$\Phi_{33}(x)=-\frac{\pi i}{2}(\mu+\alpha)(\lambda+2\mu)H_0^{(1)}(i\delta_4|x|)$$

$$\Phi_{kj}(x)=\frac{\gamma(1-i\omega\tau_1)(\nu+\beta)(\mu+\alpha)}{2}(\ln|x|+1)x_k,\quad k=\overline{1,2},\ j=4$$

$$\Phi_{44}(x)=(\mu+\alpha)(\nu+\beta)(\lambda+2\mu)(\ln|x|+\frac{3}{2})+\frac{\alpha^2}{2^5 3^2}\frac{\partial^4}{\partial x_1^2\partial x_2^2}\left\{|x|^6\ln|x|+O(|x|^8\ln|x|)\right\}$$

$$\Phi_{41}=\Phi_{42}=\Phi_{34}=\Phi_{43}=0$$

From the rule of constructions of $\Phi(x,\omega)$ and $\Phi(x)$ we find

$$\varphi(x,\omega)-\varphi(x)=O(|x|^8\ln|x|)+\text{const} \qquad (10)$$

Near $|x|=0$ we have:

$$\Phi(x,\omega)-\Phi(x)=\hat{L}\left(\frac{\partial}{\partial x}\right)[\varphi(x,\omega)-\varphi(x)]+\left[\hat{L}\left(\frac{\partial}{\partial x},-i\omega\right)-\hat{L}\left(\frac{\partial}{\partial x}\right)\right]\varphi(x,\omega)=$$

$$=\hat{L}\left(\frac{\partial}{\partial x}\right)O(|x|^8\ln|x|)+\left[\hat{L}\left(\frac{\partial}{\partial x},-i\omega\right)-\hat{L}\left(\frac{\partial}{\partial x}\right)\right]|x|^6\ln|x|. \qquad (11)$$

Difference $\hat{L}\left(\frac{\partial}{\partial x},-i\omega\right)-\hat{L}\left(\frac{\partial}{\partial x}\right)$ contains not less than fourth row production according coordinats of Decart. That's why according to (11) we have:

$$\Phi(x,\omega)-\Phi(x)=O(|x|^2\ln|x|)$$

$$\frac{\partial^2}{\partial x_k\partial x_j}[\Phi(x,\omega)-\Phi(x)]=O(\ln|x|),\quad k,j=\overline{1,2}$$

Let $\widetilde{\Phi}(x,\omega)$ - the matrix of fundamental solutions of $\widetilde{L}\left(\frac{\partial}{\partial x},-i\omega\right)$ conjugate operation (the rule of Lagranje). It is easy to check - up equation:

$$\widetilde{\Phi}(x,\omega)=\Phi^T(-x,\omega)$$

With fundamental solutions it is possible to construct new singular solutions of (2) equation.

Let's discuss the differential operator:

$$HU=(TU-\gamma Nu_4);\quad RU=\left(HU,\frac{\partial u_4}{\partial n}\right)$$

where, $N=(n,0)^T$, $n=(n_1,n_2)$

TU - is the stress operator of the couple - stress elasticity [1, 2]

$$P^{(k)}U=(HU,-(\delta_{1k}+\delta_{2k})u_3+(\delta_{0k}+\delta_{3k})(\nu+\beta)\frac{\partial u_3}{\partial_n},\ -(\delta_{1k}+\delta_{3k})u_4+(\delta_{0k}+\delta_{2k})\frac{\partial u_4}{\partial n})^T,$$

$$Q^{(k)}U=(u,(\delta_{1k}+\delta_{2k})(\nu+\beta)\frac{\partial u_3}{\partial_n}+(\delta_{0k}+\delta_{3k})u_3,\ \ (\delta_{1k}+\delta_{3k})\frac{\partial u_4}{\partial n}+(\delta_{0k}+\delta_{2k})u_4)^T$$

$$u=(u_1,u_2),\quad k=\overline{0,3},\quad P^{(0)}=R,\quad Q^{(0)}=\left\|\delta_{kj}\right\|_{4\times 4}$$

Let's construct matrices:

$$\left[\widetilde{R}\left(\frac{\partial}{\partial y},n\right)\widetilde{\Phi}(y-x,\omega)\right]^T=\left\|(\widetilde{R}\widetilde{\Phi})_{pj}\right\|^T_{4\times 4}$$

$$\left[\widetilde{P}^{(k)}\left(\frac{\partial}{\partial y},n\right)\widetilde{\Phi}(y-x,\omega)\right]^T=\left\|(\widetilde{P}^{(k)}\widetilde{\Phi})_{pj}\right\|^T_{4\times 4}$$

$$\left[\widetilde{Q}^{(k)}\left(\frac{\partial}{\partial y},n\right)\widetilde{\Phi}(y-x,\omega)\right]^T=\left\|(\widetilde{Q}^{(k)}\widetilde{\Phi})_{pj}\right\|^T_{4\times 4}$$

where, $\widetilde{R}$, $\widetilde{P}^{(k)}$, $\widetilde{Q}^{(k)}$ are conjugate operator the constructed matrices represent the solutions of the system (2). Let's call them basic singular solutions.

The constructed matrices have essential meaning in the theory of boundary value problems.

A.Razmadze Mathematical Institute
Georgian Academy of Sciences

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T.Aliashvili

Signature Method for Counting Point in Semi-Algebraic Subsets

Presented by Corr. Member of the Academy N.Berikashvili, May 7, 1996

ABSTRACT. **The theorems on counting root numbers of real polynomial endomorphism in an arbitrary semi-algebraical set are given and several cases are considered. The estimate of the computational complexity of counting root numbers is obtained. The mentioned facts generalize early results obtained by the author. Particularly the numbers of variables and equations are unrestricted.**

Let $\varphi = (f, g)$: $\boldsymbol{R}^2 \to \boldsymbol{R}^2$ be a polynomial endomorphism of the type (m, n), i.e. $\deg f = m$ and $\deg g = n$. We call such endomorphism proper if its complexification $\varphi_C = \boldsymbol{C}^2 \to \boldsymbol{C}^2$ is proper in the usual sense, i.e. if the inverse image of the compact is compact.

From the $\boldsymbol{C}$- properness follows that there exist exactly $N = mn$ common complex roots $P_j = (\alpha_j, \beta_j), j = 0, \dots, N-1$. We assume that $\beta_i \neq \beta_j$, $i \neq j$ and such map is called y - convenient.

We use the classical Hermite - Jakoby signature method [1]. Namely, to a given map we assign the following quadratic form on the auxiliary space $\boldsymbol{R}^N$:

$$Q_h^\varphi(\bar{\xi}) = \sum_{j=0}^{N-1} h(\alpha_j, \beta_j)(\xi_0 + \xi_1\beta_j + \xi_2\beta_j^2 + \dots + \xi_{N-1}\beta_j^{N-1})^2,$$

where $h \in \boldsymbol{R}_2$ and $\boldsymbol{R}_2 = \boldsymbol{R}[x, y]$ is the ring of real polynomials in two variables. If $h \equiv 1$ we write $Q_1^\varphi \equiv Q^\varphi$ and call it principal "counting" form [2]. Denote by $\# Z_C(f, g, h)$ the number of complex roots of three functions $f, g, h \in \boldsymbol{R}_2$. We will describe how to compute $Z_C(f, g, h)$.

In the article [2] Q_h^φ always was a non - degenerated form. In the present paper it may be a degenerated form.

Theorem 1. *Let $\varphi: \boldsymbol{R}^2 \to \boldsymbol{R}^2$ be any convenient C - proper polynomial endomorphism and $h \in \boldsymbol{R}_2$ then*

$$\# Z_C(f, g, h) = N - rk\,(Q_h^\varphi).$$

We remark that $\# Z_C(f, g, h) \equiv \# Z_R(f, g, h) (\mathrm{mod}\ 2)$, so that if $N - rk\,(Q_h^\varphi)$ is odd then $\# Z_R(f, g, h) \neq \varnothing$.

Moreover, we can also estimate the real root number of a polynomial endomorphism in an arbitrary semi - algebraic set. It may be done with the help of formulae of the inclusion and elimination.

Theorem 2. *Let φ be of the same type as above and $h_1, h_2 \in \boldsymbol{R}_2$ such that $Z_C(f, g, h_i) = \varnothing$. $i = 1, 2$. Then*

$$\#\,(Z_R(f,g)\cap\{h_1>0,\ h_2>0\})=\frac{s(Q^{\varphi})+s(Q^{\varphi}_{h_1})+s(Q^{\varphi}_{h_2})+s(Q^{\varphi}_{h_1h_2})}{4}$$

where $s(Q^{\varphi})$, $s(Q^{\varphi}_{h_1})$ $s(Q^{\varphi}_{h_2})$ *and* $s(Q^{\varphi}_{h_1h_2})$ *are the signatures of the forms* Q^{φ}, $Q^{\varphi}_{h_1}$, $Q^{\varphi}_{h_{21}}$ *and* $Q^{\varphi}_{h_1h_2}$ *respectively.*

($Z_R(f,g)$ is the same that $\varphi_R^{-1}(0)$).

Remark 1. Particularly, if $h = J(f,g)$ is the Jacobian of φ then it is easily seen that $Z_R(f,g,J)$ coincides with the set of multiple roots of φ_C. Thus our formulae enable to calculate multiple root number for an arbitrary polynomial endomorphism.

Now we shall consider the case when there are three conditions of the inequality type $h_1, h_2, h_3 \in R_2$. We are interested in $\#\,(Z_R(f,g)\cap\{h_1>0,\ h_2>0,\ h_3>0\})$.

In general case $\{h_1=0\}$ $\{h_2=0\}$ $\{h_3=0\}$ curves cut the plane at the eight pieces. Let $x_1, x_2, \ldots, x_8$ be the real root numbers in the corresponding domains, i.e. x_1 is the number of the roots which are in the domain $\{h_1, h_2, h_3\} > 0$ $(+\ +\ +)$, $x_2 - \{h_1>0,\ h_2>0$ and $h_3<0\}$ $(+\,+\,-)$, $x_3-(+\,-\,+)$, $x_4-(-\,+\,+)$, $x_5-(+\,-\,-)$, $x_6-(-\,+\,-)$, $x_7-(-\,-\,+)$, $x_8-(-\,-\,-)$. Let s, s_i, s_{ij} $(i,j=1,2,3;\ i\neq j)$ and s_{123} denote signatures of quadratic forms

Q^{φ}, $Q^{\varphi}_{h_i}$, $Q^{\varphi}_{h_ih_j}$ and $Q^{\varphi}_{h_1h_2h_3}$ respectively.

A simple calculation gives that

$$\#\,(Z_R(f,g)\cap\{h_1>0,\ h_2>0\ h_3>0\})=x_1=\frac{s+s_1+s_2+s_3+s_{12}+s_{13}+s_{23}+s_{123}}{8}.$$

It is possible to calculate the root number for every combination of h_1, h_2, h_3. For example,

$$x_5=\frac{s+s_1-s_2-s_3-s_{12}-s_{13}+s_{23}+s_{123}}{8}.$$

The system matrix has a special symmetry which enables us to use this method for arbitrary number of unequality condiditions. Thus we can solve the analogous problem for any semi - algebraic subsets of R^n.

Having in mind computations on personal computers it is interesting how much algebraic operations are needed for the estimation of the real root number of a polynomial endomorphism .

Using some results of the number theory we have obtained the following estimate of the computational complexity.

Proposition 1. *Let* φ *be a* C *- proper y - convenient polynomial endomorphysm of* R^2 *with the components having degrees not greater than d. Then the number of algebraic operations on the coefficients of components of the endomorphism* φ *which are needed for calculation of the signature* Q^{φ} *is less than the number*

$$2d^2(d+4)(2d)!-d(d+4)-\sum_{k=1}^{d+1}\ \sum_{i\geq\left[\frac{k+1}{2}\right]}^{d}(2i-k)!(i-1)+$$

$$+\sum_{p=1}^{2(d^2-1)} 6p\left(\left[\frac{e^{\pi\sqrt{\frac{2}{3}}\sqrt{p-\frac{1}{24}}}}{4\sqrt{3}\left(p-\frac{1}{24}\right)}\right]+1\right)+\sum_{k=1}^{d^2}\left(2k^2+8k+5\cdot\frac{(k-1)(k-2)}{2}\right)+2$$

It should be noted that this formula gives a sufficiently exact upper estimate.

It means that there exists such a choice of the coefficients that the above number may be attained.

It will be also interesting to compare this with the computational complexity of the algorithm announced by Z.Szafraniec in [3]. Unfortunately no details were published till now.

By means of the result described above and in [2] more general theorems are obtained. In this case we consider more polynomials.

Theorem 3. *If* $\varphi = (f, g)$: $\mathbf{R}^2 \to \mathbf{R}^2$ *C proper y - convenient polynomial endomorphysm and* $\mathbf{R}_2$ *then*

$$\#(Z_{\mathbf{R}}(f, g, h) = s(Q^{\varphi}) - s(Q^{\varphi}_{(h^2)}).$$

This is the criterion of existence of common zeroes of three polynomials.

For arbitrary number of polynomials we have an analogous result.

Theorem 4. *Let* φ *be the same and* $h_1, \ldots, h_l \in \mathbf{R}_2$ *then*

$$\#(Z_{\mathbf{R}}(f, g, h_1, \ldots, h_l) = s(Q^{\varphi}) - s(Q^{\varphi}_{(h^2)}),$$

where (h^2) *denotes* $\sum_{i=1}^{l} h_i^2$.

The more general case when we have $n + p$ functions of n variables is represented analogously to the preceeding case but the formula correspondingly acquires more contentional loading.

Obviously, everything here may be reduced to a calculation of Newton mixed sums [4], [5], which is a completely nonevident task.

Up-to-date discoveries concerning n - dimensional Grothendique residues [6] (cf. also [7]) enable us to perform this calculation in the general case. The precise method will be described in the next publication.

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Georgian Technical University

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MATHEMATICS

Corr. Member of the Academy N.Berikashvili

On the Criterion of the Secondary Cross Section

Presented June 13, 1996

ABSTRACT. **New formula for the secondary obstruction to Serre fibration is given.**

1. **Introduction.** Let $F \to E \to B$ be a Serre fibration with the base polyhedron and with the first obstructon zero. Hence if $\pi_i(F) = 0$, $i < q$, $i \neq p$ and $\pi_p = \pi_p(F)$, $\pi_q = \pi_q(F)$, then there exists a cross section s over q-skeleton and the so called secondary obstruction class $h^{q+1}(s^q) \in H^{q+1}(B, \pi_q)$ is defined which in contrast with the first obstruction class is not defined uniquely. The problem to decide whether the fibration has a cross section over $(q+1)$-skeleton of B in terms of secondary obstruction class and invariants of fibration was considered beginning Hopf [1] and in wide special cases was solved [1,2,3,4,5,6]. In [7] the author has constructed the "secondary obstruction functor" which provides alternative approach to the question and leads to an answer without exeption. The formulation of final answer does not use the notion of the mentioned functor and proceeds in familiar algebraic topology terminology. This formulation is given below.

2. **Statement of results.** Assume that we have a Serre fibration $F \to E \to B$ with fiber $\pi_i(F) = 0$, $i < q$, $i \neq p$. Let $\pi_p = \pi_p(F)$, $\pi_q = \pi_q(F)$. Let $K(G, n)$ denote the Eilenberg-MacLane complex. Consider the bicomplexes

$$C^*(B, C^*(K(\pi_p, p), \pi_q)) \tag{1}$$

and

$$C^*(B, C_*(K(\pi_p, p))). \tag{2}$$

They are regarded as cochain complexes of B with coefficient complexes

$$C^*(K(\pi_p, p), \pi_q) = Hom(C_*(K(\pi_p, p), \pi_q) \tag{3}$$

and

$$C_*(K(\pi_p, p)) \tag{4}$$

respectively. Of course (3) and (4) are paired by evaluation map to the group π_q:

$$C^*(K(\pi_p, p), \pi_q) \otimes C_*K(\pi_p, p)) \to \pi_q \tag{5}$$

and this pairing is differential.

The total degree in complex (2) is $p - q$ and we assume that in total complex n-cochains are given by $X^n = \prod_{p-q=n} C^p(B, C_q(K(\pi_p, p)))$. The filtration by first degree is complete in a sence of Eilenberg-Moore. The total complex is nontrivial in negative dimensions as well and our special attention is concerned especially to dimension 0. Of course

$$C^*(B, C_*(K(\pi, p))) = Hom(C_*(B), C_*(K(\pi, p))) \tag{6}$$

and hence a 0-dimensional cocycle of complex (2) is identified as a chain map

$$C_*(B) \to C_*(K(\pi, p)).$$

The complex (1) we can identify as the cell cochain complex of cartesian product of cell complexes B and $K(\pi_p, p)$:

$$C^*(B, C^*(K(\pi_p, p), \pi_q)) = C^*(B \times K(\pi_p, p), \pi_q)). \tag{7}$$

The pairing of coefficients (5) enables to define the pairing by $\cup$- product of total complexes (1) and (2) in $C^*(B, \pi_q)$:

$$C^*(B, C^*(K(\pi_p, p), \pi_q)) \otimes C^*(B, C_*(K(\pi_p, p))) \to C^*(B, \pi_q). \tag{8}$$

This pairing defines the pairing of homology

$$H^*(C^*(B, C^*(K(\pi_p, p), \pi_q))) \otimes H^*(C^*(B, C_*(K(\pi_p, p)))) \to H^*(B, \pi_q)$$

i.e.

$$H^*(B\times K(\pi_p, p), \pi_q)) \otimes H^*(C^*(B, C_*(K(\pi_p, p)))) \to H^*(B, \pi_q). \tag{9}$$

Especially any 0-dimensional cohomology class of complex (2) defines the homomorphism of groups $H^i(B\times K(\pi, p), \pi_q) \to H^i(B, \pi_q)$, $i = 0, 1, 2, 3, \ldots$

In the complex $C^*(B\times K(\pi_p, p), \pi_q)$ we have two subcomplexes

$$0 \to C^*(K(\pi_p, p), \pi_q) \to C^*(B\times K(\pi_p, p), \pi_q). \tag{10}$$

and

$$0 \to C^*(B, \pi_q) \to C^*(B\times K(\pi_p, p), \pi_q) \tag{11}$$

These subcomplexes are direct summands and hence

$$C^*(B\times K(\pi_p, p), \pi_q) = C^*(K(\pi_p, p), \pi_q) + \overline{C}^*(B\times K(\pi_p, p), \pi_q) + C^*(B, \pi_q)$$

for suitable $\overline{C}^*(B\times K(\pi_p, p), \pi_q)$. (If B has only one 0-cell $*$ then this is obvious: $* \times K(\pi_p, p) \subset B\times K(\pi_p, p)) \supset B\times *$).

Let $s^q : B^q \to E$ be a cross section over q-skeleton. Then it defines an obstruction cocycle $c^{q+1}(s^q) \in C^{q+1}(B, \pi_q)$. With this analogy we can construct a $(q+1)$-cocycle on $B\times K(\pi_q, p)$: consider induced by projection $B\times K(\pi_p, p) \to B$ from fibration E a fibration $E' \to B\times K(\pi_p, p)$; let on the $(p-1)$-skeleton $[B\times K(\pi, p)]^{p-1} = [B\times *]^{p-1} = B^{p-1}$ the cross section $s_{B\times L}$ coincide with s^q; on the p-cell $\sigma^0 \times \tau^p$ let $s_{B\times L}$ represent τ^p as element of homotopy group π_p, $\tau^p \in \pi_p$; on the p-cell $\tau^p \times \tau^0$ let $s_{B\times L}$ coincide with s^q; constructed cross section extends from p-skeleton of $B\times K(\pi_p, p)$ to q-skeleton of $B\times K(\pi_p, p)$ and we can assume that it coincides over $(B \times *)^q$ with s^q. The obstruction cocycle $c^{q+1}(s^q{}_{B\times L}) \in C^{q+1}(B\times K(\pi_p, p)\pi_q)$ has the form:

$$c^{q+1}(s_{B\times K}) = m^{0,q+1} + m^{1,q} + m^{2,q-1} + m^{3,q-2} + \ldots + m^{q-p+1,p} + \ldots + 0 + \ldots + 0 + m^{q+1,0},$$

where

$$m^{i,j} \in C^i(B, C^j(K(\pi_p, p)\pi_q))$$

and

$$m^{q+1,0} = c^{q+1}(s^q).$$

By above remark about cochain complex $C^*(B\times K(\pi_p, p)\pi_q)$ the above sum without $m^{q+1,0}$ is still cocycle. One sees that $m^{0,q+1}$ over every 0-simplex σ^0 is the k-invariant of fiber F. Without loss of generality we can assume that this cochain in $C^{q+1}(K(\pi_p, p), \pi_q)$ does not depend on σ^0 and hence $m^{0,q+1}$ is cocycle of $C^*(B\times K(\pi_p, p), \pi_q)$ too and lies in subcomplex (10) mentioned above. Thus we have

$$c^{q+1}(s_{B\times K}) = m^{0,q+1} + \overline{c}^{q+1}(s^q{}_{B\times K}) + c^{q+1}(s^q),$$

where $m^{0,q+1} \in Z^{q+1}(K(\pi_p, p), \pi_q)$ represents Postnikov invariant of fiber, $k^{q+1} = k^{q+1}(F) \in H(K(\pi_p, p), \pi_q)$, and

$$\overline{c}^{q+1}(s^q{}_{B\times K}) = m^{1,q} + m^{2,q-1} + m^{3,q-2} + \ldots + m^{q-p+1,p} \in Z^{q+1}(B\times K(\pi_p, p), \pi_q).$$

Hence on homology level we have

$$h^{q+1}(s_{B\times K}) = k^{q+1}(F) + \overline{h}^{q+1}(s^q{}_{B\times K}) + h^{q+1}(s^q).$$

This class of course is the Postnikov invariant of the fibration E.

Lemma 1. *If $q < 2p$ then the homology class $\overline{h}^{q+1}(s^q_{B\times K})$ is invariant of fibration.*

Now for the complex $C^*(B, C_*(K(\pi_p, p)))$, in the case $B = K(\pi_p, p)$ there is a characteristic class $1+\chi^0$, $1+\chi^0 \in H(C^*(B, C_*(K(\pi, p))))$, of dimension 0, the homology class of the cocycle id: $C_*(K(\pi, p)) \rightarrow C_*(K(\pi_p, p))$-. id has in $C^*(K(\pi_p, p), C_*(K(\pi_p, p)))$ the form

$$f = f^{0,-0} + f^{1,-1} + f^{2,-2} + f^{3,-3} + \ldots + f^{p,-p} + f^{p+1,-p-1} + \ldots + f^{k,-k} + \ldots$$

$$f = 1 + 0 + 0 + 0 + \ldots + 0 + f^{p,-p} + f^{p+1,-p-1} + \ldots + f^{k,-k} + \ldots$$

$$f^{i,-i} \in C^i(K(\pi_p, p), C_i(K(\pi_p, p))).$$

Homology class x^0 has the filtration p.

Let jet $x^p \in H^p(B, \pi_p)$. We consider it as a simplicial map x^p: $B \rightarrow K(\pi, p)$ and let $(x^p)^*(\chi^0)$ be image of class χ^0 in $H^0(C^*(B, C_*(K(\pi_p, p)))$.

Let now $\bar{s}_{B^q}$ be any other cross section over the q-skeleton of base and $h(\bar{s}_{B^q})^{q+1}$ its obstruction class. The principal result is

Theorem 1. *In above assumptions there is a cohomology class $x^p \in H^p(B, \pi_p)$ such that $h(s_{B^q})^{q+1} - h(\bar{s}_{B^q})^{q+1} = Op_{k^{q+1}}(x^p) + \overline{h}^{q+1}(s^q_{B\times K}) \circ (x^p)^*(\chi^0)$, where $\circ$ denotes the pairing (9) and Op_y denotes the cohomology operation defined by cohomology class $y \in H^*(K(\pi, p), \pi_q)$. For every $x^p \in H^p(B, \pi_p)$ there is a cross section $\bar{s}_{B^q}$ over the q-skeleton of base such that this equality holds.*

As immediate corollary one has the following criterion.

Theorem 2. *In above notations the cross section on the $(q+1)$-skeleton of base exists if and only if there is such a homology class $x^p \in H^p(B, \pi_p)$, that $Op_{k^{q+1}}(x^p) + \overline{h}^{q+1}(s^q_{B\times K}) \circ (x^p)^*(\chi^0) = h^{q+1}(s^q_B)$ where $\circ$ denotes the pairing (9) and Op_y denotes the cohomology operation defined by cohomology class $y \in H^*(K(\pi, p), \pi_q)$.*

A.Razmadze Mathematical Institute
Georgian Academy of Sciences

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MATHEMATICS

M.Usanetashvili

Conjugation Problem for a Second Order Hyperbolic System with Discontinuous Coefficients

Presented by Academician I.Kiguradze, July 8, 1996

ABSTRACT. **The conjugation problem is considered for a second order degenerated hyperbolic system with discontinuous coefficients. The uniqueness and existence theorems are proved.**

Let us consider the hyperbolic system of linear differential equations

$$L(u)=\begin{cases} y^m A_1 U^+_{xx} + 2y^{\frac{m}{2}} B_1 U^+_{xy} + C_1 U^+_{yy} + a_1 U^+_x + b_1 U^+_y + C_1 U^+ = F_1, & x>0 \\ y^m A_2 U^-_{xx} + 2y^{\frac{m}{2}} B_2 U^-_{xy} + C_2 U^-_{yy} + a_2 U^-_x + b_2 U^-_y + C_2 U^- = F_2, & x<0 \end{cases} \qquad (1)$$

where A_i, B_i, C_i, a_i, b_i, c_i $(i=1,2)$ are given $(n\times n)$ real matrices, F_i $(i=1,2)$ are given functions and $U^{\pm}$ are unknown n-dimensional vectors, $m>0$, $n>1$.

We assume that A_i, B_i, C_i $(i=1,2)$ are constant matrices, $\det C_i \neq 0$ $(i=1,2)$ and polynomials

$$P_1(\lambda)=\det(A_1+2B_1\lambda+C_1\lambda^2),$$
$$P_2(\mu)=\det(A_2+2B_2\mu+C_2\mu^2),$$

have only the real different roots $\lambda_1, \lambda_2,\ldots, \lambda_{2n}$ and $\mu_1, \mu_2,\ldots, \mu_{2n}$ satisfying the following conditions

$$\begin{gathered} \lambda_1<\lambda_2<\ldots<\lambda_n<0<\lambda_{n+1}<\ldots<\lambda_{2n}, \\ \mu_1<\mu_2<\ldots<\mu_n<0<\mu_{n+1}<\ldots<\mu_{2n} \end{gathered} \qquad (2)$$

In this case the system (1) is strictly hyperbolic and it has a parabolic degeneration when $y=0$. Under the given conditions the numbers $y^{\frac{m}{2}}\lambda_1,\ldots, y^{\frac{m}{2}}\lambda_{2n}$ and $y^{\frac{m}{2}}\mu_1,\ldots,y^{\frac{m}{2}}\mu_{2n}$ are roots of the characteristic polynomials of the system (1) $P_1(y,\lambda)=\det(y^m A_1+2y^{\frac{m}{2}}B_1\lambda+C_1\lambda^2)$ and $P_2(y,\mu)=\det(y^{\frac{m}{2}}A_2+2y^{\frac{m}{2}}B_2\mu+C_2\mu^2)$ respectively. In addition the characteristics of the system (1) passing through the point $P(x_0,y_0)$, $y_0>0$ satisfy the equations:

$$x+\frac{2\lambda_i}{m+2}y^{\frac{m+2}{2}}=x_0+\frac{2\lambda_i}{m+2}y^{\frac{m+2}{2}}y_0^{\frac{m+2}{2}}, \quad x_0>0, \quad i=1,2,\ldots,2n,$$

$$x+\frac{2\mu_j}{m+2}y^{\frac{m+2}{2}}=x_0+\frac{2\mu_j}{m+2}y_0^{\frac{m+2}{2}}, \quad x_0<0, \quad j=1,2,\ldots,2n.$$

Let D be *a* bounded domain in the upper half-plane $(y>0)$, and let it be surrounded by the following two characteristics of the system (1) passing through the origin $O(0,0)$

$$\gamma_1:\ x+\frac{2\lambda_n}{m+2}\, y^{\frac{m+2}{2}}\, y^{\frac{m+2}{2}}=0;\quad \gamma_2:\ x+\frac{2\mu_{n+1}}{m+2}\, y^{\frac{m+2}{2}}=0,$$

and by the two characteristics, passing through the point $O_1(0,y_0)$

$$\gamma_3:\ x+\frac{2\lambda_{2n}}{m+2}\, y^{\frac{m+2}{2}}=\frac{2\lambda_{2n}}{m+2}\, y_0^{\frac{m+2}{2}},\quad \gamma_4:\ x+\frac{2\mu_1}{m+2}\, y^{\frac{m+2}{2}}=\frac{2\mu_1}{m+2}\, y_0^{\frac{m+2}{2}},$$

here $y_0>0$ is an arbitrary fixed number. Let us denote by P_1 and P_2 the intersection points of γ_1 and γ_2, and of γ_3 and γ_4, respectively. Let $D^+\subset D$ be the domain surrounded by the curves γ_1 and γ_3 and by $x=0$, and $D^-\subset D$ – the domain surrounded by the γ_2 and γ_4 and $x=0$.

Further let us consider the following characteristic problem [1]: Find the regular solution of the system (1)

$$U(x,y)=\begin{cases} U^+(x,y), & (x,y)\in D^+,\\ U^-(x,y), & (x,y)\in D^-,\end{cases}$$

which satisfies the boundary conditions:

$$\left(y^{\frac{m}{2}} M_1\frac{\partial U^+}{\partial x}+N_1\frac{\partial U^+}{\partial y}+S_1U^+\right)\Big|_{op_1}=f_1, \tag{3}$$

$$\left(y^{\frac{m}{2}} M_2\frac{\partial U^-}{\partial x}+N_2\frac{\partial U^-}{\partial y}+S_2U^-\right)\Big|_{op_2}=f_2 \tag{4}$$

and the conjugation conditions on the segment OO_1:

$$U^+(0,y)-\Lambda_1U^-(0,y)=g_1(y),\quad 0\le y\le y_0, \tag{5}$$

$$U_x^+(0,y)-\Lambda_2U_x^-(0,y)=g_2(y),\quad 0\le y\le y_0 \tag{6}$$

here M_i, N_i, S_i, Λ_i $(i=1,2)$ are given real $(n\times n)$ matrices, Λ_i $(i=1,2)$ are constant matrices, f_i and g_i $(i=1,2)$ are given real n-dimensional vectors.

In the sequel we will provide that a_1, b_1, c_1, $F_1\in C^1(D^+)$, a_2, b_2, c_2, $F_2\in C^1(D^-)$, M_i, N_i, S_i, $f_i\in C^1(Op_i)$, $(i=1,2)$, $g_i\in C^1(00_1)$, $(i=1,2)$. Moreover, the inequalities

$$\sup_{D^+\backslash 0}\left\|y^{1-\frac{m}{2}}a_1\right\|<\infty,\quad \sup_{D^+\backslash 0}\left\|y^{1-\frac{m}{2}}a_{1x}\right\|<\infty,$$

$$\sup_{D^-\backslash 0}\left\|y^{1-\frac{m}{2}}a_2\right\|<\infty,\quad \sup_{D^-\backslash 0}\left\|y^{1-\frac{m}{2}}a_{2x}\right\|<\infty,$$

$$\sup_{D^+\backslash 0}\left\|y^{-\left(\alpha+\frac{m}{2}-1\right)}F_1\right\|<\infty,\quad \sup_{D^+\backslash 0}\left\|y^{-(\alpha-2)}F_{1x}\right\|<\infty$$

$$\sup_{D^-\backslash 0}\left\|y^{-\left(\alpha+\frac{m}{2}-1\right)}F_2\right\|<\infty,\quad \sup_{D^-\backslash 0}\left\|y^{-(\alpha-2)}F_{2x}\right\|<\infty, \tag{7}$$

$$f_i(0)=g_i(0)=0,\ (i=1,2),\ \alpha=\text{const}>0,$$

$$\sup_{Op_i\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} f_i \right\| < \infty, \quad \sup_{Op_i\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} f_i^{\prime} \right\| < \infty,$$

$$\sup_{OO_1\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} g_i \right\| < \infty, \quad \sup_{OO_1\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} g_i^{\prime} \right\| < \infty, \quad i=1,2$$

hold in D^+ and D^-.

The solution of the problem (1),(3)-(6) is sought in the class

$$\begin{cases} U^{\pm} \in C^2(D^{\pm}): U^{\pm}(0,0)=0, \quad \sup\limits_{\overline{D}^+\setminus 0}\left\| y^{-\alpha} U_x^+ \right\| < \infty, \quad \sup\limits_{\overline{D}^-\setminus 0}\left\| y^{-\alpha} U_x^- \right\| < \infty \\ \sup\limits_{\overline{D}^+\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} U_y^+ \right\| < \infty, \quad \sup\limits_{\overline{D}^-\setminus 0}\left\| y^{-\left(\alpha+\frac{m}{2}\right)} U_y^- \right\| < \infty \end{cases} \qquad (8)$$

Because of the fact that $\lambda_1, \lambda_2, \ldots, \lambda_{2n}$ and μ_1, μ_2, μ_{2n} are the simple roots of the polynomials $P_1(\lambda)$ and $P_2(\mu)$, the following equations hold

$$\dim\mathrm{Ker}(A_1+2B_1\lambda_i+C_1\lambda_i^2)=1, \quad \dim\mathrm{Ker}(A_2+2B_2\mu_j+C_2\mu_j^2)=1$$
$$1\le i,j\le 2n$$

Let us denote by v_i and v_j^* the vectors $v_i\in\mathrm{Ker}(A_1+2B_1\lambda_i+C_1\lambda_i^2)$, $\|v_i\|\neq 0$, $v_j^*\in\mathrm{Ker}(A_2+2B_2\mu_j+C_2\mu_j^2)$, $\|v_j^*\|\neq 0$, $i,j=1,\ldots,2n$ where $\|\cdot\|$ denotes the norm in R^n. Let us introduce the matrices

$$\Gamma_i(M_i,N_i), \quad i=1,2, \quad K=\begin{pmatrix} K_1 \\ K_2 \end{pmatrix}=(V_1, V_2), \; K^*=\begin{pmatrix} K_1^* \\ K_2^* \end{pmatrix}=(V_1^*, V_2^*),$$

$$K=\begin{pmatrix} v_1,\ldots,v_{2n} \\ \lambda_1 v_1,\ldots,\lambda_{2n} v_{2n} \end{pmatrix}, \quad \widetilde{K}=\begin{pmatrix} y^{-\frac{m}{2}} v_1,\ldots,y^{-\frac{m}{2}} v_{2n} \\ \lambda_1 v_1,\ldots,\lambda_{2n} v_{2n} \end{pmatrix},$$

$$K^*=\begin{pmatrix} v_1^*,\ldots,v_{2n}^* \\ \mu_1 v_1^*,\ldots,\mu_{2n} v_{2n}^* \end{pmatrix}, \quad \widetilde{K}^*=\begin{pmatrix} y^{-\frac{m}{2}} v_1,\ldots,y^{-\frac{m}{2}} v_{2n} \\ \lambda_1 v_1,\ldots,\lambda_{2n} v_{2n} \end{pmatrix},$$

$$\widetilde{K}_1=y^{-\frac{m}{2}} K_1, \quad \widetilde{K}_2=K_2, \quad \widetilde{K}_1^*=K_2^*, \quad G_1=\begin{pmatrix} O \\ E \end{pmatrix}, \quad G_2=\begin{pmatrix} E \\ O \end{pmatrix}.$$

Theorem. *Let the conditions*

$$\det(\Gamma_1\times V_1)(x,y)\neq 0, \qquad (x,y)\in OP_1,$$
$$\det(\Gamma_2\times V_2^*)(x,y)\neq 0, \qquad (x,y)\in OP_2,$$
$$\det\begin{pmatrix} \widetilde{K}_2 G_1 - \Lambda_1 \widetilde{K}_2^* G_2 \\ \widetilde{K}_1 G_1 - \Lambda_2 \widetilde{K}_1^* G_2 \end{pmatrix}(x,y)\neq 0, \; (x,y)\in(0,0_1).$$

be fulfilled.

Then there exists a positive number depending only on the coefficients of the system in case if $\alpha>\alpha_0$. The problems (1)- (6) have a unique solution in the class of vector-functions introduced above.

I.Javakhishvili Tbilisi State University

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MATHEMATICS

N.Skhirtladze

Some Properties of Belman's and Loo's Transformations

Presented by Academician L.Zhizhiashvili, May 23, 1996

ABSTRACT. **Some properties of Belman's and Loo's trasformations and invariant classes for this transformations are studied.**

I. Let f belong to class $C(I)$. It means that f is 2π-periodic and continuous function. Imply that $\delta\in]0, 2\pi[$ and

$$\Delta_n^k (f; x)=\sum_{i=0}^{k} (-1)^{k-i}\binom{k}{i} f(x+ih).$$

Expression [2], $\omega^{(k)}(\delta ; f)_C = \sup\limits_{|h|\leq\delta}\left\|\Delta_h^k(f;.)\right\|_C$

is modulus of fluency of order $-k$

Imply that ω is modulus of continuity [2].

H^{ω} - defines the class of functions

$H^{\omega}= \{f;\omega(\delta;f)_C\leq A(f)\omega(\delta)\}$ where $A(f)\in]0, \infty[$.

In future we need some conditions of N.K.Bari and S.B.Stechkin [3] on continuity:

Exactly $\displaystyle\int_0^{\delta} t^{-1}\omega(t)dt+\delta\int_{\delta}^{\pi} t^{-2}\omega(t)dt \leq A\omega(\delta)$.

II. If $f\in L(T)$ and is even, then its Fourier series of function is

$$\sigma[f]=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx. \tag{1}$$

And if f is odd function

$$\sigma[f]=\sum_{n=1}^{\infty} b_n \sin nx. \tag{2}$$

Imply that Fourier series of function f is given by (2) and for every natural number n the following value is considered

$$B_n=\sum_{k=n}^{\infty}\frac{b_k}{k} \tag{3}$$

This expression is called transformation of Bellman. Loo [5] considered function - ψ. Imply that $f\in L\, ln^+L(T)$ and f is odd function. If

$$\psi(x) \equiv \psi(x;f)=\frac{1}{tg\, x/2}\int_0^x f(t)dt, \tag{4}$$

where $\psi(0) = \psi(\pi) = 0$, then ψ is called transformation of Loo.

If we consider the conjugative function $\overline{\psi}$, of function ψ [6], it is unknown whether ψ is integrable on T, or not.

Following theorem is correct

Theorem 1. *Let* $f \in (ln^{+}L)^{2}(T)$, *then* $\overline{\psi}$ *is integrable on T.*

If $b_k \geq 0$ $(k=1,2,3,...)$, *then from* (3) $(B_k - \frac{b_k}{2k})_{k\geq 1}$ *sequence tends to* -0 *with monotony.*

N.K.Bari and S.B.Stechkin's conditions can be written as follows:

$$\frac{1}{n}\sum_{k=1}^{n}\omega\left(\frac{1}{k}\right) \leq A\,\omega\left(\frac{1}{n}\right). \tag{5}$$

Theorem 2. *Let* $\sigma[f]$ *be given by* (2) *and* $b_k \geq 0$, $(k = 1,2,3...)$.

Imply that ω *is modulus of continuity for which following condition* (5) *is correct. If*

$$\sum_{k=n}^{\infty}\left(B_k - \frac{b_k}{2k}\right) = 0\left(\omega\left(\frac{1}{n}\right)\right), \text{ then } f \in H^{\omega}$$

Theorem 3. *Let* $\sigma[f]$ *be given by* (2), *and* $b_k \geq 0$ $(k=1,2,3...)$

If $\sum_{k=n}^{\infty}(B_k - \frac{b_k}{2k}) = 0(n^{-1})$

then f is continuous of bounded variation.

Therem 4. *Let* ω *be a modulus of continuty, if* $f \in H^{\omega}$, *then* $\psi \in H^{\omega}$.

By this theorem H^{ω} class is invariant to ψ -transformation of Loo.

I.Javakhishili Tbilisi State University

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MATHEMATICS

G.Oniani

On Possible Meanings of Upper and Lower Derivatives

Presented by Academician L.Zhizhiashvili, November 27, 1995

ABSTRACT. **In the paper there is investigated the behaviour of upper and lower derivatives of the integrals with respect to the differentiation bases formed of intervals. The main result consists in that the analogy of the famous theorem of Besicovich generally is not true for translation invariant Busemann-Feller differentiation bases formed of intervals.**

1. A mapping B defined on $\mathbb{R}^n$ is called a differentiation basis in $\mathbb{R}^n$, if for every $x\in\mathbb{R}^n$ $B(x)$ is a collection of bounded open subsets of $\mathbb{R}^n$ containing x such that there is a sequence $\{R_k\}\subset B(x)$ with diam $R_k\to 0$ $(k\to\infty)$.

For $f\in L_{loc}(\mathbb{R}^n)$ numbers

$$\overline{D}_B\left(\int f,x\right)=\overline{\lim_{diamR\to 0,R\in B(x)}}\frac{1}{|R|}\int_R f \text{ and } \underline{D}_B\left(\int f,x\right)=\underline{\lim_{diamR\to 0,R\in B(x)}}\frac{1}{|R|}\int_R f$$

are called respectively upper and lower derivatives of the integral of f at a point x. If the upper and lower derivatives are equal, then their common meaning is called a derivative of the integral f, at a point x and it is denoted by $D_B\left(\int f,x\right)$. The basis B is said to differentiate the integral of f, if for almost every x

$$D_B\left(\int f,x\right)=f(x).$$

B is called a subbasis of B' (entry: $B\subset B'$) if $B(x)\subset B'(x)$ $(x\in\mathbb{R}^n)$. B is said to be a Buseman-Feller basis (BF - basis) if for every $R\in\overline{B}=\bigcup_{x\in\mathbb{R}^n}B(x)$ we have, that $R\in B(y)$for every $y\in R$. BF - basis is called homothety invariant (briefly: a HI - basis), if for $R\in\overline{B}$ $\overline{B}$ contains every set homothetical to R. Basis B is called translation invariant (briefly: a TI-basis), if $B(x)=\{x+R : R\in B(O)\}$ $(x\in R^n)$, where O is the origin of the coordinate system in $\mathbb{R}^n$.

Let B_2 denote the basis in $\mathbb{R}^2$, for which $B_2(x)$ $(x\in\mathbb{R}^2)$ consists of all twodimentional intervals containing x.

Let us agree $I^n=(0,1)^n$ and $f\in L(I^n)$ if $f\in L(\mathbb{R}^2)$ and $supp f\subset I^n$.

2. According to the famous theorem of Besicovitch (cf. [1] or [2], p.96) about upper and lower derivaties for every function $f\in L(\mathbb{R}^2)$ both of sets

$$\{x\in\mathbb{R}^2: f(x)<\overline{D}_{B_2}\left(\int f,x\right)<\infty\},$$

$$\{x\in\mathbb{R}^2: -\infty<\underline{D}_{B_2}\left(\int f,x\right)<f(x)\}$$

have a measure equal to zero.

Guzman and Menarges [2, p.100] established some generalization of this result. From their theorem follows that the analogy of Besicovich's statement is true for homothety invariant BF - bases $B \subset B_2$.

The question arises: may or not Besikovich's theorem be extended on translation invariant BF-bases $B \subset B_2$?

The negative answer on this question gives the following:

Theorem 1. *There is a translation invariant BF-basis $B \subset B_2$, for which there is a function $f \in L(I^2)$, $f \geq 0$ such that*

$$f(x) < \overline{D}_B\left(\int f, x\right) < \infty$$

almost everywere on I^2.

Here it must be noted: it is not difficult to prove that if $B \subset B_2$ is a translation invariant basis, then for every $f \in L(\mathbb{R}^2)$ inequalities are fulfilled almost everywere

$$\underline{D}_B\left(\int f, x\right) \leq f(x) \leq \overline{D}_B\left(\int f, x\right).$$

The question arises: if the analogous fact is true for all bases $B \subset B_2$?

The negative answer on this question gives:

Theorem 2. *There is a BF-basis $B \subset B_2$ for which there is a function $f \in L(I^2)$, $f \geq 0$ such that*

$$f(x) < \underline{D}_B\left(\int f, x\right) = \infty$$

almost everywhere on I^2.

Tbilisi State University

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MATHEMATICS

S.Topuria, N.Chikobava

Summability of the Fourier Series over Generalized Spherical Functions by the Abel and (C, α) Methods

Presented by Academician L.Zhizhiashvili, May 2, 1996.

ABSTRACT. **Some theorems on Abel and (C, α) summability of Fourier series over generalized spherical functions are proved.**

1. Notations and definitions. S^3 is a unit sphere on R^3 of the centre in origin of coordinates; (x, y) is scalar product of the vectors x and y; $D(x; h) = \{y: y \in S^3, (x, y) \geq \cos h, 0 < h \leq \pi\}$, $|D(x; h)|$ - area of the surface $D(x; h)$.

Definition 1. Let $f \in L(S^3)$. A point $x \in S^3$ will be called a L - point of the function f, if

$$\lim_{h \to 0} \frac{1}{h^2} \int_{D(x;h)} |f(y) - f(x)| ds(y) = 0.$$

Definition 2. A point $x \in S^3$ will be called a D - point of the function f, if

$$\lim_{h \to 0} \frac{1}{h^2} \int_{D(x;h)} [f(y) - f(x)] ds(y) = 0.$$

Definition 3. A point $x \in S^3$ will be called a $\widetilde{D}$ - point of the function f, if

$$\lim_{h \to 0} \frac{1}{h^2} \int_0^h \left| \int_{(x,y)=\cos\gamma} [f(y) - f(x)] dt(y) \right| d\gamma = 0.$$

Definition 4. A point $x \in S^3$ will be called a $L^*[D^*, \widetilde{D}^*]$ - point of the function f, if x and x^* are simultaneously $L[D, \widetilde{D}]$ - points of f; x^* being the opposite point to x.

It is known that if $f \in L(S^3)$ then almost every point of S^3 is its L^* - point.

The Fourier series over generalized spherical functions of the function $f \in L(S^3)$ is said to be the series

$$f(x) = f(\vartheta, \varphi) \sim \sum_{=|m|}^{\infty} I_\nu^{(m)}(f; \vartheta, \varphi), \tag{1}$$

Where

$$I_\nu^{(m)}(f; \vartheta, \varphi) = \frac{(-1)^m}{4\pi}(2\nu + 1) \int_{S^3} f(y) e^{-im(\varphi_1 + \varphi_2)} P^\nu_{m,m}(\cos\gamma) ds(y) =$$

$$= \frac{(-1)^m}{4\pi}(2\nu + 1) \int_0^\pi \int_0^{2\pi} f(\vartheta', \varphi') e^{-im(\varphi_1 + \varphi_2)} P^\nu_{m,m}(\cos\gamma) \sin\vartheta' d\vartheta' d\varphi',$$

$$\cos\gamma = \cos\vartheta \cos\vartheta' - \sin\vartheta \sin\vartheta' \cos\beta, \; \beta = \pi + \varphi' - \varphi, \; m = 0, \pm 1;$$

$$\operatorname{tg}\varphi_1 = \frac{\sin\beta \sin\vartheta'}{\cos\vartheta \sin\vartheta' \cos\beta + \cos\vartheta' \sin\vartheta}; \; \operatorname{tg}\varphi_2 = \frac{\sin\beta \sin\vartheta'}{\cos\vartheta' \sin\vartheta \cos\beta + \cos\vartheta \sin\vartheta'};$$

$$P^{\nu}_{m,n}(\mu) = \frac{(-1)^{\nu-m} i^{m-n}}{2^{\nu}(\nu-m)!}\sqrt{\frac{(\nu+m)!(\nu-m)!}{(\nu+n)!(\nu-n)!}}(1-\mu)^{\frac{n-m}{2}}(1+\mu)^{-\frac{m+n}{2}} \times$$

$$\times\frac{d^{\nu-m}}{d\mu^{\nu-m}}[(1-\mu)^{\nu-n}(1+\mu)^{\nu+n}],\ \mu = \cos\vartheta.$$

The system of generalized spherical functions coinsides with the system of common spherical functions [2] when $m = 0$, and (1) - with the Fourier - Laplaces series on sphere.

From the relation [3]

$$p^{\nu}_{1,1}(\mu) = p^{\nu}_{-1,-1}(\mu) = \frac{1}{2\nu+1}\cdot\frac{d}{d\mu}[p_{\nu+1}(\mu) - p_{\nu-1}(\mu)] - \frac{1}{2\nu+1}\cdot\frac{1}{1+\mu}[p_{\nu+1}(\mu) - p_{\nu-1}(\mu)],$$

due to [4].

$$\frac{d}{d\mu}[p_{\nu+1}(\mu) - p_{\nu-1}(\mu)] = (2\nu+1)p_{\nu}(\mu),$$

We obtain

$$p^{\nu}_{1,1}(\mu) = p_{\nu}(\mu) - \frac{1}{1+\mu}\int_{-1}^{\mu} p_{\nu}(t)dt,$$

i.e.

$$p^{\nu}_{1,1}(\cos\gamma) = p_{\nu}(\cos\gamma) - \frac{1}{1+\cos\gamma}\int_{\gamma}^{\pi} p_{\nu}(\cos t)\sin t dt, \tag{2}$$

where $p_{\nu}(\mu)$ is Legandres polynomial.

It is obvious from (2) that $p^{0}_{11}(\cos\gamma) = 0$. Hence, the series (1), when $m = \pm 1$, may be written as

$$f(\vartheta,\varphi) \sim \sum_{\nu=0}^{\infty} I^{(m)}_{\nu}(f;\vartheta,\varphi), \tag{3}$$

where

$$I^{(m)}_{\nu}(f;\vartheta,\varphi) = -\frac{2\nu+1}{4\pi}\int_0^{\pi}\int_0^{2\pi} f(\vartheta',\varphi')e^{-im(\varphi_1+\varphi_2)}[p_{\nu}(\cos\gamma) -$$

$$-\frac{1}{1+\cos\gamma}\int_{\gamma}^{\pi} p_{\nu}(\cos t)\sin t dt]\sin\vartheta' d\vartheta' d\varphi'.$$

2. **Summability of the Fourier series over generalized spherical functions by the Abel method**. The Abel means of (3) are denoted by

$$U(f;x) = U(f;\vartheta,\varphi) = \sum_{\nu=0}^{\infty} I^{(m)}_{\nu}(f;x)r^{\nu} =$$

$$= -\frac{1}{4\pi}\sum_{\nu=0}^{\infty}(2\nu+1)r^{\nu}\int_{S^3} p^{\nu}_{m,m}(\cos\gamma)f(y)e^{-im(\varphi_1+\varphi_2)}dS(y) =$$

$$= -\frac{1}{4\pi}\int_{S^3}\left[\sum_{\nu=0}^{\infty}(2\nu+1)p_{m,m}^{\nu}(\cos\gamma)r^{\nu}\right]f(y)e^{-im(\varphi_1+\varphi_2)}dS(y) =$$

$$= -\frac{1}{4\pi}\int_{S^3}p^*(r,\gamma)f(y)e^{-im(\varphi_1+\varphi_2)}dS(y),$$

where

$$p^*(r,\gamma) = \sum_{\nu=0}^{\infty}(2\nu+1)\,p_{m,m}^{\nu}(\cos\gamma)r^{\nu} =$$

$$= \sum_{\nu=0}^{\infty}(2\nu+1)p_{\nu}(\cos\gamma)r^{\nu} - \frac{1}{1+\cos\gamma}\int_{\gamma}^{\pi}\left[\sum_{\nu=0}^{\infty}(2\nu+1)p_{\nu}(\cos t)r^{\nu}\right]\sin t dt =$$

$$= p(r,\gamma) - p_1(r,\gamma).$$

$p(r, \gamma)$ is Poissons kernel to Fourier-Laplace series, i.e.,

$$p(r,\gamma) = \sum_{\nu=0}^{\infty}(2\nu+1)p_{\nu}(\cos\gamma)r^{\nu} = \frac{1-r^2}{(1-2r\cos\gamma+r^2)^{3/2}},$$

and

$$P_1(r,\gamma) = \frac{1}{2\cos^2\frac{\gamma}{2}}\int_{\gamma}^{\pi}P(r,t)\sin t dt.$$

We say that point $x(r, \vartheta, \varphi) \to x_0(1, \vartheta_0, \varphi_0)$ along paths nontangential to the sphere and write this circumstance as $x \xrightarrow{\Lambda} x_0$ if the point x tends to x_0 remaining all the time inside some cone with vertex at x_0 and angle $2\alpha < \pi$ whose axis coinsides with Ox_0 (O is a centre of a sphere).

The series (3) is called summable by the Abels method at the point $x_0(1, \zeta_0, \varphi_0)$ to the number S (or A - summable to the number S), if

$$\lim_{r\to 1-}U(f;r,\vartheta_0,\varphi_0) = S.$$

The series (3) is called summable by the A^* at the point x_0 to the number S if

$$\lim_{x\xrightarrow{\Lambda}x_0}U\ (f;\,r,\,\vartheta,\,\varphi) = S.$$

Theorem 1. *If $f(x)$ is continuous at some point $x_0(1, \vartheta_0, \varphi_0)$, then*

$$U(f;\,r,\,\vartheta,\,\varphi) \to f(\vartheta_0,\,\varphi_0)$$

howerer the point $x(r, \vartheta, \varphi)$ tends to x_0, remaining inside the sphere S^3.

Theorem 2. *If $f\in L(S^3)$, then series (3) is summable by the A^* method to $f(x)$ at all L - points of f.*

Theorem 3. *If $f\in L(S^3)$, then series (3) is summable by the Abel method to $f(x)$ at all D - points of f.*

3. **Summability of the Fourier series over generalized spherical functions by the**

(C, α), $\alpha>-1$, **method**. Denote by $\sigma_n^{\alpha}(f;x)$ the Cesaro means (C, α), $\alpha>-1$, of series (3), i.e.

$$\sigma_n^\alpha(f;x) = \frac{1}{A_n^\alpha}\sum_{j=0}^{n} A_{n-j}^{\alpha-1} S_j(f;x) =$$

$$= -\frac{1}{2\pi}\int_0^\pi\int_0^{2\pi} f(\vartheta',\varphi')e^{-im(\varphi_1+\varphi_2)}\Phi_n^\alpha(\gamma)\sin\vartheta' d\vartheta' d\varphi',$$

where

$$\Phi_n^\alpha(\gamma) = K_n^\alpha(\gamma) - N_n^\alpha(\gamma),$$

$$K_n^\alpha(\gamma) = \frac{1}{A_n^\alpha}\sum_{j=0}^{n}\left(j+\frac{1}{2}\right)A_{n-j}^\alpha P_j(\cos\gamma),$$

$$N_n^\alpha(\gamma) = \frac{1}{2\cos^2\frac{\gamma}{2}}\int_\gamma^\pi K_n^\alpha(t)\sin t dt,$$

and $S_j(f;x) = S_j(f;\vartheta,\varphi)$ is the partial sum of series (3). $K_n^\alpha(\gamma)$ is a kernel of (C,α) means to Fourier - Laplace series [1].

Theorem 4. *Let $f \in L(S^3)$. Then the equality*

$$\lim_{n\to\infty}\sigma_n^\alpha(f;x) = f(x)$$

is fulfilled: 1) at all $\tilde{D}^*$ - points of f, if $\frac{1}{2} < \alpha < 1$ and

2) at all $\tilde{D}$ - points of f, if $\alpha \geq 1$.

Georgian Technical University

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CYBERNETICS

Z.Piranashvili

On the Extension of Kotelnikoff-Shannon Formula

Presented by Academician V.Chavchanidze, June 10,1996.

ABSTRACT. **The extension of Kotelnikoff-Shannon formula which gives the possibility to increase interpolation step on the discreet points together with process value up to some order considering derivative value is given in the paper.**

The extension of results of the work [1] is given in the paper, namely it holds:

Theorem. If the conditions of theorem 1 in [1] is fulfilled, then for almost every sample distribution functions of random process $\xi(t)$ one has:

$$\xi(t)=\sum_{k=-\infty}^{\infty}\left\{\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!\,\alpha^{N-r}}\left[\sum_{m=0}^{\tau}\frac{\left[\alpha\left(t-\frac{k\pi}{\alpha}\right)\right]^{N-m}}{(\tau-m)!}A_{m\tau n}\times\right.\right.$$

$$\left.\left.\times\sum_{j=0}^{m}\frac{\left(t-\frac{k\pi}{\alpha}\right)^{j}\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!}\right]\varphi_{\tau N}(t;k,q,\alpha,\beta)\right\}\left[\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right]^{N+1} \qquad (1)$$

for every $\alpha > \sigma/(N+1)$ and positive $\beta < [(N+1)\,\alpha-\sigma]/q$, where N and q are some fixed non-negative integers, and

$$A_{m\tau N}=\lim_{x\to 0}\frac{d^{\tau-m}}{dx^{\tau-m}}\left(\frac{x}{\sin x}\right)^{N+1},$$

$$\varphi_{\tau N}(t;\,k,\,q,\,\alpha,\,\beta)=\lim_{\varsigma\to\frac{k\pi}{\alpha}}\frac{d^{N-\tau}}{d\varsigma^{N-\tau}}\left(\frac{\sin\beta(\varsigma-t)}{\beta(\varsigma-t)}\right)^{q}.$$

The proof is similar to the one of theorem 1 in [1] and it is based on the following estimate: if $f(z)$ is integral function of exponential type with exponent σ, bounded on the real line, then for it the following estimater holds

$$\left|f(z)-\sum_{k=-n}^{n}\left\{\sum_{\tau=0}^{N}\frac{1}{(N-\tau)!\,\alpha^{N-\tau}}\left[\sum_{m=0}^{\tau}\frac{\left[\alpha\left(z-\frac{k\pi}{\alpha}\right)\right]^{N-m}}{(\tau-m)!}A_{m\tau N}\times\right.\right.\right.$$

$$\times \left(\sum_{j=0}^{m} \frac{\left(z-\frac{k\pi}{\alpha}\right)^j f^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!} \right) \Bigg] \varphi_{\tau N}(z;k,q,\alpha,\beta) \Bigg\} \left[\frac{\sin\alpha\left(z-\frac{k\pi}{\alpha}\right)}{\alpha\left(z-\frac{k\pi}{\alpha}\right)} \right]^{N+1} \Bigg| \le$$

$$\le \frac{L_f \cdot C_{qN}(z)}{[(N+1)\alpha-\sigma-q\beta\,]\cdot n^{q+1}}. \qquad (2)$$

for every non-negative integers N and q and for any fixed z under sufficiently big n, where the function

$$C_{qN}(z) = \left(\frac{2\alpha}{\pi\beta}\right)^{q+1}\left(\frac{2}{1-e^{-\pi}}\right)^{N+1}\beta\cdot e^{q\beta|y|}\,|\sin^{N+1}\alpha z|$$

is finite in every bounded variation

$$z=x+iy\,,\; L_f = \sup_{-\infty<x<\infty} |f(x)|.$$

We obtain estimate (2) if for integral

$$\frac{1}{2\pi i}\int_{C_n}\frac{f(\zeta)}{\sin^{N+1}\alpha\zeta}\left(\frac{\sin\beta(\zeta-z)}{\beta(\zeta-z)}\right)^q\frac{d\zeta}{\zeta-z},$$

where C_n is circle $|\zeta| = (n+1/2)\,\pi/\alpha$, N and q are some non-negative integers, employ Cauchy theorem about residues and estimate the mentioned integral.

When $N=0$, then $\tau=0$, $m=0$ and from (1) we obtain formula [1]

$$\xi(t)=\sum_{k=-\infty}^{\infty}\xi\left(\frac{k\pi}{\alpha}\right)\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\left[\frac{\sin\beta\left(t-\frac{k\pi}{\alpha}\right)}{\beta\left(t-\frac{k\pi}{\alpha}\right)}\right]^q, \qquad (3)$$

which is valid for every $\alpha>\sigma$ and positive $\beta<(\alpha-\sigma)/q$.

When $q=0$ from (3) we obtain known Kotelnikoff -Shannon formula

$$\xi(t)=\sum_{k=-\infty}^{\infty}\xi\left(\frac{k\pi}{\alpha}\right)\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}, \qquad (4)$$

where α is arbitrary fixed number more then σ.

When $N=1$, then from (1) we obtain formula

$$\xi(t)=\sum_{k=-\infty}^{\infty}\left\{\left[\left[(1+q)\frac{\sin\beta\left(t-\frac{k\pi}{\alpha}\right)}{\beta\left(t-\frac{k\pi}{\alpha}\right)}-q\cos\beta\left(t-\frac{k\pi}{\alpha}\right)\right]\xi\left(\frac{k\pi}{\alpha}\right)+\right.\right.$$

$$+\left(t-\frac{k\pi}{\alpha}\right)\frac{\sin\beta\left(t-\frac{k\pi}{\alpha}\right)}{\beta\left(t-\frac{k\pi}{\alpha}\right)}\xi'\left(\frac{k\pi}{\alpha}\right)\Bigg]\left[\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right]^2\cdot\left[\frac{\sin\beta\left(t-\frac{k\pi}{\alpha}\right)}{\beta\left(t-\frac{k\pi}{\alpha}\right)}\right]^{q-1}, \qquad (5)$$

where $\alpha > \sigma/2$, $0 < \beta < (2\alpha - \sigma)/q$.

When $q = 0$, then from (5) we obtain

$$\xi(t)=\sum_{k=-\infty}^{\infty}\left[\xi\left(\frac{k\pi}{\alpha}\right)+\left(t-\frac{k\pi}{\alpha}\right)\xi'\left(\frac{k\pi}{\alpha}\right)\right]\left[\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right]^2, \qquad (6)$$

when $\alpha > \sigma/2$.

When $q = 0$, then from (1) we have

$$\xi(t)=\sum_{k=-\infty}^{\infty}\left\{\sum_{m=0}^{N}A_{mNN}\frac{\left[\alpha\left(t-\frac{k\pi}{\alpha}\right)\right]^{N-m}}{(N-m)!}\left(\sum_{j=0}^{m}\frac{\left(t-\frac{k\pi}{\alpha}\right)^j\xi^{(j)}\left(\frac{k\pi}{\alpha}\right)}{j!}\right)\right\}\times$$

$$\times\left[\frac{\sin\alpha\left(t-\frac{k\pi}{\alpha}\right)}{\alpha\left(t-\frac{k\pi}{\alpha}\right)}\right]^{N+1}$$

where $A_{mNN}=\lim\limits_{x\to 0}\frac{d^{N-m}}{dx^{N-m}}\left(\frac{x}{\sin x}\right)^{N+1}$. $\alpha > \sigma/(N+1)$ is any fixed number, N is some non-negative integer (when $N=0$, Kotelnikoff-Shannon formula (4) is obtained from (7)).

The formula (7) is given in [2] continuous functions with finite Fourier transform.

Observe that comparing formulae (1) and (7) with Kotelnikoff-Shannon formula, (1) and (7) gives possibility to increase the distance $h = \pi/2$ between points of interpolation $(N + 1)$-times and in particular cases for formulae (5) and (6) - twice. Moreover increasing q the speed of convergence of the above given expansions increased. Observe also that these expansions are valid for particular cases of spectral representation of covariance functions, namely for Karhunen-Loeve and Khinchin-Bochner representation. (Corollary 1,2,3 of theorem 1 [1]).

Institute of Cybernetics
Georgian Academy of Sciences

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PHYSICS

N.Kekelidze, R.Charmakadze, R.Alania, D.Kakushadze, A.Berdzenishvili, Z.Tkeshelashvili

Investigation of the Electroluminescent Properties of Gallium Nitride Structures Implanted with Zn and Mg Ions

Presented by Corr. Member of the Academy T.Sanadze, August 6, 1996

ABSTRACT. **Epitaxial GaN layers which always contain a great number of donor type defects were strongly compensated by doping in the growth process with further Zn and Mg ion implantation.**

Electroluminescent structures with new and important properties were obtained. Intensive light emission was produced in practically entire range of visible and near-ultraviolet light spectrum. The emission colour changes with variation in the degree of doping and the voltage applied to the structure. "Oscillation" of the current-voltage characteristic not observed in other structures was identified. Strong polarization of the short-wave band with a maximum at 3.29eV was found.

INTRODUCTION

Gallium nitride, a III-V semiconductor with a wide band gap (Eg=3.425 eV) [1] is very important and promising material. In particular, GaN light-emitting structures (LES) can be obtained for practically entire visible range including the near-UV region.

However today wide application of GaN is limited mainly due to the fact that GaN crystals and layers contain an extremely large number of native defects. Among these defects N vacances playing the role of donors are likely to be predominant [2]. The number of donor vacancies in GaN is so great that the electron concentration in undoped material may have the value $n=(1\div8)\cdot10^{19}cm^{-3}$. Owing to the indicated defective character of GaN, it is very difficult to obtain p-type material of adequate quality and thus a p-n junction which is so necessary for creation of highly efficient semiconductor devices. Therefore, at present, the development of production techniques of highquality GaN layers and crystals, their effective doping, etc. is of great importance. This is not feasible without fundamental investigations and the establishment of the character of defect formation in the material under discussion.

The present paper gives the results of technological and scientific investigations allowing construction of LES-s with unique characteristics on the basis of GaN.

EXPERIMENT

Radiative structures were grown on (1120) sapphire substrates using methods developed by Marushka [3] with some modifications in this regime.

Initially, GaN layers were grown on the substrate: they were undoped and thus had a very high carrier concentration of $8\cdot10^{19}cm^{-3}$. This layer is designated as n^+-GaN and is of 20μm thick (Fig. 1). Then during the growth in the chloridehydride flow Zn was introduced. This layer is designated as n-GaN. The layer thickness is 2÷3μm, $n=10^{18}cm^{-3}$. Then the Zn flow was interrupted and the substrate growth continued until

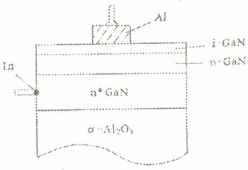

Fig.1. n^+-n-i structure of GaN on a sapphire substrate.

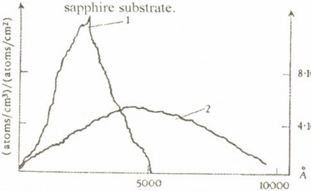

Fig.2. Depth distribution of Zn atoms in the epitaxial GaN layers (1 – before annealing, 2 – after annealing).

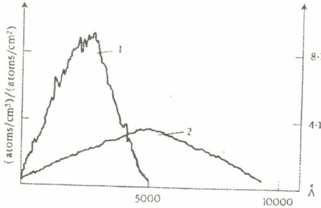

Fig.3. Depth distribution of Mg atoms in the epitaxial GaN layers (1 – before annealing, 2 – after annealing).

a layer of 0.8μm thick was obtained. The carrier concentration in n-GaN was approximately $3 \cdot 10^{18} cm^{-3}$.

An additional amount of Zn and Mg was introduced into the resultant structure by means of ion implantation. Profiles of zinc and magnesium distribution over the GaN layer thickness were recorded. The results are shown in Figs.2 and 3. All processes of the implantation of the gallium nitride layers were carried out in the same manner as in [4,5].

After Zn or Mg implantation, the GaN structures were subjected to multi-step annealing. Different techniques of annealing were used, such as photon, laser and temperature treatment. The typical disribution of Zn and Mg atoms over the thickness of the GaN epitaxial layers after annealing is shown in Figs.2 and 3 (curves 2).

Studies of zinc (magnesium) distribution conducted additionally by means of a scanning microscope showed the presence of a concentration gradient in the layers.

The quality of the layers was assessed by means of a diffractometer with GUR-5 goniometer and GP-4 attachment to DRON-1. The measurements showed a coarse-mosaic character of the epitaxial layers.

The mosaic character was studied by finding the difference between the reflections from a plane of (hh2he) and (hoho) type. The coarse mosaic character of the layers was identified by recording a doublet on the diffractometer at the frequency 1000 pulses/s with a counter rotational speed $\theta/2\theta \approx (1/16)$deg. at scanning (here θ is the Wolf-Bregg angle). The coarse-mosaic character of the layers was also revealed by X-ray using the Berg-Barret method in NiK_β line emission.

It should be noted that before ion implantation under the applied voltage of the polarities ("–" i-GaN) the structure emitted light in the wide spectral range, but for the given srtucture there is only one colour. The emission was unstable and of low intensity.

It should be emphasized that this structure can be essentially used as a source of near-UV emission with a maximum at 3.29 eV (300K). This happens with the change in the voltage polarity and when the concentration in the i-layer is of the order of $10^{17}cm^{-3}$. In such structures with near-UV emission, a high quantum efficiency of 0.3% is achieved.

RESULTS AND DISCUSSION

The end distribution of the charge carrier concentration (average) in the GaN layers before and after implantation and annealing is shown in Fig.4. The curves are obtained on the basis of measurements of electrical characteristics with the subsequent electrochemical etching of layers in combination with measurements of voltage-capacity characteristics. These measurements were made at certain structure depths after which the curves given in Fig.4 were constructed by means of approximation.Curve 4-1 corresponds to the initial impurity distribution, while curve 4-2 – to the distribution after implantation and annealing. The electron concentration

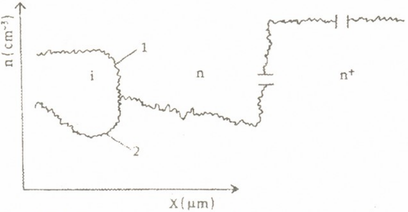

Fig.4. Free charge carrier concentration before implantation (1) and after implantation and annealing (2).

in n^+-GaN was $8\cdot10^{19}cm^{-3}$, the average electron concentration in the i-layer after annealing was $4\cdot10^{17}cm^{-3}$.

The peculiarity of the obtained structure is the presence of a compensated region (Fig.1 i-GaN) with a specific shape of the concentration gradient (curve 4-2).

When the voltage of 4V is applied (i.e. "+" on i-layer), the structure emits light from red to blue depending on the implantation degree and the i-layer thickness.

If the structure emits red light under the applied voltage, the increase of the voltage results in a sharp change first to yellow and then to green. If the structure emits yellow light, the increase of the voltage leads to green and then to blue light emission.

A typical emission spectrum for one of the samples is given in Fig.5. This blue emission band is characterized by quite a narrow spectrum half-width (of the order of 230 nm).

In such structures very high quantum efficiency (0.8-1.0%) is obtained for emission with a maximum at 2.89eV.

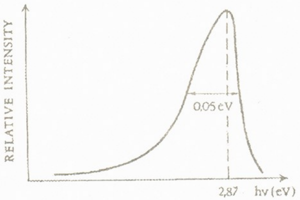

Fig.5. A typical emission spectrum for one of the samples.

A jumpwise character of electroluminescence is correlated somehow with the nature of I-V characteristics. Earlier, such voltage dependence of current was not observed in other structures. In particular, let us consider the structure with an "end" luminescence in the blue spectral region. Such structures, as a rule, begin to gleam at a voltage of the order of 4.5 V (point "a" in Fig.6). In this case the light is yellow (2.1 eV). With the further voltage increase, the

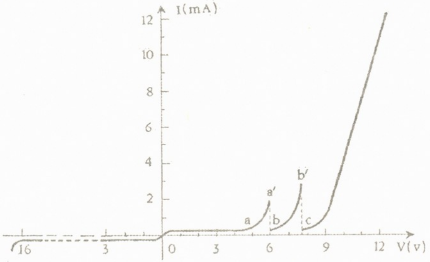

Fig.6. Current-voltage characteristic

yellow band intensity increases up to a certain level. When the voltage corresponds to point "a'", the luminescence actually goes out.

With a voltage increase from point "b", the structure begins to emit a green light (2.5eV), whose intensity rises up to the voltage corresponding to point "b'". At this point, luminescence actually goes out again.

A further voltage increase (from point "c") results in blue-light emission (2.87 eV); its intensity increases with a further increase in voltage. At large currents (of the order of 60 mA), a breakdown takes place in the structure.

To explain the processes occuring in electroluminescent structures various mechanisms, sometimes wholly or partially opposite to each other, are discussed in the literature [6-10]. Based on our experimental results and taking into account the date and considerations presented in [1-12], we think that when an electric field of sufficient strength is applied to the structure, electrons move from the n^+ layer to the n-layer and hence to the i-layer. On the other hand, under the influence of the electric field electrons flow from the i-layer into the metal. These electrons will originate mainly from Zn atoms in the proximity of the metal, whose energy levels lie in the forbidden region. In Fig.7 a schematic diagram (of the Zn levels) is shown: a) when no

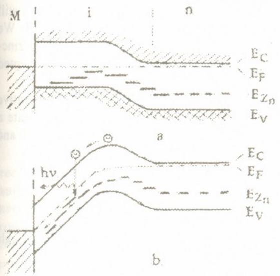

Fig.7. Schematic diagram

no voltage is applied, b) when a voltage is applied. On the Zn level the remaining holes move into the i-layer depth.

It can be shown that the migration of holes in the i-layer requires a high electric field, that is why i-layer must be thin.

Electrons transferred from the n-layer into the i-layer recombine there with the holes resulting in the emission of visible light [8]. As it has been mentioned, ion implantation resulted in a concentration gradient, i.e. Zn levels are disposed at the various distance from the top of the Ev band.

When the external field of sufficient strength is applied, electrons passing from the n-layer into the i-layer recombine on the most deep Zn level.

With increasing voltage the electron free path increases and hence the recombination already proceeds both energetically and geometrically on Zn levels situated in the different place. Exactly this circumstance causes the change in the emission spectrum towards the short-wave region with increasing voltage.

Production of descrete colours in a single structure was protected by the author certificate [10].

Taking the capacity-voltage characteristics of the emmiting crystal a diffusion length of carriers can be found. In our case it turned out to be approximately 1μm.

Initial GaN contains an abundance of native defects, mainly the vacances of nitrogen-V which are donors. We think that it is precisely these vacancies that play an important role both in undoped and doped gallium nitride as well as in degradation processes. In addition to nitrogen vacancies, there will be a very large number of bivacancies-2V in GaN.

When Zn or Mg atoms are introduced into GaN, they interact primarily with the vacancies forming the acceptor centre Zn+V. But the Zn+V centre will not be very stable.The activator complex Zn plus the bivacancy (Zn+2V) appears to be much more stable. Though Zn+V centres are formed in large numbers, they decay easier. Therefore, the concentration of Zn+2V complexes can be quite considerable, as well as their optical efficiency. Being introduced into a bivacancy, Zn forms a dumbbell-loke complex, i.e. an assymetric centre of dipole type.

Zn+2V+impurity (in particular Zn+2V+C, Zn+2V+0), Zn+V+impurity, Zn+3V should also be effective complexes. It is precisely these complexes that play a decisive role in the formation of intensive bands of GaN:Zn. It can be suggested here that the complex Zn+2V is resposible for the formation of the intensive band at 3.29eV characterised by a high degree of polarization. The complex Zn+2V+0 is probably associated with light emission at 2.55 eV.

Naturally, Ga vacancies, Zn and Ga interstitials, their substitutions and various associations with defects and impurities, which may appear in various processes, will be present in the material.

The polarization effect (60-70%) of the electroluminescent emission of GaN:Zn in the band with a maximum at 2.55 eV was first found in [13]. We have detected a still higher polarization (70-80%) in the short-wave band with a maximum at 3.29eV. We connect such high degrees of polarization with the presence of a great amount of zinc-bivacancy complexes of dipole type. Apparently, small regions of equally oriented dipole centres may also exist in the disordered GaN.

To conclude, it should be mentioned that the presence of great amount of native nitrogen vacancies as well as a high doping degree and implantation effects create a very complex situation manifesting itself in various properties of the material and requiring a further detailed investigation.

I.Javakhishvili Tbilisi State University

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PHYSICS

Z.Kachlishvili, K.Jandieri

Continuous Oscillations in Compensated Semiconductor under Impure Electric Breakdown

Presented by Corr. Member of the Academy N.Tzintzadze, June 1, 1996

ABSTRACT. **On the basis of mathematical model describing generation - recombination processes from impurity levels, dielectric relaxation of electric field and delay of electron temperature T_e relatively to the change of electric field necessary and sufficient condition of appearing persistant oscillations in compensated semiconductor under impure electric breakdown is established. This condition is reduced to rather simple form after a number of transformations. It contains the delay time of electron temperature τ_D as an important parameter characterizing the behavior of the system. The dependence of τ_D from T_e is studied. It is shown that while regarding the dynamics of the system τ_D with good accuracy can be considered constant.**

In the present paper we investigated the conditions of constant oscillations in extensively homogeneous, partially compensated semiconductor under impurity electric break down. The mathematical models used in [1,2,3] have been referred to. Relaxation equation average energy of charge carrier given in [1] was replaced by delay equation of electron temperature which allows to single out delay time τ_D as one of the most important parameters in determining system behaviour and to obtain simple analytic criterion of continuous oscillations beginning. Along with it examination of electron temperature delay unlike [2,3] gives the possibility to account the delay not only of the impact ionization coefficient A_I, but also the delay of μ mobility and thermal recombination coefficient B_T.

The dynamics of the system is characterized by the equations which describe:

1) generation and recombination process from impurity level;

$$\frac{dn}{dt} = (j\sigma + A_T)(N_D - N_A - n) + A_I(N_D - N_A - n)n - B_T(N_A + n)n \qquad (1)$$

2) dielectric relaxion of the electric field;

$$\frac{dE}{dt} = \frac{4\pi}{\varepsilon SR}(U - LE - e\mu SRnE) \qquad (2)$$

3) and delay of dimensionless electron temperature $Z = T_e / T$ relatively to the change of electric field (T is the temperature of lattice).

$$\frac{dZ}{dt} = \frac{Z - Z_0(E)}{\tau_D} \qquad (3)$$

Where N_D, N_A, n are donor concentrations, acceptors and free electrons correspondingly; $j\sigma$ and A_T are optic and thermal ionization rates from donor levels; E is electric field voltage; R is resistance of loading, switched in series with the sample; U is emf of continuous current source; L is the length of sample allong the field; S is the cross section of the sample; ε is dielectric penetration of the sample; z_0 is stationar meaning of z defined from the energy balance equation [4] for each fixed E, where along with phonon and ionized mechanism of energy relaxation unlike [1] the energy losses on $1S-2P$ excitation of impurity atoms have been taken into consideration.

In order to study the system of equations (1) - (3) let's introduce new variables: $n = N_D X$, $U = E_B\zeta$, $E = E_B y$, $a_I = A_I N_D$, $b_T = B_T N_D$, where E_B is the meaning of the field in breaking down point.

According to Rauss - Hurvits criterion the equlibrium point of the system (x^*, y^*, z^*) is saddle - focus type point, if $pq - r < 0$, where p, q and r are the coefficients of corresponding characteristic equation $\Delta^3 + p\Delta^2 + q\Delta + r = 0$, which are always positive in our case. Then both continuous and chaotic oscillations can appear in the system. Thus, the condition of continuous oscillations is as follows:

$$\eta^2 + (2Wx^*\mu + F + \frac{1}{\tau_D} + \frac{z'_{0y}}{1+F\tau_D} Wx^*y^*\mu'_z)\eta +$$

$$+\frac{F}{\tau_D} + W\mu x^*\{\frac{1}{\tau_D} + \frac{z'_{0y}}{\tau_D(1+F\tau_D)} y^*\frac{\mu'_z}{\mu} +$$

$$+F + W\mu x^*(1 + \frac{z'_{0y}}{1+F\tau_D} y^*\frac{\mu'_z}{\mu}) - \frac{z'_{0y}}{1+F\tau_D} y^*(b'_z - a'_z x^*)\} < 0 \qquad (4)$$

where $\eta = 4\pi L/\varepsilon SR$, $W = 4\pi e N_D/\varepsilon$, $F = \sqrt{b^2 + 4ad}$, $b = -\gamma - b_T c + a_I(1-c)$, $a = a_I + b_T$, $d = \gamma(1-c)$, $\gamma = j\sigma + A_T$, $c = N_A/N_D$ - the compensation rate.

η and the coefficient standing before it are always positive. Therefore the fulfilment of (4) demands the negativity of free member. This final condition in its turn provides the fulfilment of (4) with the help of corresponding parameters L, S, R, ζ selection.

Thus the sufficient and necessary condition of the continuous oscillations is reduced to the negativity condition of free member (4). Calculations show that the last is carried only in rather narrow changing interval of E in the neighbourhood of break down point (as in [5,6]).

If scattering of impulse takes place on one of the defects (dynamic or static) then the corresponding mobility can be expressed in such a way: $\mu = \mu_0 z^m$ (where $m = -^1/_2$, 0, $^3/_2$ under scattering of impulse by acoustic phonons, by neutral and ionized atoms of impurity correspondingly). Let's regard the case of underlight absence; because of low tempratures we neglect thermal generator (i.e. $\gamma = 0$. It is easily seen that in this case the appearance of continuous oscillations is possible only when $E > E_B$ [3]). If we regard the behaviour of free member (4) in the neghbourhood of break down, the necessary and sufficient condition of continuous oscillations appearing will be as follows:

$$\tau_D b_T^B c(1 - \frac{1}{\beta}(y^* - 1)) > \frac{1+\alpha m}{2\alpha} \qquad (5)$$

where $\beta = \frac{\alpha}{1+\alpha m} \frac{a_I^B}{W\mu_0 z^{*m} c}$ and $\alpha = \frac{z'_{0y}(y^*)}{Z^*} y^*$ and index "B" indicates that the corresponding meaning is taken in break down point. Of course, to satisfy (5) it is necessary to fulfil the condition $y^* - 1 < \beta$, which defines this interval of changing y^*, in which continuous oscillations can appear. For the following analysis let's assume that $y^* - 1 = {}^{\beta}/_2$. Then if we take into consideration that according to our denotations $b_T^B = B_T^B N_D$, the condition (5) is reduced to:

$$\tau_D B_T^B N_D c > \frac{1+\alpha m}{\alpha} \tag{6}$$

Thus , a rather simple criterion is observed, which allows to determine those meanings of parameters entered in it, for which the appearance of continuous oscillations is expected. As it is seen, among those parameters delay time τ_D plays an important role. The final part of the work is dedicated to the study of relation τ_D/τ_e (τ_e - is energy relaxation time) dependence on electron temperature. τ_e is defined from the equation: $KT(z_0 - 1)/\tau_e = g_{z0} = r_{z0}$, where $g = e\mu(z)E^2$ is acquisition power, and r is summed rate of energy loss [7]. The delay time τ_D is defined from the equation: $\tau_D = KT(z - z_0)/(r - g)$.

For different mechanisms of energy and impulse scattering we'll have the following various cases:

1. The energy is scattered on acoustic phonons: (for the corresponding rate of energy loss we use well-known Shokli's formula [8]), and impulse is scattered on phonones, on ionized and neutral atoms of impure. Correspondingly we'll have: (the notion $\Delta z = z - z_0$ is introduced)

$$\frac{\tau_D}{\tau_e} = \sqrt{1+\frac{\Delta z}{z_0}}\left(2-\frac{1}{z_0}+\frac{\Delta z}{z_0}\right)^{-1}; \quad \frac{\tau_D}{\tau_e} = Z_0\left(1+\frac{\Delta z}{z_0}\right)^{-1/2};$$

$$\frac{\tau_D}{\tau_e} = \left(1+\sqrt{1+\frac{\Delta z}{z_0}}\right)\left(2+\sqrt{1+\frac{\Delta z}{z_0}}+\frac{\Delta z}{z_0}-\frac{1}{z_0}\right)^{-1} \tag{7}$$

2. The energy is scattered on impurity atoms excitation [4]. For above given mechanisms of impulse scattering we'll have: (here and in the following case the results correspond to small deviations, when $\Delta z \le z_0$ are presented)

$$\frac{\tau_D}{\tau_e} \approx \frac{z_0+\alpha}{(z_0-1)}\left(\frac{\alpha^2}{z_0^2}+\frac{\alpha}{z_0}+1\right)^{-1}; \quad \frac{\tau_D}{\tau_e} \approx \frac{z_0+\alpha}{(z_0-1)}\left(\frac{\alpha^2}{z_0^2}-\frac{\alpha}{z_0}-1\right)^{-1};$$

$$\frac{\tau_D}{\tau_e} \approx \frac{z_0+\alpha}{(z_0-1)}\left(\frac{2\alpha^2}{z_0^2}+\frac{\alpha}{z_0}+1\right)^{-1} \tag{8}$$

where $\alpha \equiv Tex/T$ (Tex is impure excitation temperature).

3. The energy is scattered on the ionization of impurity atoms

$$\frac{\tau_D}{\tau_e} \approx \frac{1}{(z_0-1)(A-B)}; \quad \frac{\tau_D}{\tau_e} \approx \frac{(A-B-2/z_0)^{-1}}{(z_0-1)}; \quad \frac{\tau_D}{\tau_e} \approx \frac{(A-B-1/2z_0)^{-1}}{(z_0-1)};$$

where $A \equiv 2\{\beta \ln[0{,}63(1+4z_0/\beta)]\,[1+4z_0/\beta+\sqrt{1+4z_0/\beta}\,]\}^{-1}$

$$B \equiv \beta E_i(-\beta/z_0)\{Z_0^2[\exp(-\beta/z_0)+\beta/z_0 E_i(-\beta/z_0)]\}^{-1}. \tag{9}$$

Here $\beta \equiv Tion/T$, where $Tion$ is impure ionization temperature.

For general case, when all enumerated mechanisms of impulse and energy scattering are regarded joinly, the dependence of relation τ_D/τ_e on electron temperature was obtained with the help of machine calculations in case of both small and arbitrary deviations. The obtained data show that approximate formulae describe the real dependence well if $\Delta z \leq 0,1z_0$. Under larger deviations for relation τ_D/τ_e they give high meanings. Figures 1 and 2 illustrate the dependence $\tau_D/\tau_e = f(z_0)$, obtained with the help of machine calculations in *n - Ge* at temperature $T = 4,2$ K for two different meanings of compensation rate and for different consentrations of donor impurity.

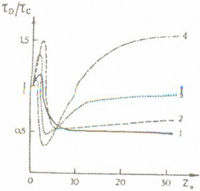

Fig. 1. τ_D/τ_e as a function z_0 in case of small deviations; $c = 0.9$, and N_D, takes the following meanings: 1.10^{13} cm^{-3}, 2.10^{14} cm^{-3}, 3.10^{15} cm^{-3}, 4.10^{16} cm^{-3}

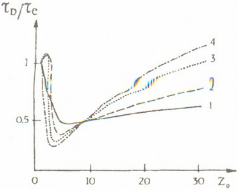

Fig. 2. τ_D/τ_e as a function z_0 in case of small deviations; $c = 0.1$, and N_D, takes the following meanings: 1.10^{13} cm^{-3}, 2.10^{14} cm^{-3}, 3.10^{15} cm^{-3}, 4.10^{16} cm^{-3}

At the end it should be noted: that oscillations appear in narrow interval of changing z^* in the neighbourhood of breakdown point z_B, $z_B >> 1$, if only the smallest meanings of compensation are not regarded. Besides the amplitude of oscillation is small relatively to z^*. Proceeding from this $\Delta z << z_0$. As it is seen from the given formulae when $\Delta z >> 1$ and $\Delta z << z_0$, the relation τ_D/τ_e is slightly depended on z and z_0 (the given Figures testify it also). Besides in the marked interval the change of τ_e is slight. All this gives the ground while regarding the dynamics of the system to consider τ_D as constant parameter (for the given C and N_D).

I.Javakhishvili Tbilisi State University

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PHYSICS

E.Khutsishvili, M.Paghava, A.Kuchashvili

Thermal Expansion Coefficient of Gallium Arsenide at High Temperatures

Presented by Corr. Member of the Academy G. Tsagareishvili, June 22, 1996

ABSTRACT. **Thermal exspansion coefficient of single and polycrystals gallium arsenide has been investigated at the temperature range from 300 to 900 K.**

Good agreement of gallium arsenide thermal expansion coefficient temperature dependence with that of heat capacity of semiconductors was found.

The effect of annealing at 823 and at 873 K of gallium arsenide is expected to be connected with the process of arsenic impoverishment of the material.

Gallium arsenide for its physical properties is widely used for many fields of semiconductor technical applications. One of the most necessary characteristics applied material in the fabrication of devices is the thermal expansion coefficient.

Few reports, however, have been published on the thermal expansion coefficient of gallium arsenide [1-7]. There are some discrepancies in the data of references. At the same time data given in references [1-7] belong to the low temperatures (< 300K). It will be necessary to extend the measurements of thermal expansion coefficient of gallium arsenide to higher temperatures.

In the present paper thermal expansion coefficient (α) of gallium arsenide single crystals (with [III] orientation) and polycrystals at the high temperatures in the range 300-900K have been investigated. We have carried out measurements in air using quartz dilatometer. The measurements were performerd on gallium arsenide samples before and after thermal annealing at 823K (for 2 hrs) and 873K (for 1 hr) in hydrogen atmosphere. The experimental temperature dependence of thermal expansion coefficient of gallium arsenide is plotted in Figure.

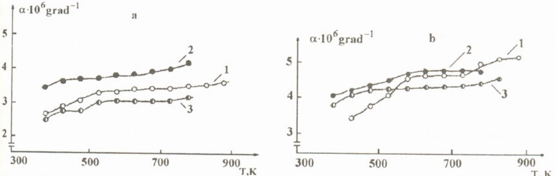

Fig.1. Temperature dependence of thermal expansion coefficient of gallium arsenide single crystals (a) and polycrystals (b); 1 - initial state; 2 - annealing at 823K two hrs; 3 - annealing at 873K one hr

The increase of thermal expansion coefficient of gallium arsenide with temperature is the same for both single crystals and polycrystals: in the range 300-600K α increases relatively faster than at higher temperatures, where this increase is much less and α approaches to the saturation. This dependence is similar to temperature change of heat capacity characteristic of semiconductors over the whole investigated temperature range.

As to the annealing our experimental results have shown that the influence of annealing on the thermal expansion coefficient of gallium arsenide polycrystals is not observed. However first annealing causes weak increase of thermal expansion coefficient of gallium arsenide single crystals. After the second annealing the results of thermal expansion coefficient approach to the values of unannealed samples.

If we assume that the obtained results are connected with gallium arsenide belonging to chemical compounds with break structure we can presume that the break annealing causes arsenic losses from the material.

F.Tavadze Institute of Metallurgy
Georgian Academy of Sciences

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PHYSICS

A.Rukhadze, M.Chogovadze, L.Chachkhiani

Radiative Instability in Ideal Liquid with the Tangential Discontinuity

Presented by Corr. Member of the Academy N. Tsintsadze, June 27, 1996

ABSTRACT. **The nature of the tangential discontinuity in the ideal liquid has been investigated. New type of instability - radiative instability stipulated by motion of a light liquid above the surface of heavy one has been discovered. This radiative instability should be considered as stimulated Cherenkov sound wave radiation from discontinuity surface.**

The theory of instability of the tangential discontinuity in the ideal liquid [1] is generalized by accounting the radiation from its surface. Contrary to the well-known instability of discontinuity related to the surface waves excitation by the inhomogeneity of stream velocity (Kelvin-Helmholts instability [2]) the considered radiative instability is stipulated by the supersonic motion of a light liquid above the surface of heavy one.

1. The equation of motion of the ideal liquid [1]

$$\operatorname{div} \mathrm{H} = 0, \quad \operatorname{rot}[\mathbf{v} \mathrm{x} \mathbf{H}] = \frac{\partial \mathbf{H}}{\partial t}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla P}{\rho} + \frac{1}{4\pi\rho}[\operatorname{rot}\mathbf{H}\times\mathbf{H}] \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}\rho\mathbf{v} = 0, \qquad \frac{\partial P}{\partial \rho} = c_o^2$$

admit the solution with inhomogeneous $\rho_o(x)$, $P_o(x)$, $\mathbf{v}_o(x)$ and $\mathbf{H}_o(x)$ when $\mathbf{v}_o \parallel \mathbf{H}_o \parallel OZ$ and the relation

$$P_o(x) + \frac{H_o^2(x)}{8\pi} = \text{const} \tag{2}$$

is satisfied. However it is well-known that such motion in general occurs to be unstable. Usually two types of instability are considered: the volumetric one stipulated by the smooth inhomogeneities of the equilibrium parameters of liquid and surface one which takes place when inhomogeneities are sharp and there exists a discontinuity surface in the liquid. In the first case the system of equations (1) for the small volumetric perturbations is analyzed with the given boundary conditions, whereas in the second case the surface type perturbations located near the tangential discontinuity are considered. It will be shown below that these two types of instabilities have one nature and at the same time the second one represents the limiting case of the first.

Besides the theory of instability tangential discontinuity is generalized by accounting the radiation from the discontinuity surface. Such instability takes place when the light liquid is moving with supersonic velocity above the surface of heavy liquid. It is obvious that it can be considered as stimulated Cherenkov radiation of sound waves from the surface of discontinuity.

2. The most complete analysis of the instabilities of tangential discontinuity related to the excitation of surface waves was done in the paper by V.G.Kirtskhalia [3]. In accordance to this paper the system of equations (1) can be reduced to one equation for quantity

$$y = \frac{v_{1x}}{\omega - k_z v_o}$$

Supposing that $y \sim \exp(-i\omega t + ik_z z)$ this equation will be

$$\frac{d^2y}{dx^2} + \{\frac{[(\omega - k_z v_o)^2 - k_z^2 v_A^2][(\omega - k_z v_o)^2 - k_z^2 c_o^2]}{(\omega - k_z v_o)^2 (v_A^2 + c_o^2) - k_z^2 v_A^2 c_o^2}\} y = 0, \qquad (3)$$

where v_o is the stream's velocity, c_o is the sound velocity, $v_A = H/(4\pi\rho)^{1/2}$ is Alfen speed.

This equation is valid in both sides of the discontinuity surface $x = 0$, on which the quantities c_o^2, v_A^2, and v_o are jumping, but the relation (2) takes place.

The boundary conditions for (3) can be obtained by integration of equations (1) near the surface $x = 0$, assuming that the quantities $\mathbf{H}$ and ρ are finite:

$$\{y\}_{x=0} = 0, \quad \{\frac{(\omega - k_z v_o)^2 (v_A^2 + c_o^2) - k_z^2 v_A^2 c_o^2}{(\omega - k_z v_o)^2 - k_z^2 c_o^2} y'\}_{x=0} = 0. \qquad (4)$$

The notation $\{A\}_{x=o}$ means the jump of A at x=0.

Supposing that $y(x) \to 0$ at $x \to \pm\infty$ from (3) and (4) in the paper [3] the following dispersion equation was obtained

$$\beta_1 \chi_2 + \beta_2 \chi_1 = 0 , \qquad (5)$$

$$\chi_i^2 = \frac{[(\omega - k_z v_{oi})^2 - k_z^2 c_{oi}^2][(\omega - k_z v_{oi})^2 - k_z^2 v_{Ai}^2]}{k_z^2 c_{oi}^2 v_{Ai}^2 - (\omega - k_z v_{oi})^2 (v_{Ai}^2 + c_{oi}^2)},$$

$$\beta_i = (\omega - k_z v_{oi})^2 - k_z^2 v_{Ai}^2 \qquad (6)$$

where (i = 1,2)

In [3] it was supposed that $\chi_i^2 > 0$ that corresponds to the exponentially damping solutions y(x) at $x = \pm\infty$. However, it is easy to show that the equation (5) stays valid even when χ_i^2 is complex with $\mathrm{Re}\chi_i^2 > 0$. The solutions y(x) then correspond to the volumetric waves propagating from the discontinuity surface at x = 0. Such solutions are considered below.

3. Let us remind some results of instability theory of tangential discontinuity for the surface waves, i.e. when $\chi_i^2 > 0$. We'll restrict by the simplest case when only the stream's velocity is gamping, or

$$v_o(x) = \begin{cases} +v_o, & x > 0, \\ -v_o, & x < 0. \end{cases} \qquad (7)$$

For noncompressing liquid, when $c_o^2 \to \infty$ (more correspondingly $c_o^2 >> v_A^2, v_o^2$) from (6) it follows that $\chi_i^2 = k_z^2 > 0$. The dispersion equation (5) then has the exact solution

$$\omega^2 = k_z^2 (v_A^2 - v_o^2). \qquad (8)$$

We see that this solution corresponds to the unstable oscillations ($\omega^2 < 0$) if $v_o^2 > v_A^2$. Thus, in the absence of external magnetic field, $\mathbf{H}_o \to 0$, the tangential discontinuity occurs to be unstable [2]. The external magnetic field with $v_A^2 > v_o^2$ stabilizes this instability, as it was first shown by S.I.Syrovatsky [4].

Strictly speaking all the above statements are correct only for the surface type oscillations. Now we also generalize the results for the volumetric oscillations.

4. For clarifying the nature of considered instability we analyze again the stability of smoothly inhomogeneous, noncompressing liquid system with velocity profile $v_o(x)$ in the absence of external magnetic field. Then from (1) in linear approximation one can easily obtain

$$\frac{d^2 v_{1x}}{dx^2} - \left[k_z^2 - \frac{k_z v_o''(x)}{\omega - k_z v_o(x)}\right] v_{1x} = 0. \qquad (9)$$

This equation must be completed by the following boundary conditions

$$v_{1x}\Big|_{x=\pm a} = 0. \qquad (10)$$

First of all it is obvious, that the instability is possible only if in the considered region $-a \le x \le a$ there exists a point where $v_o''(x) = 0$. But this condition is not satisfactory. For example, if the stream velocity $v_o(x)$ is symmetric, the instability is impossible. Really if $v_o(x) = u_o x^2/a_o^2$ (the homogeneous part of $v_o(x)$ can be easily excluded) then from (9) the longwave perturbations, $|k_z|a \ll 1$, it follows that

$$v_{1x}'' - \frac{2}{x^2 - \dfrac{\omega a_o^2}{k_z u_o}} v_{1x} = 0. \qquad (11)$$

Introducing $\xi = x(k_z u_o / \omega a_o^2)^{1/2}$ and $u = v_{1x}'$ the equation (11) can be reduced to the standard Legandre equation

$$(\xi^2 - 1)u'' + 2\xi u' - 2u = 0 \qquad (12)$$

the general solution of which looks like

$$u = C_1 P_1(x(k_z u_o/\omega a_o^2)^{1/2}) + C_2 Q_2(x(k_z u_o/\omega a_o^2)^{1/2}) \qquad (13)$$

Substituting this solution into the boundary condition (10) we obtain the dispersion equation describing time evolution of small initial perturbations. It is easy to show that this dispersion equation has only real solutions $\omega(k_z)$. This means that the perturbations are stable.

Quite another situation arises if $v_o(x)$ is an old function of x with $v_o''(x) \ne 0$ in the region $-a \le x \le a$. Thus if $v_o(x) = u_o x^2/a_o^2 + \beta x$ then from (9) one can obtain the standard Bessel equation

$$v_{1x}'' - \left\{k_z^2 - \frac{2}{[x + (-\omega a_o^2/k_z u_o)^{1/2}]^2}\right\} v_{1x} = 0, \qquad (14)$$

where $a_o^2\beta/2u_o \equiv (-\omega a_o^2 / k_z u_o)^{1/2}$. This equation was very carefully investigated in [5]. It was shown that it has unstable solutions with $\mathrm{Im}\,\omega = k_z u_o[(3)^{1/2}/6]a^2/a_o^2$. Namely this instability corresponds to the well-known Kelvin-Helmholts instability [2] and represents the genesis of the surface waves instability considered above.

5. Finally let us consider the possibility of existence of the radiative instability in the ideal liquid with tangential discontiniuty. For simplicity we restrict ourselves by the consideration of the situation when a light liquid stream is moving above the surface of heavy one with velocity profile

$$v_o(x) = \begin{cases} v_o, & x > 0, \\ 0, & x < 0. \end{cases} \qquad (15)$$

and the external magnetic field absents. Then in accordance with equilibrium $c_{01}^2 >> c_{02}^2$. Therefore the light liquid can be considered as noncompressible, $c_{01}^2 \to \infty$. As a result we obtain $\chi_1^2 = k_z^2 > 0$. This means that in the light liquid the perturbations occur to be of screening, or surface type. On the contrary in the heavy liquid they occur to be volumetric with $\mathrm{Re}\,\chi_2^2 < 0$ and slightly damping $0 < \mathrm{Im}\chi_2^2 << |\mathrm{Re}\chi_2^2|$. Then from the dispersion equation (5) one can easily find the following solutions

$$\omega = k_z v_o \pm i\,|k_z|\,c_{o2} \qquad (16)$$

The solution with $\mathrm{Im}\omega > 0$ corresponds to the unstable perturbations. It is easy to understand that this instability represents the stimulated Cherenkov radiation of sound waves exciting by the light liquid stream, moving above the surface of heavy liquid.

In conclusion it should be noted, that Cherenkov type instability as the above considered (8) can be stabilized by the action of external magnetic field if $v_A^2 > c_o^2$.

Georgian Technical University

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ANALYTICAL CHEMISTRY

O.Manjgaladze, M.Gverdtsiteli

Application of Theory of Graphs in Systemic-Structural Analysis of Educational Material

Presented by Academician G.Tsintsadze, June 17, 1996

ABSTRACT. **The method of systemic analysis is considered. The mathematical criterion of efficiency of this method, relative information, is devised in terms of graphs theory.**

Modern pedagogics pay particular attention to systemic approach of educational process. The methods of systemic analysis,i.e. logical structuration and logical dosage were elaborated by O.Zaitsev [1,2] and E.Mishina [3]. According to this approach the concepts are used as structural elements and logical connections between them as system-generators for educational material. Three types of logical connections are distinguished: genetic, subordinate and co-subordinate.

The genetic connections indicate the essence of event and stages of its development. For example: oxidizer (1) promotes process of oxidation (2). The connection (1)-(2) is genetic and is directed from cause (1) to effect (2).

The subordinate connections are directed from family concept to species concept. For example: chemical reactions (3) are devided into combination (4), decomposition (5), substitution (6) and metathesis (7) processes. (3)-(4), (3)-(5), (3)-(6) and (3)-(7) connections are subordinate.

Co-subordinate connections exist among the concepts of one family and they have no directions. In our example co-subordinate connections are: (4)-(5), (4)-(6), (4)-(7), (5)-(6), (5)-(7) and (6)-(7).

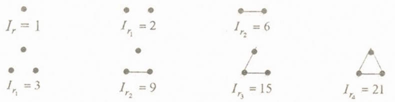
This method was modernized and applied to construct the course of theoretical principles of analytical chemistry by O.Mandjgaladze [4,5].

Mathematical criterion of efficiency of the logical-structural method, the concept of relative information (I_r), was elaborated in terms of theory of the graphs [6]:

$$I_r = N(\sum_i \deg V_i + 1)$$

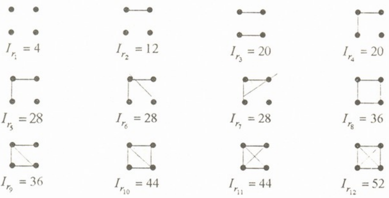
where: N is a number of vertexes in graph (the number of structural elements in the system); $degV_i$ - the degrees of vertexes of graph (numbers of logical connections among structural elements).

The values of I_r for one-, two-, three- and four-term systems are brought below:



$I_r = 1$ $\quad I_{r_1} = 2$ $\quad I_{r_2} = 6$

$I_{r_1} = 3$ $\quad I_{r_2} = 9$ $\quad I_{r_3} = 15$ $\quad I_{r_4} = 21$



As we see for two-term system in a case of maximum connection (full-graph) I_{r_2} is 3 times as large as in case of non-connection (zero-graph); for three-term system I_{r_4} / I_{r_1}=7; for four-term system $I_{r_{12}} / I_{r_1}$=13. So, I_r increases with increase of structural elements numbers in system and numbers of logical connections among them. This result is universal for every *N*.

By modernizing this approach I_r can be calculated for the systems with non-equivalent concepts (painted graphs) and non-equivalent connections (orientaited graphs).

I.Javakhishvili Tbilisi State University

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ORGANIC CHEMISTRY

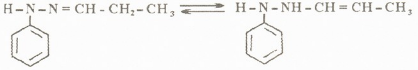
M.Gverdtsiteli, N.Samsonia

Algebraic Investigation of Fisher's Reaction

Presented by Corr. Member of the Academy L.M.Khananashvili, June 29, 1995

ABSTRACT. **The algebraic method of recording of organic molecules and reactions in terms of ANB - matrices is presented. Their diagonal elements are the atomic numbers of chemical elements, whereas nondiagonal ones - the multiplicities of chemical bonds. Algebraic investigation of Fisher's reaction is given in terms of this method.**

Fisher's reaction is an important method for synthesis of homologues of indole. The main stage of this complex process (according Robinson) is rearrangement of tautomeric form of phenylhydrazone, which proceeds analogous to benzidine rearrangement [1]:



Theoretical investigation of this rearangement was carried out in terms of algebraic chemistry [2].

Contiguity matrices and their various modifications are efficiently used in modern theoretical organic chemistry for characterization of molecules and their transformation [2,3]. One type of such matrices are ANB - matrices. Their diagonal elements represent atomic numbers of chemical elements, whereas nondiagonal ones - the multiplicities of chemical bonds [4]. For arbitrary ABC molecule ANB - matrix has a form:

$$\overset{1}{A}\overset{2}{B}\overset{3}{C} \qquad \left\| \begin{matrix} Z_A & \Delta_{AB} & \Delta_{AC} \\ \Delta_{AB} & Z_B & \Delta_{AC} \\ \Delta_{AC} & \Delta_{BC} & Z_C \end{matrix} \right\|$$

where: Z_A, Z_B, Z_C are atomic numbers of A, B and C chemical elements; Δ_{AB}, Δ_{AC}, Δ_{BC} represents multiplicities of chemical bonds between A and B, A and C, B and C.

The modelling reaction of phenylhydrazone rearrangement and its notation in ANB - matrices form are brought below:

$$H_2N{-}N = CH{-}CH3 \rightleftarrows H_2N{-}NH{-}CH = CH_2$$

$$\left\|\begin{matrix}7&1&0&0&0&0&0&0&1&1\\1&7&2&0&0&0&0&0&0&0\\0&2&6&1&0&0&0&1&0&0\\0&0&1&6&1&1&1&0&0&0\\0&0&0&1&1&0&0&0&0&0\\0&0&0&1&0&1&0&0&0&0\\0&0&0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&1&0&0\\1&0&0&0&0&0&0&0&1&0\\1&0&0&0&0&0&0&0&0&1\end{matrix}\right\| \longrightarrow \left\|\begin{matrix}7&1&0&0&0&0&0&0&1&1\\1&7&1&0&1&0&0&0&0&0\\0&1&6&2&0&0&0&1&0&0\\0&0&2&6&0&1&1&0&0&0\\0&1&0&0&1&0&0&0&0&0\\0&0&0&1&0&1&0&0&0&0\\0&0&0&1&0&0&1&0&0&0\\0&0&1&0&0&0&0&1&0&0\\1&0&0&0&0&0&0&0&1&0\\1&0&0&0&0&0&0&0&0&1\end{matrix}\right\|$$

Let's consider the expression;

$$\Delta_r = \Delta_f - \Delta_i$$

where: Δ_i - is the value of determinant of ANB - matrix for the initial form; Δ_f - for final form; Δ_r - change of the value of determinant in the result of rearangement.

As calculations show:

$$\Delta_r = 444 - 416 = 28$$

Thus, the algebraic criterion of this process is changing the determinant value of ANB - matrices.

I.Javakhishvili Tbilisi State University

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ORGANIC CHEMISTRY

M.Sturua, T.Vephkhvadze, R.Ziaev, D.Tsakadze, Sh.Samsonia, A.Abdusamatov

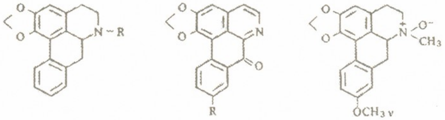
Alkaloids of Magnolia (*Magnolia obovata Thunb*)

Presented by Corr. Member of the Academy D.Ugrekhelidze, July 21, 1996

ABSTRACT. **From magnolia (*Magnolia obovata Thunb*) gathered in Batumi Georgia) well-known alkaloids remerin, anonain, liriodenin and lanuginozin were isolated and identified. By modern methods the structure of fifth alkaloid was ascertained - N-oxide of isolaurelin, which is isolated from plants for the first time.**

The plant from genus *Magnolia* is related to family of *Magnoliaceae* and is represented by about 70 species. It is known that genus Magnolia contains considerable amount of alkaloids. Bis-benzylisokhnolic [1], aporphic alkaloids and their oxo - derivatives [2,3] were isolated from some species.

Leaves of *Magnolia obovata Thunb* (gathered in Batumi) were investigated. By extraction with chloroform 0.48% sum of alkaloids were received. It was devided in two parts - phenolic and nonphenolic. From phenolic part lireodenin [1] was isolated. On chromatographic column with silicagel from mother liquor - anonain (II), remerin (III), liriodenin (I), lanuginozin (IV) and new base (V) were isolated.



II R = H
III R = CH_3

I R = H
IV R = OCH_3

Base (V) has composition $C_{19}H_{19}NO_4$. Melting point is 138 - 140^{0}C (acetone). This base is low - soluble in organic solvents and high - soluble in water. IR spectrum of (V) contains maximums of absorption at 223, 283 nm (lg ε 4.35; 4.18); in PMR - spectrum signals appear at: from protons of N - methylic group (2.90 m.p. 3H, singlet), methoxyl - (3.86 m.p., 3H singlet), methylendioxi - (5.95 and 6.10 m.p. single - proton dublets with J = 1.5 Hz), and signals of four aromatic protons. In mass-spectrum there are peaks with m/z 325 (M^+, 3.5%), 309 $(M - 16)^+$, 308 $(M - 17)^+$, 307 $(M - 18)^+$, 294 $(M - 31)^+$, 292, 267, 266 (100%), 265, 251, 235.

High solubility in water, characteristic shift of signals of N-methyl groups protons to faint field [4], faint intensity if peak of molecular ion and presence in mass - spectrum of (V)peaks of ions $(M - 16)^+$, $(M - 18)^+$ indicate that base (V) is N-oxide of isolaurelin [5]. Reduced by zinc in 10% of muriatic acid, compound identical to isolaurelin was isolated.

In this way five bases were isolated from leaves of *Magnolia obovata Thunb*. N-oxide of isolaurelin is found in plants for the first time.

In chromatography silicagel L (Czechoslovakia) 5/40 mkm was used with addition of 5% of gypsum (as a fixative) - TLC, 40/100 mkm (column chromatography). Benzen-methanol (4:1) was used as a solvent. IR spectrum were taken on spectrometer UR - 20 in KBr: PMR - on JNM - 4H - 100/100 mHz (δ - scale, CD_3OD, MDS); mass-spectrum - on instrument MX - 1310.

Isolation of alkaloids. 2 kg of dry leaves of *Magnolia obovata Thunb* were soltened by 5% solution of ammonia and alkaloids were extracted by chloroform. Extract was treated by 10% solution of sulphuric acid. The solution was alkalizated by 25% solution of ammonia and alkaloids were comprehensively extracted by chloroform. 9.6 g or 0,48% sum of alkaloids from dry mass was obtained.

The alkaloids sum was treated by 10% of sulphuric acid. The solution was alkalizated by 25% of ammonia and alkaloids were extracted by ether. Extract (0,5 l) was treated by 5% solution of caustic soda and washed by water. After destilling ether 5.85 g of nonphenolic part of alkaloids sum was isolated. Alkalizated solution was accidified by muriatic acid (1:1), again alkalizated by 25% solution of ammonia and extracted by ether. 2.1 g of phenolic bases were obtained.

Nonphenolic fraction was treated by chloroform and 0,9 g of liriodenin was isolated. Mother liqour was chromatographed on column with silicagel. By eluting with mixture benzene - methanol (99:1, 98:2, 95:5), anonain (0.056 g), remerin (0.15 g), liriodenin (0.22 g), lanuginozin (0.15 g) and N-oxide of osolaurelin (0.08 g) were isolated.

Well known alkaloids were identified by direct comparison with variable samples of alkaloids, isolated from *Liriodendron tulipifera* L from family *Magnoliaceae*.

N-oxide of isolaurelin. By treatment of benzene - methanol eluate (95:5) with aceton the base with composition $C_{19}H_{19}NO_4$ was isolated. Melting point 138 - 140^{0}C, Rf 0.18.

Reduction of N-oxide of isolaurelin. 0.04 g of base was isolated in 10% solution of sulphuric acid; zinc dust was added and kept for twenty four hours. The mixture was filtrated and alkalizated by 25% of amonia and extracted by ether. By elutriation of ether the base was isolated with Rf 0.64, equal to Rf of isolaurelin.

I.Javakhishvili Tbilisi State University Tashkent Agricultural Institute

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ORGANIC CHEMISTRY

M.Kezherashvili, N.Lekishvili, L.Asatiani, B.Butskhrikidze, L.Kipiani

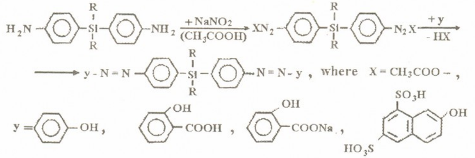
The Silicon-Containing New Bisazodyes

Presented by Corr. Member of the Academy L.Khananashvili, May 20, 1996

ABSTRACT. **The silicon-containing new bisazodyes were synthesized and studied. Some of the synthesized dyes were investigated for dyeing natural silk fibres. It was established that the colours of obtained dyes on natural silk is characterized by brightness and high fastness to different treatment.**

It is known from literature that introduction of dialkyl (aryl) silil groups into molecular composition of organic azodyes increases their thermostability and improves their hydrophobic properties, etc. [1]. We carried out synthesis of new bisazodyes containing silicium atoms, which have been based on the Bis - (Para - aminophenil) silane as initial diamine [2].

We obtained bisazodyes by the reaction of tetranitrogenation and by the subsequent azocoupling of the obtained tetraazonium salt with various azoic coupling components. The reaction may be expressed by the following scheme:



We conducted the reaction of tetranitrogenation in the area of acetic acid at about 0-5°C temperature, according to the method described in [3]. For example, we added previously prepared suspension (0.5 mole initial diamine in the 300 ml water) the 0.2 mole acetic acid as 25% (per) aqueous solution of 0.1 mole $NaNaO_2$. We continued to stir it for about 40-45 minutes, added it 0.1 mole sodium salt of salicile acid, 0.25 mole $NaHCO_3$ and stirred reaction mixture for about 60 minutes. At the end of reaction we filtered and washed the educt sediment by cold water. We purified the obtained product using extraction method in the Sohxlet's apparatus by mixtures of ethanol and dimethyl sulfoxide with their equal molar ratio (1:1). After removed of solvent from the filtrate we dried the sediment (yellow crystals) in vacuum($P_{Rem} \approx 0.25$-0.26 kp).

Composition of synthesized dyes have been established by the element analysis (table) and by IR spectroscopic analysis. In the IR spectra absorption bands were not detected in the regions of 3390-3340 cm^{-1} which is characterized for $-NH_2$ groups. In the IR spectra the absorption bands were observed in the regions of 1650-1660 cm^{-1}, which is characterized for $-N{=}N-$ groups.

It should be noted that yields of bisazodyes depend upon conditions of tetranitrogeneration stages. The research shows that with temperature increase the yields of dyes decrease due to partial destruction of intermediate tetraazonium salt and formation of corresponding bisphenol.

From obtained dyes some of them have been used for dyeing natural fibres. The dyeing was carried out in aqueous solutions by direct method. It was stated that obtained dyes at dyeing natural fibres is characterized by brightness and high fastness to different treatment.

Table

The yields, element analysis and molar mass of obtained dyes

Azoic coupling component	Formula of dyes	Yields %	Molar mass (Ebulli-oscopy)	Found % / Calculated %			
				C	H	N	Si
I	$C_{13}O_2SiH_{26}N_4$	60	514	70.5 70.37	5.00 5.06	10.00 10.89	5.05 5.45
II	$C_{33}O_6SiH_{26}N_4$	73.5	602	62.5 62.78	3.92 4.32	8.47 9.30	4.25 4.65
III	$C_{33}O_6SiN_4H_{24}Na_2$	85	646	59.90 60.30	3.35 3.72	8.35 8.67	4.02 4.33
IV	$C_{33}O_{14}SiN_4H_{30}S_4$	65	934	51.20 50.11	2.30 3.21	6.50 6.00	3.25 3.00

I.Javakhishvili Tbilisi State University

N.Muskhelishvili Kutaisi Technical University

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ORGANIC CHEMISTRY

N.Kobakhidze, Z.Machaidze, M.Gverdtsiteli

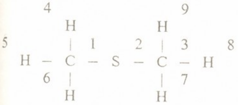
Algebraic Investigation of Thioethers

Presented by Corr. Member of the Academy L.Khananashvili, June 12, 1996

ABSTRACT. **Algebraic investigation of thioethers was carried out in terms of ANB - matrices method. Their diagonal elements represent atomic numbers of chemical elements, whereas nondiagonal ones - the multiplicities of chemical bonds.**

Contiguity matrices and their various modifications are effeciently used in modern theoretical organic chemistry [1,2]. One type of such matrices are ANB - matrices: their diagonal elements represent atomic numbers of chemical elements, whereas nondiagonal elements - the multiplicities of chemical bonds [3].

The graphic formula of dimethylsulphide (2-thiapropane) with numeration of atoms and corresponding ANB - matrices are brought below:



$$\begin{Vmatrix} 6 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 16 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 6 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{Vmatrix} \quad (1)$$

The first column of the matrix corresponds to atom numbered in graphic formula by cipher "1" (carbon-atom); second column corresponds to atom numbered by cipher "2" (sulphur-atom) etc. The first column begins with cipher "6" - the atomic number of carbon; cipher "1" means, that the bond between carbon and sulphur is simple; cipher "0" means, that carbon "1" is not bonded to carbon "3"; three ciphers "1" mean that carbon "1" and hydrogenes "4", "5" and "6" are single bonded; the column ends with three ciphers "0" - they mean that "1" carbon is not bonded with hydrogenes "7", "8" and "9". Other columns of matrix are constructed analogously.

The values of determinants of ANB - matrices do not depend on the numeration of atoms in molecules (they are invariants of molecular graphs).

For simple calculations hydrogen atoms are not often taken into account (so called "molecule skeleton" should be considered). The molecule skeleton of dimethylsulphide and corresponding simplified ANB - matrix (pseudo ANB - matrix) are brought below:

$$\overset{1}{C}-\overset{2}{S}-\overset{3}{C} \qquad \begin{Vmatrix} 3 & 1 & 0 \\ 1 & 16 & 1 \\ 0 & 1 & 3 \end{Vmatrix} \qquad (2)$$

In this case the range of ANB - matrix reduces by six units (total number of hydrogene atoms in dimethylsulphide). From diagonal elements of (1) matrix the numbers of hydrogene atoms which were bonded to corresponding atoms are substracted. It was confirmed that $\Delta(1) = \Delta(2)$.

The values of determinants of ANB - matrices for thioethers and corresponding standard entropies [4] are listed in the Table.

Table

The values of determinants (Δ) of ANB - matrices for thioethers and corresponding standard entropies (S°_{298}).

Thioether	S°_{298}	Δ
2-Thiapropane	68.32	138
2-Thiabutane	79.62	508
2-Thiapentane	88.84	1894
3-Thiapentane	87.96	1870
2-Thiahexane	98.43	7068
3-Thiahexane	98.97	6972
3,3-Dimethyl-2-Thiabutane	89.21	6264
4-Thiaheptane	107.22	25994
4-Thiaoctane	117.90	97004
5-Thianonane	125.84	361998
5-Thiadecane	136.52	1350988

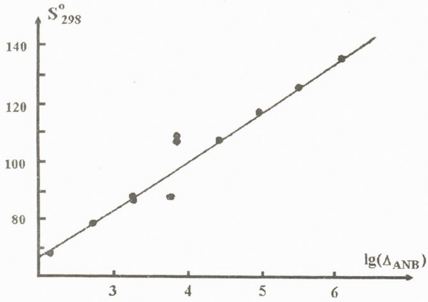


Fig.1. The plot $S^{\circ}_{298} = 17.14 \lg \Delta + 32.10$ for thioethers.

$S^{\circ}_{298} \sim \lg \Delta$ plot is constructed on the computer (Fig. 1). The equation describing this dependence has the form:

$$S^{\circ}_{298} = 17.14 \lg \Delta + 32.10 \quad (3)$$

The correlation coefficient r was calculated by formula:

$$r = \left(\sum_i y_i y_i'\right)^2 / \sum_i y_i^2 \sum_i y_i'^2 \quad (4)$$

where: y_i are values from correlation plot, y_i' - experimental values. r is equal to r = 0.999, so excellent correlation was observed.

Thus we can consider $\lg(\Delta_{ANB})$ as the topologic index [2] for "stucture - properties" correlation in homologous series of thioethers.

I.Javakhishvili Tbilisi State University

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PHYSICAL CHEMISTRY

K.Amirkhanashvili, L.Rapoport, Academician T.Andronikashvili

Studies of Sorbent-Sorbate Adsorption Interaction on Capillary Chromatography

Presented September 2, 1996

ABSTRACT. A review is made on the capillary columns used in gas-liquid chromatographic investigation. The preference utilization of quartz capillary columns is shown.

1. Selection of a Capillary Column.

Gas-liquid chromatographic studies of components based on multiple recurrence of single acts of separation with complete phase inversion is an efficient method of investigation of physico-chemical interactions. If it is assumed that a single act of evaporation-condensation takes place on the sorbent layer within the length of one theoretical plate (Th.P.), then the investigation of physico-chemical interactions must require the conditions leading to homogeneity of Th.Ps that can be essentially characterized as:

1) minimal difference in the thickness of adsorbent layer;
2) minimal influence of the carrier on the sorption mechanism;
3) minimal difference in packing density of the sorbent lengthwise chromatography column.

While selecting the system between packed and capillary columns the pointed out requirements are complied with capillary columns thanks to the fact that a capillary wall with its small surface and limited activity serves as a sorbent carrier and the flow pipe has a large specific midship section in respect to the area occupied by the absorbent stationary liquid phase (SLPh), that makes for a relatively low fall lengthwise the column. Mentioned peculiarities quite favourable for reliable representation of physico-chemical interactions on the resulting chromatogram should make the choice unique in favour of capillary columns, but for the limitations connected with the formation of continuous, uniform and stable layer of SLPh on the column wall. As surface-tension of the layer SLPh is comparable with adhesive forces on a small radius of the capillary curvature and increases considerably for polar liquids, not all the SLPh can be applied to the capillary. They held long on its surface without disturbing continuity and formation of defects affecting the quality of the capillary columns.

Thus development of suitable technological procedures is important to overcome the mentioned obstructions.

Since columns from metal, polymers, glass and quartz have been used in capillary analytical chromatography practice [1], we should consider their suitability for performing physico-chemical studies on absorbent-sorbate systems.

Metal columns as a rule are from stainless steel or tombac, more rarely of nickel, aluminium and copper. Disadvantage of these columns is their high activity, hence metrologically non-controllable results of physico-chemical measurements. At the same time practically any SLPh can be applied to them under favourable conditions with limited but real period of column lifetime. Such columns make it possible to perform qualitative and not quantitative investigation of physico-chemical interactions.

Columns from polymeric materials are of limited use under special conditions where columns from other materials are of little use. Application of these columns in individual cases guarantee possibility of complying special requirements to the separation process on them. Columns of this type are rare, made to special order and are not on sale as a rule.

Glass columns are from plants for stretching capillaries. They are used not only for producing chromatography columns but in laboratories of educational institutions and enterprises as well wide application with gas-liquid chromatography. Application of SLPh on glass column is partially limited in respect to polarity but is quite available, though it requires considerable efforts sometimes leading to desirable reproducibility of column characteristics. Industrially made serial columns have broad nomenclature in respect to sorbents. The main disadvantage of glass columns is their fragility. Due to this disadvantage they are not applied in practice. The use of glass capillary columns in physico-chemical investigation will apparently remain limited until this disadvantage is overcome.

Quartz capillary columns were first developed by Ettre [2], who prepared and tested them in Perkin-Elmer firm. The peculiarity of such capillaries is a tube with the thickness of walls not exceeding several mkm being drawn out of quartz soft pipe of high purity and twisted in coil after cooling the melt. Thus the capillary is in stressed state, having an elliptic section differing slightly from the circular one.

To prevent germs of quartz defects growing into cracks, the outer surface of quartz is covered with polyamide coating while preparing columns. In the process of polycondensation this coating blockades the germs of defects, hindering their growth in the stressed state of the capillary walls. Besides this coating insulates the capillary from exposure to external action and gives it high flexibility.

One of the major disadvantages of such a capillary is a considerable activity of its inner surface connected with isolation of water at polycondensation of polyamide on its external surface and its diffusion to the inner surface. The only way of removing the water from the quartz wall is a blow-down of the capillary with thermal stability of the coating. Desiccation of such a capillary at an accessible temperature gives positive results only after its prolonged treatment. SLPh is applied to the inner surface by dynamic or static method [3]. The layer of SLPh may remain applied, but due to its weak adhesion to the wall, as a rule, it gains defects considerably reducing the efficiency and corrupts the nature of sorbate-absorbent interaction. In those cases, when applied layer of SLPh can be stitched, it forms immobilised layer onto the inner surface of the capillary. Its chemical energy links are enough to retain the layer as a uniform flexible formation.

Of rather wide choice of SLPh used in analytical capillary chromatography mainly methyl, methylphenyl and silicone elastomers, as well as polymethyleneglycols are amenable to immobilization. As a rule, thermal stitching has been used with peroxides of benzoyl, diphenyl or other initiators. Limitation of usable SLPh as in the general case is residual activity of the inner surface of the capillary.

P.Melikishvili Institute of Physical and Organic Chemistry
Georgian Academy of Sciences

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GEOLOGY

Academician I.Gamkrelidze, D.Shengelia, G.Chichinadze

Macera Nappe in the Crystalline Core of the Greater Caucasus and its Geological Significance

Presented August 14, 1996

ABSTRACT. **It is indicated that in the Elbrus subzone of the Main Range zone of the Greater Caucasus are preserved separate fragments of once indivisible Macera nappe which was overthrusted from south (from the Pass subzone) during the sudetic phase. Due to this conclusion division of metamorphic complexes of the Greater Caucasus into series, their interrelations and questions of its age succession is considered in a new light.**

Since early 70-ies when North Caucasian geologists G.Baranov, I.Grekov and S.Kropachev had stated existence of Paleozoic nappes within the Forerange zone of the Greater Caucasus [1], tectonic structure of the pre-Jurassic basement of the Greater Caucasus attracted particular attention.

At the same period (1974) nappe structures were revealed in the Main Range zone of the Greater Caucasus. G.Baranov and I.Grekov and irrespectively I.Gamkrelidze and G.Dumbadze indicated that mica schists of Buulgen series are tectonically overlapped by migmatites of Macera series and the authors of this paper ascertain that the Damkhurts suite of the Laba series is tectonically overlapped by Macera series [2]. G.Baranov and S.Kropachev stated the presence of "pregranitic tectonic discontinuity" between the intra- and suprastructures of the crystalline core of the Greater Caucasus within the Northern subzone of the Main Range [1]. An idea of allochtonous nature of the Laba series and its separate parts was also expressed (G.Baranov, I.Grekov, D.Shengelia, Sh.Adamia, G.Chichinadze et al.).

Data obtained by authors during last years allow to assume, that all the stratified and nonmigmatised metamorphic suites of the Elbrus subzone represent separate fragments of once united megaslab-large nappe (Fig.1). This nappe is overthrusted on unstratified and intensively migmatized infrastructure of the Greater Caucasian crystalline core which by its primary disposition was disjoined from it. On the one hand this assumption confirms the idea that the pre-Jurassic basement of the Greater Caucasus is of imbricated structure and on the other hand allows to consider in a new light division of metamorphic complexes into series, their interrelations and age succesion.

For the overthrusted metamorphic formations we have presented the same name that was suggested by G.Baranov in 1980-"Macera nappe". Under this name he united following outcrops in the Main Range zone; between Bolshaia Laba and Mali Zelenchuk riverheads, and more to the east in Aksaut, Teberda, Baksan and Chegem riverheads.

We attribute these at present separate fragments of considerable size and also comparatively smaller ones to the Macera nappe (Fig.1). Metamorphites of all these fragments almost didn't differ in composition and degree of metamorphizm; they are

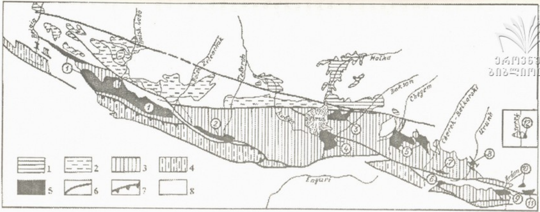

Fig.1 Scheme of disposition of Macera nappe fragments in the Main Range zone of the Greater Caucasus.
Exposures of crystalline complexes of the Greater Caucasus:
1 - in the Bechasin zone; 2 - in the Forerange zone; 3 - in the Elbrus subzone of the Main Range zone; 4 - in the Pass subzone; 5 - fragments of the Macera nappe (figures in circles): 1 - Tsakhvoa-Arkasara, 2 - Aksaut, 3 - Kirtik, 4 - Baksan (Donguzorun and Jusengi sheets), 5 - Bezengi (Koru and Siliksu sheets), 6 - Kharves, 7 - Akhsu, 8 - Matsuta, 9 - Buron, 10 - Unal, 11 - Phiagdon, 12 - Khde (Dariali); 6 - faults; 7 - exposure of Paleozoic subduction zone; 8 - Post-Paleozoic and Mesozoic nonmetamorphosed sedimentary cover.

correspondingly featured by similar mineral paragenesises and apparently in their primary disposition they occupied approximately one and the same stratigraphic level.

Geological position of separate fragments varies at present: tectonic overlap of crystalline rocks of infrastructure by metamorphities which compose them is here observed; though in some places they are izolated from each other; sometimes these fragments due to more later tectonic movements are in tectonic contact with the post-Paleozoic deposts which more often as well as Middle Carboniferous-Permian formations transgressively overlie metamorphites of separate fragments of the Macera nappe. In some places these metamorphites are tectonically layered.

In addition, in the contact zone of the Macera nappe fragments with infrastructure are stated: 1) considerable difference in the degree of their regional metamorphism; 2) development of intense cataclasis and mylonitization processes in contact roks; 3) very often disposition of post-metamorphic bimicaceous and microline granitic and granodioritic (mainly the Ulukam type) bodies in its contact area; 4) intrusions of syn- and premetamorphic granitoids are not observed in metamorphites that compose the Macers nappe; these bodies are widespread in gneiss-migmatitic infrastructures. When these bodies are in contact with the overthrusted metamorphites both granitoids and metamorphites are broken down and mylonitized and their contact is tectonic.

The Macera nappe is represented by Dombai and Arkasara (M.Somin), or Donguzorun, Dupukh, Kti-Teberda and Kurganshichat (S.Baranov) suites and also by their analogues (Bezengi, Kirtik and Buron suites). According to our data all these suites by rock and mineral paragenesises and also their metamorphic character are analogous to terrigane micaceous schists which are developed in the upper part of the Buulgen series. They or their analogues are following quite gradually the Klich suite of Buulgen series, which is underlaid by Gwandra suite. This part of metamorphites of Buulgen series is represented by nonmimatized Sisina (Ladevali, Vertskhlis tba, perhaps Uluchiran) suites. M.Somin attributes to them Dombai and Arkasara suites as well. He indicates that the Dombai suite without any break gradually continues the Klich one [3]. To our data the same picture is in interrelations of the Klich and overlying Sisina and Ladevali suites [4, 5]. According to G.Baranov the Buulgen series is terminated by Uluchiran suite which analogues he indicates in the lower part of the Macera nappe too. It's noteworthy that stratigraphic analogues of the Dombai and Arkasara suites are met both in the Elbrus and in the Pass subzone.

Thus, all the fragments of the Macera nappe corresponds to apparently once very thick upper part of the Buulgen section, and are overthrusted from south to north, from the Pass subzone into Elbrus subzone.

Taking into account that Late Hercinian granitic bodies are intruded between the infrastructure of the Greater Caucasus and Macera nappe and apparently Middle Paleozoic rocks composimg this nappe are overlapped by Middle Carboniferrous-Permian sediments, then overthrusting of the Macera nappe should be connected with Early Hercynian (Bretonic) phase, or most probably, like other Paleozoic nappes of the Greater Caucasus, with the sudetic phase, which was basically revealed here in the Lower-Middle Visean (G.Baranov et al. [9] assume the pre-Hercynian age of Macera nappe). At the same time tectonic overlap mainly on the mafic part of Buulgen series and Laba series with the infrastructure rocks of the Greater Caucasus, as it was mentioned earlier, shows that a part of these rocks at the same time are subject to underthrusting (subduction) under the island arc of the Greater Caucasus [2, 6]. (Some authors consider them an accretionary prism [7].) Besides, subduction was primary process and more prolonged in contrast to overthrusting (obduction) which had lightning speed in the geological sence [2]. We should consider that this overthrusting coinsides with the closure of marginal sea of the Southern slope of the Greater Caucasus promoted with the sudetic phase, when from the Hypsomatrically uplifted suture zone the upthrusted plate suffers gravitational sliding on the continental margin. For example, similar event took place in the process of sliding of fragments of the Patagonian back-arc basin crust in Middle Cretaceous on the continental margin [8].

Alongside with this as it was already noted, some investigators and the authors of this paper among them, consider that in recent structure Laba series is of allochtonous-imbricated composition. Here in Lashtrak, Ajara and damkhurts independent nappes are distinguished. Their primery stratigraphic succession is not clear. In first two nappes the samenamed suites are united, but in the Damkhurts nappe two suites- Damkhurts and Mamkhurts are distinguished. Analogues of the Lashtrak and Ajara suites (nappes) are known neither in stratified metamorphic complexes of the Main Range nor in Buulgen series and within the Macera nappe. In G.Baranov's opinion possible analogue of the Mamkhurts suite is Donghuzorun (Dupukh) suite but M.Somin doesn't share this opinion [3]. We think it possible that the unity Klich suite and of the certain part of micaceous schists is analogue of the Mamkhurts suite, or also Mamkhurts and Damkhurts suites, which correspond to the upper part of Gwandra suite and Klich suite. So, to our mind full synchroniety of Laba and Buulgen series and their unification into one series is inexpediently and we consider it natural, that lower parts of Buulgen series are older than Middle Paleozoic. Some authors completely attribute them to Proterozoic [9]. We can therefore conclude, that stratified members of metamorphic complex of Elbrus and also of the Pass subzone excluding Ajara and Lashtrak suites of Laba metamorphic complex before the overthrusting represented parts of indivisible thick stratigraphic section.

It follows from the all abovementioned that the old notion of "Macera series" looses its essence, as to our mind the traditional name "Macera series" unites two different geological bodies: as it seems mainly Precambrian gneiss-migmatitic infrastructural complex of Elbrus subzone of the Greater Caucasus and stratified mainly terrigene suites of Buulgen series, overthrusted from the Pass subzone which may be of Middle Paleozoic age.

In order to define tectonic position of metamorphic complexes of the Greater Caucasian crystalline core it is essential to make comparative petrological characteristic of granitoids which had developed within the limits of these complexes. It is quite natural that granitoids which developed in different structural zones, and hence, in geological retrospective in different geodynamic conditions and in the basement of various composition, essentially differ from each other.

In conditions of selective melting of sialic infrastructure of Elbrus subzone (island arc of the Greater Caucasus) which took place immediately after overthrusting of Macera nappe, in conditions of deep subsidence of this subzone were formed K-rich granites; their considerable part emplacemented in the contract zone of the infrastructure and the Macera nappe and "heal" their tectonic contact. It's quite natural that these granites cut syn- and premetamorphic granitoids which are widespread in the infrastructure and their pebble are observed in Upper Carboniferrous and Permian sediments.

In the same time interval (sudetic phase) in the Pass subzone in lower part of Buulgen series and mainly in underlying, deeper horizons magmatites of low-K quartz-diorite-plagiogranitic series are formed. Through the whole section of Buulgen series diorite-plagiogranitic series are formed. Through the whole section of Buulgen series their active contact whith metamorphites is observed. Varieties of low-K postmetamorphic granitoids are not found within the Macera nappe and post metamorphic high-K granites whith form in infrastructure are not found in metamorphits of Buulgen series.

As for syn- and premetamorphic granitoids in Elbrus zone they exist in the infrastructure. Synmetamorphic granitoids are represented by Early Hercynian plagiogneisses, plagiogranites, plagiomigmatites, microclinised porphyroblastic granites. Their pebble appear in Middle Carboniferous sediments. Orthogneisses of granodioritic composition belong to premetamorphic (pre-Hercynian) granitoids. In the Pass subzone the premetamorphic granitoids (gneissic diorites, quartz diorits, plagiogranites, plagiogranodiorites) occur in the lower and middle parts of the Buulgen series section, and Synmetamorphic granitoids are represented by various para- and orthogneisses and granitoids.

In Laba metamorphic complex are notable premetamorphic, apparently Lower Paleozoic Beshta and Kamenistaia plagiogranodioritgneisses, whose analogues are unknown in the Main Range zone. They are located within the Lashtrak and Ajara nappes [5]. Whereas to synmetamorphic granitoids belong plagiogneissic and plagiogranitic sheets of the Mamkhurts suite and gneissic quartz dioritic and autochthonous conformable bodies of plagiogranites of the Damkhurts suite.

Finally, if we look through the present day data on tectonic structure of the Greater Caucasian pre-Jurassic basement, we should share the idea that it represents a typical tectonically layered complex [10]. Such horizontal tectonic layering of pre-Jurassic basement not only in the Greater Caucasus, but also through the whole Caucasus and its adjacent regions was indicated before [6]. At the same time its striking that most of the overthrusted slabs of the Greater Caucasus were formed in different geodynamic conditions, particullarly, here are combined formational complex of palezoic island arc and marginal sea and various parts of oceanic crust of Paleotethys basin (in Forerange zone). If we also take into account, that most of the overthrusted slabs were moving on the serpentinite "lubricant" (in their contact zones bodies of ultramafites are very frequent), we should consider, that here is real vertical-accretionary complex which consists of separate fragments of exotic terranes (classic example of such accretion is apparently Brooks Range in the northern part of Alaska). In other words, during the Hercynian tectogenesis, and may be partially during Early Cimmerian folding took place mainly vertical and not lateral accretion of the earth crust; that naturally complicates to define primary (predeformational) spatial disposition of separate terranes. This problem requires special analyses and is not a subject of this paper.

A.Janelidze Institute of Geology

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H.Mikadze

Method of Compact Limestones Treatment for Micropaleontological Investigation

Presented by Academician I.Gamkrelidze, June 06, 1996

ABSTRACT. **A new method is applied for separation of microforaminifers from the Upper Cretaceous compact limestones, glauconitic sandstones and clay and marlaceous limestones of the Gagra-Djava zone.**

Upper Cretaceous deposits of the Gagra-Djava zone are represented by carbonaceous rocks; variegated limestones containing frequent nebses and intercalation of variegated flints.

Many investigations suggested various zonal subdivisions of the Upper Cretaceous sediments of the region. In most cases foraminifers were studied either in petrographic sections or in the samples from the soft rocks. But in petrographic sections precise identification of taxons is impossible as the shell are of diverse orientation and the section crossouts them in different planes.

Thus resulting from the above mentioned it is indispensable to work out a new method of foraminifer separation from compact rocks. Collection of foraminifers from the Upper Cretaceous (Abkhazis-Racha and Odishi-Okriba) facies type sediments are used for treatment. Samples were taken in the proportions from 30 cm up to 2 m and more, according to the character of rocks; 1000 samples alltogether.

Techniques of Sample Treatment

Fig.1. Marginotruncana pseudolinneiana Pessagro, Georgia (Samegrelo), the Tskhemura river (left tributary of the river Tekhura), Lower Turonian

For the first time in Georgia we have applied threestaged method of sample treatment allowing to desintegrate compact rocks and to separate foraminifer shells.

Main point of the method: on its first stage 200-300 g of the rocks is pestled in the iron mortal; size of fractions shouldn't exceed 0.1-0.5 mg. Solution of sodium carbonate is poured on the obtained powder 1 tablespoon full of sodium carbonate $(Na_2CO_3 10 \cdot H_2O)$ to 250 g of water.

We keep this powder in solution during 24 hours, then we heat it until its evaporation and exsiccation.

The second stage is treatment of the powder in the hydrous sodium sulphate. Hydrous sodium sulphate is heated beforehand. During calcination it extracts water and starts boiling. In the process of boiling the powder produced at the first stage is added. This porridge like mass is heated and cooled for several times (5-10 times) then it is washed in water and dried. It is washed off in metal or in some other fireproof vessel.

Fig.2. Globotruncana bulloides Volger, Georgia (Samegrelo), the Tskhemura river, (left tributary of the Tekhura river), Lower Santonian.

At the third stage rocks are treated with glacial acetic acid and copper vitriol. Copper vitriol (5-10 g) on each sample is treated with fire (only enamel vessels are used), till it becomes white hot and water molecules completely evaporate.

On the second stage preliminarily washed and dried mass is put into the tin. Worked up copper vitriol is added here and afterwards 20-25 mg of glacial acetic acid is added and the tin is tightly closed.

As we know the glacial acetic acid freezes under +17°C. During 3-4 days this solution is alternately kept in comparatively cool (17°C) and warm (17°C) place. After that the powder is washed out in water, till it becomes transparent.

After each stage obtained mass is washed out in warm water.

As we know carbonate sediments as well as shells of foraminifers are dissoluted under the influence of acid. In order to choose the most optimal time for keeping the powder in the glacial acetic acid and cooper vitriol without distroying the shells several tests are carried out.

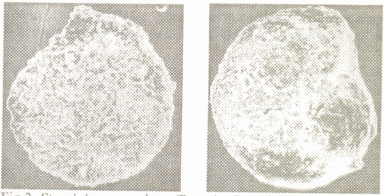

Fig.3. *Stensioina oxsculpta* (Reuss), Georgia (Samtredia) the Tskhemura river, (left tributary of the Tekhura river), Upper Coniacian
a - dorsal view
b - ventral view

The most suitable sample is pestlled and put into several tins (the weight of powder doesn't matter) .Then we repeat the third stage of washing out. But time interval is different for each portion of the powder. First portion is washed out after 12 hours; the second one - after 24 hours, the third - after 2 days and the fourth - only after 4 days. After each washing out the powder is studied with ocular and after these tests it turned out that the optimum time for keeping the powder in acid without distroying the shells is more than 4 days for very compact rocks and 1-2 days for the soft ones.

Basing on the above mentioned method used on the samples taken from several sections of the Gagra-Djava zone we can make the following conclusions:

1. For glauconitic sandstones that are mainly observed in the Cenomanian, first stage of the method must be used.

2. For the clay and marlaceous limestones that don't comprise flinty intercalations - first and second stages are forbidden, as water and hydrous sodium sulphate will distroy the shells of foraminiferes (shells and rocks are of the same chemical composition). Thus only the third stage of the method can be used here.

3. All three stages can be applied to compact flinty limestones. It should be noted that after the second stage of washing, heary concentrate must be studied under the binocular for sampling of those foraminifers which do not require further processing. The rest of the material is treated according to the third stage.

Thus the new method of washing out of the heavy concentrate allows to distinguish well preserved abundant planctonic and benthonic foraminifers.

The Caucasian Institute of Mineral Resources

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PETROLOGY

A.Okrostsvaridze, D.Shengelia

New Data on the Formation of Microcline Granites of Dzirula Salient

Presented by Corr. Member of the Academy G.Zaridze July 8, 1996

ABSTRACT. **Considering structure of Dzirula salient characteristics of binary granites formation of inner Massif of the Caucasus it is assumed that during Tournasion-early-middle Visean period the rocks of sialic profile (the substratum of microcline granitoids) were overthrust by allochthons of femic profile. The thickening of the crust caused the change in PT gradient which stimulated the process of selective melting in sialic rocks and formation of S-type granites. The trace of tectonic transport of allochton was closed and thus erased by granitoid magma activity (late Visean-Bashkirian) in major areas.**

The Caucasus is spread from the Caspian Sea to the Black Sea and represents a connective segment between the European and Asian parts of the Alpine-Himalayan folded system. It is situated between comparatively static Russian and Arabic plates, moving to the North - North-West direction. It consists of three major geostructural units: the folded systems of the Lesser and Greater Caucasus and inner Caucasion Massif.

The inner Caucasion Massif is situated in the area between the Lesser and Greater Caucasus. It consists of pre-Alpine crystalline basement and Alpine Sedimentary cover. In the latter one there are three principal salients: Dzirula, Khrami and Loki.

Dzirula crystalline salient is situated in the central part of the inner Caucasian massif and is a complex geological formation. In the Dzirula salient two-age and genetic groups of rocks are distinguished: prehercynian and hercynian formations. Prehercynian formations occupy the upper structural floor and are mainly represented by crystalline shales, plagiogneisses, amphibolites, quartz-diorite gneisses. In this association of rocks dominates the last one. Numerous inclusions of different shape and size of gabbroes, gabbro-diorites and diorites are observed in them with diameters in the range of 3-60 cm and in some places comprise the half of the outcropped area. In tectonic scales of the farther east of Dzirula salient assosiation of serentinites gabbro-amphibolites and diabase porphyrites are distinguished. They are considered as ophiolitic formations [1,2]. It should be mentioned that in 1953 G.Zaridze and N.Tatrishvili considered femic rocks of Dzirula salient as residues of Paleozoic geosynclinal substratum and that quarz-diorite gneisses were formed as a result of metasomatic granitization [3]. This idea was approved in geological circles and in fact it gave start to the principles of metasomatic granitization. I.Khmaladze and K.Chikelidze [4] considered the above mentioned quartz-diorite-gneisses as magmatic formations explained by hypodiomorphic structures, zonal plagioclasses and unchangable mineral and chemical composition and disoriented xenolithes. According to our data quarz-diorite gneisses of Dzirula salient are hybrid formations formed under the influence of magmatic material on progressive sedimentary sheet by the so-called melting. According to Rb/Sr and U/Pb dating the absolute age of quartz-diorite gneisses of Dzirula salient is evaluated as 762±222 Ma [5].

The rocks described are crossed by microcline granites, which form sectioning bodies of different thickness. In fact all the old formations are full of the bodies of such veins and injections: as a result crystallin plates plagiogneisses, plagiomigmatites, tonolites and metabasites undergo microclinization of different quality. In fact microclinization gives the ultimate geological picture of Dzirula salient. The sectioning bodies are represented by anatectic massive equally-grained biotite-muscovite granites, alaskites, aplites and pegmatoids. The main rock-constructing minerals are quartz, acid plagioclase, microcline biotite and muscovite. According to Rb/Sr and U/Pb the absolute age of binary granites of Dzirula salient is 330-311 Ma [5].

Petrochemical study of quartz-diorites gneisses and microcline granites and also the data of discriminant diagram by Hassan and McAlister [6] show (Fig.1) that quartz-diorites of hybrid genesis are attributed to granitoids of I-type while anatectic Microcline-granites represents granitoids of S-type.

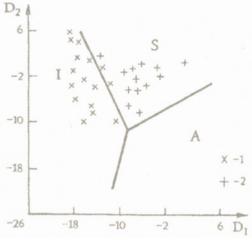

Fig. 1. Prejurassic granitoids of Dzirula salient on the discriminant I, S, A diagram [6].

D_1=0,76Al_2O_3+5,96TiO_2+2,91MnO–1,93Na_2O+1,95K_2O–18,50P_2O_5

D_2=0,37 Al_2O_3+7,25TiO_2–54,04MnO–4,28Na_2O–0,55K_2O+45,81P_2O_5

1. Quartz-diorite gneisses, 2. Microcline Granitoids

As we can see quartz-diorite gneisses and microcline granites are separated not only in time but they are genetical disconnected as well. The first ones are prehercynian hybrid formations formed by complex interaction between the mantle and crust by the so called "progressive melting" and the others are formed in the late Hercynian period by typical anatectic melting of sialic material.

Thus the rocks of femic complex of Dzirula salient occupying, the upper floor of the structural section can be considered as the residue of the oceanic crust as a result of tectonic and metasomatic processes; here ophiolitic scales are distinguished, but the formations of gabboro-diorite-quartzdiorite series prevail with fragments of regionally metamorphosed sedimintary and volcanic sedimentary rocks.

But microcline-granites that occupy the lower floor of the section are apparently the result of continental type sediments melting.

It should be mentioned that none of the concerning works can explain the existing picture of formation of granitoids. According to the present view two-mica potassium granitoids should be formed as a result of selective melting of poor in potassium gabboro-diotite-quartz diorate series, or of deeper basalts and ultrabasits of the oceanic crust; this is impossible even by the results of experimental data. For real picture the only scheme is to admit the reverse variant of the construction of the crust of Dzirula salient of the Caucasus inner Massif. Here the upper sections of searm outcrop is occupied by the oceanic crust formation and lower - by the continental crust. Following from this we admit that the rocks of salic profile (granitoid substratum) were covered by a thick overthrust - sheet of femic profile.

The period of formation of charriage is the same as of Chorchano-Utslevi allochthon complex: allochthon complex: Tournasion-early-middle-Viseon, because, according to all features they are the result of one and the same tectonic process. The thickness of allochton reached several kilometers and thickening of the crust established high PT gradient and stimulated in sialic rocks the process of selective melting and the formation of two-mica granites. Their formation took place already in late-Visean-Bashkirion period, because they have intrusive contacts with Chorchano-Utslevi allochthon, but do not cross upper carbonate, neoautochton. It should be mentioned that activity of granite magmo healed and erased the zone of allochthon transport.

Besides the above mentioned data the following facts indicate allochthonous character of the femic complex of Dzirula salient bulge:

1. Metabasites analogical to intensively tectonized ophiolites (the second and third layers of the oceanic crust) or paleooceanic crust of middle oceanic range type are widely spread in them [7].
2. Close space and genetic relation of the rocks of femic complex with the allochthonous scales of Chorchano-Utslevi.
3. The different structure and style of the rocks of femic and sialic profil [2].

Thus we have an interesting picture of formation of microcline granites in Dzirula salient of inner Massif of the Caucasus. Namely during Tournasion-early-middle-Visean period formation of chariage of thick femic rocks (the crust of oceanic type) over the rocks of sialic profile (the crust of continental type) took place, which caused thickening of the earth crust in this area and estableshed high PT gradient. This process stimulated selective melting in sialic rocks and formation of microcline S-type granites, which during late Visen-Bashkirian period crossed and metasomatically changed the upper layer of the crust of oceanic type.

A.Janelidze Institute of Geology
Georgian Academy of Sciences

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BULLETIN OF THE GEORGIAN ACADEMY OF SCIENCES, 154, №1, 1996

A.Sulamanidze

The Way of Boundary Effect Detection at Conductor Setting

Presented by Academician T.Loladze, June 27, 1996

ABSTRACT. **An original way of detection of thermoelectromotive force originated on the border of solid and liquid phases of current conducting material is proposed. A simple and accessible scheme is worked out for registration of the mentioned effect.**

Practical application of the method opens possibilities of quantitative estimation of boundary effect of conductors, fixing their place in thermoelectric metal series and of consecutive control of thermal processes in the zone of unipolar current pulse resistance welding and of crystallization process control.

As far back as 1956 A.Ioffe [1] expressed an idea and then in 1957 W.Pfann, K.Benson and J.Wernick [2] proved experimentally that thermoelectric phenomenon initiated on the border of solid and liquid states of the same conductor must have a definite effect on metal recrystallization process. Namely, heat removal emitted at this time occurs usually not only with heat conduction, but with thermoelectric effect electron flow as well. Therefore the mentioned thermoelectric effect can be used as means of thermal processes regulation in the zone of unipolar current pulse of electric resistance welding and also for crystallization process control.

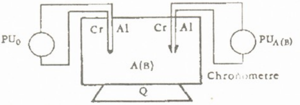
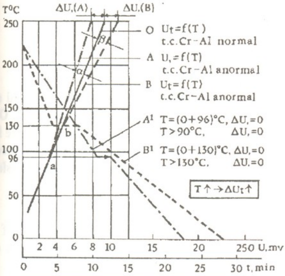


Fig. 1. Practical scheme of the detection effect.

For practical realization of the latter notion and considering that nowadays thermoelectric process occurring on the border of metal phase transformation is not almost investigated, the author has developed an original scheme (Fig. 1). It suggests placement of the investigated material, the alloys A and B, in fireproof porcelain melting-pot. The chemical composition of these materials are shown in Fig. 2, Then the material is heated with external heat source (Q) above melting temperature for (100-150°C). Chromel-alumel (two) thermocouples, one of which (left) is normal and the second (right) has shared ends are placed in melted material. The readings of thermocouples are simultaneously registered with two millivoltmeters. The left one (PU) registers the readings of normal thermocouple and the right one (PU) the readings of the abnormal one.

Investigation process is carried out first for one (A) and then for the second (B) material. When the temperature of melted material becomes 250^0C, chronometer is switched on and measurement results (temperature, voltage fall) are in equal time intervals fixed in tables. Chromel-alumel thermocouple characteristics (straight line) known from literature is at first marked down on the corresponding reference axes prepared on the basis of table data (Fig.2). Then diagram (A) is plotted for material A using the data obtained with shared ends thermocouple (millivoltermeter PU) and for the same material A, the setting diagram (A) is plotted basing on chronometric data. Thus for material A we have three graphs (O, A, A), wherefrom it is evident that when it transfers from liquid to solid state to setting, the data of two millivoltmeters are intershifted for a definite angle (α) (called below as effect characterizing angle) and their intercoincidence begins from the point which corresponds to setting temperature (96^{0}C) of this material.



n\c	Sn	Pb	Cd	Bi	Zn	In
A	18,75	31.25	—	50,0	—	—
B	52,0	30.0	13,0	5,0	—	—

Fig. 2. Diagrams of material setting and thermoelectric characteristics of the effect.

Quantitatively different, but qualitatively analogous picture is observed for material B - three graphs (O, B, B) and characteristic angle (β). Intercoincidence of (O) and (B) graphs and the existence of divergence angle (β) ceases when melting temperature of material reduced to setting temperature (130°C).

The obtained results prove the considerations expressed by the author as a result of his recent works [3, 4].

It should be mentioned that the so-called effect characterizing angle detected by this method allows to register the border effect of a conductor, quantitatively estimate and to compose thermoelectric series of conductors and semiconductors according to potential gradients that occur on the border of solid and liquid phases of separate materials.

Georgian Technical University

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HYDRAULIC ENGINEERING

D.Namgaladze, V.Nanitashvili

Nonstationary Process Control in Supply Pipelines by the Finite Control Method

Presented by Academician G.Chogovadze, June 22, 1996.

ABSTRACT. **In the paper problem of pressure control at the beginning and at the end of supply pipeline is considered. The method of finite control is used. On the basis of differential equations describing the nonstationary operation regime in the pipeline and using optimal control theory, the control functions are obtained. They allow to reduce pressure oscillations to a minimum in the pipeline at transfer from one stationary regime to another.**

A supply pipeline is a complex engineer construction. Reliable operation of a pipeline depends on a number of factors and is strongly connected with strength of largediameter pipes operating under high pressures.

At present strength calculation in supply pipelines is performed by the limiting state method considering tensile strength under the static internal pressure. This calculation does not allow possible inhomogeneity of stress distribution on the pipe wall caused by deviation of the pipe profile from the right geometrical shape due to the bead, its edge shift and profile ovality, as a whole.

In practice there is a certain failure percentage of pipes, satisfying static strength conditions while they wear.

The most probable cause of these failures is accumulation of damages and development of initial defects resulting in appearance and spreading of cracks due to repeated effects in the process of operation. Thus according to [1], supply pipeline undergo approximately 500÷600 cycles of repeated loadings a year caused by various technological and operation factors (pumping station switching off, caused by failures in electric and mechanical equipment, changes in pumping regime, etc.). For the estimated service period of a pipeline (≈20 years) total number of loading cycles can excess over 7000÷9000 cycles.

Let us consider the transfer of the supply pipeline from one stationary regime to another. In this case pressure and discharge fluctuations occur. Therefore such pressure variation with time at the pumping station should be chosen, when these fluctuations might be removed in the shortest possible time. The optimal control theory offers several ways to solve this problem, in particular, the finite control method [2] which is developed for a specific problem [3].

Let us consider pressure distribution in the pipeline, with respect to the new stationary state, as a perturbation occuring instantly at the moment of change of the pipeline operation regime.

The non - stationary regime of the pipeline operation is described by a linearized set of I.A.Charnyi equations[4]:

$$\left.\begin{aligned} -\frac{\partial P}{\partial x} &= \frac{\partial v}{\partial t} + 2av; \quad 0 \le X \le L \\ -\frac{\partial P}{\partial t} &= C^2 \frac{\partial v}{\partial x}; \quad\quad t \ge 0 \end{aligned}\right\}, \tag{1}$$

where P and v - the pressure and mass velocity, respectively;
c - the velocity of shock wave propagation;
a - the linearization constant.
Initial conditions are:

$$\left.\begin{aligned} P(x,0) &= P_H(x) = P_H + \frac{x}{L}(P_0 - P_H) \\ \frac{\partial P}{\partial t}(x,0) &= 0 \end{aligned}\right\}, \tag{2}$$

where P_H - the pressure at the beginning of the run in the initial stationary regime;
P_0 - the pressure at the end of the run in the initial stationary regime.

After a new stationary regime has been established, pressure distribution in the pipeline is described by the expression:

$$P_k(X) = P_k + \frac{x}{L}(P_0' - P_k), \tag{3}$$

where P_k and P_0' - pressures at the beginning and at the end of the pipeline, respectively.

Such situation can be observed when an additional pump is connected to the line.

Boundary conditions are chosen as follows:

$$\left.\begin{aligned} P(0,t) &= P_k + U(t) \\ P(L,t) &= P_0' + F(t) \end{aligned}\right\} \tag{4}$$

where $U(t)$ and $F(t)$ are the control functions chosen so that for a finite time T a new stationary regime might be established in the pipeline.

The stationary condition

$$\frac{\partial P(x,T)}{\partial t} = 0 \tag{5}$$

is added to the condition (3).

From Equation (1) one can easily obtain:

$$\frac{\partial^2 P}{\partial t^2} + 2a\frac{\partial P}{\partial t} - c^2\frac{\partial^2 P}{\partial x^2} = 0 \tag{6}$$

If a new function $R(x,t) = P(x,t) - P_k(x)$ is introduced it will be evident that

$$\frac{\partial^2 R}{\partial t^2} + 2a\frac{\partial R}{\partial t} - c^2\frac{\partial^2 R}{\partial x^2} = 0 \tag{7}$$

If we denote $\Delta p = P_H - P_k$; $\Delta p' = P_0' - P_0$; $\Delta p'' = \Delta p - \Delta p'$, it can be obtained:

$$R(x,0) = \Delta p''\left[1 - \frac{x}{L}\right] - \Delta p'. \tag{8}$$

New condition take the form

$$\left.\begin{array}{ll} \dfrac{\partial R(x,0)}{\partial t}=0; & R(0,t)=U(t); \\ R(L,t)=F(t); & R(x,T)=0; \\ \dfrac{\partial R(x,T)}{\partial t}=0; & \beta=\dfrac{aL}{\pi c}. \end{array}\right\}. \quad (9)$$

Physically minimum time for solution of the set problem is $T = 2L/c$.

Let us introduce dimensionless coordinates $\xi = \pi x/L$ and $\tau=\frac{\pi ct}{L}-\pi$., as well as a new function $Z(\xi,\tau)=\frac{R(x,t)+\Delta p'}{\Delta p''}$. Then we obtain the following problem:

$$\frac{\partial^2 Z}{\partial \tau^2}+2\beta\frac{\partial Z}{\partial \tau}-\frac{\partial^2 Z}{\partial \xi^2}=0 \quad (10)$$

$$\left.\begin{array}{ll} Z(\xi,-\pi)=1-\dfrac{\xi}{\pi}; & \dfrac{\partial Z(\xi,-\pi)}{\partial \tau}=0; \\ Z(0,\tau)=\dfrac{U(t)+\Delta p'}{\Delta p''}; & Z(\pi,\tau)=\dfrac{F(t)+\Delta p'}{\Delta p''}; \\ Z(x,\pi)=\dfrac{\Delta p'}{\Delta p''}; & \dfrac{\partial Z(x,\pi)}{\partial t}=0. \end{array}\right\} \quad (11)$$

Hereafter we shall write x and t instead of ξ and τ.

Let us denote

$$(t)=\frac{U(t)+\Delta p'}{\Delta p''} \quad \text{and} \quad M(t)=\frac{F(t)+\Delta p'}{\Delta p''} \quad (12)$$

Introduce a new function $S(x,t)$ instead of $Z(x,t)$

$$Z(x,t) = S(x,t)e^{-\beta(t+\pi)}. \quad (13)$$

Then our problem will be changed into the following one:

$$\frac{\partial^2 S}{\partial t^2}-\beta^2 S(x,t)-\frac{\partial^2 S}{\partial x^2}=0\,, \quad (14)$$

$$\left.\begin{array}{l} S(0,t)=Z(0,t)e^{\beta(t+\pi)}=V(t)e^{\beta(t+\pi)} \\ S(\pi,t)=Z(\pi,t)e^{\beta(t+\pi)}=M(t)e^{\beta(t+\pi)} \end{array}\right\} \quad (15)$$

Let us denote

$$\left.\begin{array}{l} r(t)=V(t)e^{\beta(t+\pi)} \\ z(t)=M(t)e^{\beta(t+\pi)} \end{array}\right\} \quad (16)$$

Since $\tau=\dfrac{\pi ct}{L}-\pi$, at $t = 0$, $\tau = -\pi$, i.e. at $\tau < -\pi$ or $t < -\pi$ the problem becomes meaningless. Therefore with $t < -\pi$, $S(x, t) = 0$.

That is at $t < -\pi$ the function $S(x,t)$ can be continued with zero. Therefore equation (14) will be written down as follows:

$$\frac{\partial^2 S}{\partial t^2} - \beta^2 S - \frac{\partial^2 S}{\partial x^2} = \left(1 - \frac{x}{\pi}\right)\left[\beta\delta(t+\pi) - \frac{d}{dt}\delta(t+\pi)\right] \tag{17}$$

Let us make Fourier transformatin of equation (17) over t variable; we obtain

$$-\omega^2 \widetilde{S}(x,\omega) - \beta^2 \widetilde{S}(x,\omega) - \frac{d^2\widetilde{S}(x,\omega)}{dx^2} = \left(1 - \frac{x}{\pi}\right)e^{i\omega\pi}(\beta + i\omega) \tag{18}$$

The boundary conditions:

$$\left.\begin{aligned} \widetilde{S}(0,\omega) &= \widetilde{r}(\omega) \\ \widetilde{S}(\pi,\omega) &= \widetilde{\tau}(\omega) \end{aligned}\right\} \tag{19}$$

The solution of equation (18)can be written as

$$\widetilde{S}(x,\omega) = c_1 h_1(x,\omega) + c_2 h_2(x,\omega) + \widetilde{F}(x,\omega), \tag{20}$$

where $h_1(x,\omega) = e^{i\sqrt{\omega^2+\beta^2 x}}$ and $h_2 = e^{-i\sqrt{\omega^2+\beta^2 x}}$.

$$F(x,\omega) = -\sum_{k=1}^{2} h_k(x,\omega) \int_0^x \frac{W_k\left[h_1(\xi,\omega), h_2(\xi,\omega)\right]}{W\left[h_1(\xi,\omega), h_2(\xi,\omega)\right]}(\beta + i\omega)e^{i\omega\pi}\left(1 - \frac{\xi}{\pi}\right)d\xi.$$

Here $W[h_1,h_2]$ is Wronsky determinant; $W_k[h_1,h_2]$ are the determinants obtained from the Wronsky determinant by replacing the k - column with the column (0;1). After calculating W_k determinants and making integrations we obtain

$$\widetilde{F}(x,\omega) = \frac{1}{\lambda^2}(\beta + i\omega)e^{i\omega\pi}\left[\frac{x}{\pi} - 1 + \cos\lambda x - \frac{\sin\lambda x}{\pi\lambda}\right] \tag{21}$$

where $\lambda = \sqrt{\omega^2 + \beta^2}$.

Integration constants c_1 and c_2 should be chosen so, that the boundary conditions might be fulfilled. Therefore we obtain:

$$\left.\begin{aligned} c_1 h_1(0,\omega) + c_2 h_2(0,\omega) + \widetilde{F}(0,\omega) &= \widetilde{r}(\omega) \\ c_1 h_1(\pi,\omega) + c_2 h_2(\pi,\omega) + \widetilde{F}(\pi,\omega) &= \widetilde{\tau}(\omega) \end{aligned}\right\} \tag{22}$$

Solving this system, we obtain:

$$\widetilde{S}(x,\omega) = \frac{1}{\Delta(\omega)}\left[h_1(x,\omega)\Delta_1(\omega) + h_2(x,\omega)\Delta_2(\omega)\right] + \widetilde{F}(x,\omega), \tag{23}$$

where

$\Delta(\omega) = -2i\,\sin\lambda\pi$

$$\left.\begin{aligned} \Delta_1(\omega) &= \widetilde{r}(\omega)e^{-i\lambda\pi} - \widetilde{\tau}(\omega) + \frac{(\beta + i\omega)e^{i\omega\pi}}{\lambda^2}\left[\cos\lambda\pi - \frac{\sin\lambda\pi}{\lambda\pi}\right] \\ \Delta_2(\omega) &= \widetilde{\tau}(\omega) - \frac{(\beta + i\omega)e^{i\omega\pi}}{\lambda^2}\left[\cos\lambda\pi - \frac{\sin\lambda\pi}{\lambda\pi}\right] - \widetilde{r}(\omega)e^{i\lambda\pi} \end{aligned}\right\} \tag{24}$$

Since the $S(x,t)$ function must be finite over the t variable (different from idential zero at $|t| < \pi$), according Vinier-Pely theorem its Fourier representation $\widetilde{S}(x,\omega)$ at $x \in [0;\pi]$ must satisfy three requirements:

1. $\widetilde{S}(x,\omega)$ can be analytically continued over the whole complex plane $z = \omega + iy$ (i.e. $\widetilde{S}(x, z)$ function must be an integral function over the complex variable z);
2. $\widetilde{S}(x, z)$ must have the degree no more than π;

3. $\widetilde{S}(x,\omega)$ must be integrable with the square on the real axis $z = \omega$.

To fulfill the requiment it is necessary and sufficient that $\Delta_1(z_k) = 0$ or $\Delta_2(z_k) = 0$ at all points, where $\Delta(z_k) = 0$. Thus from equation (23) we obtain $\sin\sqrt{z_k^2+\beta^2}\pi = 0$ or $z_k = \pm\sqrt{k^2-\beta^2}$.

Theorefore the first requiment takes the form of the following interpolation problem for the functions:

$$e^{-i\lambda\pi}\tilde{r}(\omega) - \tilde{\varkappa}(\omega) = \frac{(\beta+i\omega)e^{i\omega\pi}}{\lambda^2}\left[\frac{\sin\lambda\pi}{\lambda\pi} - \cos\lambda\pi\right] \tag{25}$$

$$e^{i\lambda\pi}\tilde{r}(\omega) - \tilde{\varkappa}(\omega) = \frac{(\beta+i\omega)e^{i\omega\pi}}{\lambda^2}\left[\frac{\sin\lambda\pi}{\lambda\pi} - \cos\lambda\pi\right] \tag{26}$$

or

$$(-1)^k\tilde{r}\left(\pm\sqrt{k^2-\beta^2}\right) - \tilde{\varkappa}\left(\pm\sqrt{k^2-\beta^2}\right) = -(-1)^k\frac{\beta\pm i\sqrt{k^2-\beta^2}}{k^2}e^{\pm i\sqrt{k^2-\beta^2}\pi}. \tag{27}$$

Let us write equation (27) for even and odd k and obtain

$$\tilde{r}\left(\pm\sqrt{k^2-\beta^2}\right) - \tilde{\varkappa}\left(\pm\sqrt{k^2-\beta^2}\right) = -\frac{\beta\pm i\sqrt{k^2-\beta^2}}{k^2}e^{\pm i\sqrt{k^2-\beta^2}\pi}, \tag{28}$$

$$-\tilde{r}\left(\pm\sqrt{k^2-\beta^2}\right) - \tilde{\varkappa}\left(\pm\sqrt{k^2-\beta^2}\right) = \frac{\beta\pm i\sqrt{k^2-\beta^2}}{k^2}e^{\pm i\sqrt{k^2-\beta^2}\pi}. \tag{29}$$

To find functions $\tilde{r}(z) - \tilde{\varkappa}(z)$ and $\tilde{r}(z) + \tilde{\varkappa}(z)$ use the interporation Lagrange formula ($\sum_k{}'$ and $\sum_k{}''$ symbols denote summation over even and odd k).

$$\tilde{r}(z) - \tilde{\varkappa}(z) = -\sum_k{}'\left[\frac{\beta+i\lambda_k}{k^2}e^{i\lambda_k\pi}\frac{\tilde{\varphi}(z)}{\tilde{\varphi}'(\lambda_k)(z-\lambda_k)} + \frac{\beta-i\lambda_k}{k^2}e^{-i\lambda_k\pi}\frac{\tilde{\varphi}(z)}{\tilde{\varphi}'(-\lambda_k)(z+\lambda_k)}\right] +$$

$$+\tilde{\gamma}(z)\tilde{\varphi}(z), \tag{30}$$

$$\tilde{r}(z) + \tilde{\varkappa}(z) = -\sum_k{}''\left[\frac{\beta+i\lambda_k}{k^2}e^{i\lambda_k\pi}\frac{\tilde{\varphi}(z)}{\tilde{\varphi}'(\lambda_k)(z-\lambda_k)} + \frac{\beta-i\lambda_k}{k^2}e^{-i\lambda_k\pi}\frac{\tilde{\varphi}(z)}{\tilde{\varphi}'(-\lambda_k)(z+\lambda_k)}\right] +$$

$$+\tilde{\gamma}(z)\tilde{\varphi}(z), \tag{31}$$

where $\lambda_k = \sqrt{k^2-\beta^2}$,

$\tilde{\varphi}(z)$ - the integral function of z argument,

$\tilde{\varphi}(\pm\lambda_k) = 0$,

$\tilde{\varphi}'(\pm\lambda_k) \neq 0$,

$\gamma(z)$ - the integral function of zero degree.

For satisfaction of other requirements of the Vinier – Pely theorem and for the functions to be finite on the segment $[-\pi, +\pi]$, it is sufficient to take the function $\varphi(z)$

$$\tilde{\varphi}(z)=\frac{\sin\sqrt{z^2+\beta^2\pi}}{\sqrt{z^2+\beta^2}} \tag{32}$$

then

$$\tilde{\varphi}'(\pm\lambda_k)=\frac{\pm\pi\,\lambda_k(-1)^k}{k^2} \tag{33}$$

With allowance for equations (32), (33) and equations (30), (31) takes the form:

$$\tilde{r}(z)-\tilde{\pi}(z)=-\left[\sum_k{}'\left(\frac{\beta+i\lambda_k}{z-\lambda_k}\frac{e^{i\lambda_k\pi}}{\pi\lambda_k}-\frac{\beta-i\lambda_k}{z+\lambda_k}\frac{e^{-i\lambda_k\pi}}{\pi\lambda_k}\right)+\tilde{\gamma}(z)\right]\tilde{\varphi}(z), \tag{34}$$

$$\tilde{r}(z)+\tilde{\pi}(z)=\left[\sum_k{}''\left(\frac{\beta+i\lambda_k}{z-\lambda_k}\frac{e^{i\lambda_k\pi}}{\pi\lambda_k}-\frac{\beta-i\lambda_k}{z+\lambda_k}\frac{e^{-i\lambda_k\pi}}{\pi\lambda_k}\right)+\tilde{\gamma}(z)\right]\tilde{\varphi}(z). \tag{35}$$

Equations (34) and (35) for z = ω are represented as follows:

$$\tilde{r}(\omega)-\tilde{\pi}(\omega)=\left[-\sum_k{}'\tilde{\Phi}_k(\omega)+\tilde{\gamma}(\omega)\right]\tilde{\varphi}(\omega) \tag{36}$$

$$\tilde{r}(\omega)+\tilde{\pi}(\omega)=\left[\sum_k{}''\tilde{\Phi}_k(\omega)+\tilde{\gamma}(\omega)\right]\tilde{\varphi}(\omega). \tag{37}$$

To determine the inverse Fourier transformation for these functions let us use the convolution theorem. We have

$$\varphi(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\sin\left(\pi\sqrt{\omega^2+\beta^2}\right)}{\sqrt{\omega^2+\beta^2}}e^{i\omega t}d\omega=$$

$$=\begin{cases}\frac{1}{2}I_0\left[\beta\sqrt{\pi^2-t^2}\right]; & |t|<\pi,\\ 0; & |t|>\pi\end{cases} \tag{38}$$

$$\left.\begin{aligned}&\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{\omega\pm\lambda_k}d\omega=\frac{1}{2}e^{\mp i\lambda_k t}\,sign\,t\\ &sign\,t=\begin{cases}1; & t>0\\ -1; & t<0\end{cases}\end{aligned}\right\} \tag{39}$$

And finally

$$r(t)-\pi(t)=-\frac{1}{2\pi}\sum_k{}'\int_{-\pi}^{+\pi}\left[\frac{\beta}{\lambda_k}\sin\lambda_k(\pi+t-\tau)+\cos\lambda_k(\pi+t-\tau)\right]\times$$

$$\times\,sign\,(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau+\frac{1}{2}\int_{-\pi}^{+\pi}\gamma(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau, \tag{40}$$

$$r(t)+\mathfrak{r}(t)=-\frac{1}{2\pi}\sum_{k}{}''\int_{-\pi}^{+\pi}\left[\frac{\beta}{\lambda_k}\sin\lambda_k(\pi+t-\tau)+\cos\lambda_k(\pi+t-\tau)\right]\times$$

$$\times\, sign\,(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau+\frac{1}{2}\int_{-\pi}^{+\pi}\gamma(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau\,. \qquad (41)$$

Adding and substracting (40) and (41) for the control functions we obtain:

$$r(t)=-\frac{1}{2\pi}\sum_{k=1}^{\infty}\int_{-\pi}^{+\pi}\left[\frac{\beta}{\lambda_k}\sin\lambda_k(\pi+t-\tau)+\cos\lambda_k(\pi+t-\tau)\right]\times$$

$$\times\, sign\,(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau+\frac{1}{2}\int_{-\pi}^{+\pi}\gamma(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau\,, \qquad (42)$$

$$\mathfrak{r}(t)=\frac{1}{4\pi}\sum_{k}{}'\int_{-\pi}^{+\pi}\left[\frac{\beta}{\lambda_k}\sin\lambda_k(\pi+t-\tau)+\cos\lambda_k(\pi+t-\tau)\right]\times$$

$$\times\, sign\,(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau\,-$$

$$-\frac{1}{4\pi}\sum_{k}{}''\int_{-\pi}^{+\pi}\left[\frac{\beta}{\lambda_k}\sin\lambda_k(\pi+t-\tau)+\cos\lambda_k(\pi+t-\tau)\right]\times$$

$$\times\, sign\,(t-\tau)I_0\left(\beta\sqrt{\pi^2-\tau^2}\right)d\tau\,. \qquad (43)$$

The transfer to the initial control functions occurs for the dependencies:

$$\left.\begin{aligned}V(t)&=r(t)e^{-\beta(t+\pi)}\\ M(t)&=\mathfrak{r}(t)e^{-\beta(t+\pi)}\end{aligned}\right\}, \qquad (44)$$

$$\left.\begin{aligned}U(t)&=V(t)\Delta p''-\Delta p'=r(t)\Delta p''e^{-\beta(t+\pi)}-\Delta p'\\ F(t)&=M(t)\Delta p''-\Delta p'=\mathfrak{r}(t)\Delta p''e^{-\beta(t+\pi)}-\Delta p'\end{aligned}\right\}. \qquad (45)$$

It is quite natural that for physically minimum time the control is not the only one. Arbitrariness in the choice of the function $\widetilde{\varphi}(z)$ and the integral function $\widetilde{\gamma}(z)$ allows to impose some restrictions on the control. Therefore at passive control (a control system without feedback) the problem is practially reduced not to the complete removal of oscillations, but to the reduction of their amplitude up to the most optimal value.

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BULLETIN OF THE GEORGIAN ACADEMY OF SCIENCES, 154, №1, 1996

ELECTROTECHNICS

A.Zerekidze, T.Natenadze

To the Applied Programmes Packet of DC Traction Machines Automation Design

Presented by Corr. Member of the Academy I.Jebashvili, July 4, 1996

ABSTRACT. **We report on the necessity of d.c traction electric machines automation design complex approach. The work suggests a packet of applied programmes and determines the application of each programme**.

It is known that design is an intermediate link between science and production. The application of different constructional design , change of materials and design methods significantly stipulates technical level and production quality.

The search and analysis of construction - technological solutions satisfying different demands to the designed products require complex account of all the factors determining the quality of the design.

EC provides with the possibility of saying "no" to the suggestion that all dimensions of designed electric machines and all characteristics of applied materials are always single-valued.

All the products properties and characteristics should be known with rather high accuracy only before designing experimental spesimens.

The traditional division of separate electric machines design and production doesn't satisfy up-to-date demands. Even the most perfect design fails to give us the desired results unless the demands of technology and possibilities of production have not been taken into consideration.

Should the influence of technology and quality of materials on electronic machines output parameters be envisaged, then its high quality can be guaranteed, of course on the definite level of technological culture and conformity of materials with the demands of standards.

The creation of safe machine with optimal parameters output characteristics of which are in accordance with the demands of exploitation put forward the necessesity of solving various problems of design. Thus we have to create complex system automatic design taking into account technological processes and organization of production of design.

D.c. traction machines as any other kinds of machines require programme packet creation for their automation calculations.

Such packet of programmes should at least imply the following applied programmes:

1. Parameter optimization.
2. Calculation of electromagnetic characteristics by one of the methods.
3. Calculation of possible characteristics deflections.
4. Determination of non-sparking commutation zones.

The first programme is to solve the question of chosing basic dimensions and sections of magnetic conductor and winding. With the help of the second programme which determines magnetic field distribution along all parts of the machine, electromagnetic characteristics of traction machines set with high acuracy.

If first two programmes they operate with nominal meanings of dimensions and averaged properties of magnetic and electronic conduction materials, the possible deflactions of output characteristics are defined in the third programme.

The fourth programme belongs checking calculations, when the designed machine is checked by thermal parameters.

If the second programme gives a clear picture of potential conditions on the collector, then commutational possibilities of traction machines are determined with the help of the fifth programme. Commutation qualities of d.c. machines are estimated best of all by determining their non - sparking zones.

The problem of electric machine optimal design is multicriterion, as a lot of technical and economic demands should be taken into consideration. Not only multicriterion determines the difficulties of optimal design but also unsufficient study of limited permissible quantities and interrelations of many variable parameters being in complex and contradictory dependence. That's why, for instance, in cybernetics the method of analysis and synthesis of traction electric machines [1], the way of mathematical formalization and variable agregation are chosen.

In order to use the method of mathematical formalization and generation aiming build as multicriterion corrections [2] mentioned above time on the problem deformalization and further detail design will be needed.

Optimization algorithms worked out the base of traditional methods of calculations were used, for example [3]. Some reasons within limits of permissible quantities and interrelations for such algorithm are also given.

Chosing the method in the majority of cases guided by the volume of the programme complexity of mathematical expressions and desirable accuracy of the results.

To rise the accuracy of determination characteristics for the chosen optimal variant traction, it is necessary to make calculation on strict theoretical base. The use of the electromagnetic field theory for electromagnetic calculations allows to account the satiration of separate parts of machine magnetic curcuit, their dependence and influence on each other with more accuracy. It also makes possible to arrange electric machine characteristics precisely, as they are integral characteristics of the field. This problem is solved by numeral methods in computer.

Electric machines refer to the class of products in productions of which statistic variation and probability process play essential role.

All types of electric machines including d.c. traction machines after output have been experimentally registered under test stand different from calculated level output characteristics. Moreover output characteristics of scattering meanings can be considerable.

Non homogeneity of output traction machines on technical specifications, parameters, quality indeces appear due to a number of reasons. Firstly, while producing any electrotechnical product oscillations of technological process regimes are ineritable. Secondly, in conditions of mass or serial production the oscillations of quality of rair material, half - finished products and completed products are also ineretable.

Motor output parameters (such as traction, stating moment, idle current, coefficient of performance and so on) depend on accidental meanings of all geometric dimensions and characteristics of used materials within the limits of their real scatterings, defined by fields of technological admit combination of these parameters for each specimen.

In conclusion, the results presented here indicate that calculation programmes breaking point deflactions characteristics must exist.

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AUTOMATIC CONTROL AND
COMPUTER ENGINEERING

Z.Babunashvili, G.Popov

Automatic System for Watching Movement of Stormclouds

Presented by Academician M.Salukvadze, June 21, 1996

ABSTRACT. **Proposed system for watching movement of stormclouds together with the other methods of meteorological researches enable us to be prepared long before the storm and increase considerably the efficiency of hailprevention system.**

Our knowledge about lightning strokes based on numerous researches enables us to explain physical mechanism, character of lightning, determine trajectories of stormclouds and foresee places of strokes on to the ground [1,2].

Present work describes the automatic system for watching movements of stormclouds. The principle of detection of magnetic waves emanated from lightning is used in the system.

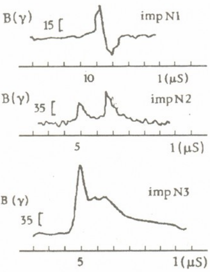

Fig.1

Fig. 1 shows oscillating currents of magnetic impulses emanated from lightning [3]. The impulses NN 1 and 2 are the results of electrical discharges of clouds. The impulse 3 corresponds to the discharge between the stormcloud character and its duration makes $20 \cdot 10^{-6}$ - $30 \cdot 10^{-6}$ sec. Frequently there were registered unipolar impulses (impulse 2) that preceed bipolar ones.

In a number of countries scientists invented similar systems reacting electrical discharges between stormclouds and the ground. For watching the front of the storm we need the information not only about strokes to the ground but also about discharges inside stormclouds and between separate clouds which are much more frequent. For this purpose we use the multichannel kilometer-length-wave detector.

The system is intended for watching the movement of the stormclouds in regions where such atmospheric phenomena as hail, storm, squale etc. are frequently observed. Such system is needful in well developed agricultural lands such as wine-growing regions in Georgia and Bulgaria.

Fig. 2 shows the scheme of the system for watching the storm front. The system consists of several multichannel detectors disposed on the protected area. Distance between detectors and their quantity depends on the precision of needed localization. For example for an area of 200-250 square km four detectors disposed in 15-20 km from each other are enough.

Each detector sends data to the dispatcher that by calculating them determines the location of storm front. This information can be used by many services such as

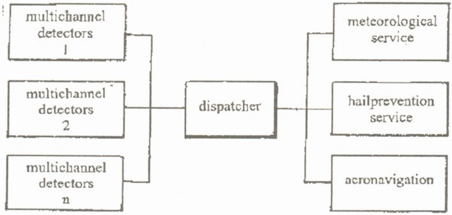

Fig.2. Scheme of the system for watching the storm front

meteorological service, dispatcher service of large energetical systems, hailprevention service, navigation, aeronavigation etc.

Each multichannel detector consists of 5 channels with different sensitivity towards east-west and north-south (Fig. 3). The system reacts to electrical discharges up to 250 kilometers away.

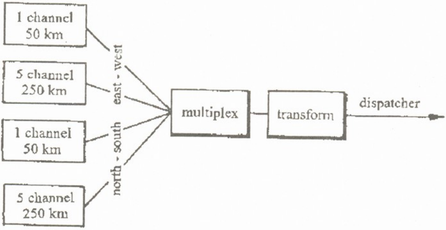

Fig.3. Scheme of the multichannel detector

The scheme of channels of the multichannel detectors is shown on the Figures 4 and 5. It presents one of the various detectors invented by our group [4].

The signal amplified by bicascade selective amplifier after straightening proceeds to the input to the comparator that determines the treshold of sensitivity of this channel. The monostable relaxation scheme excludes influence of feeble return impulses of lightning discharges. Each channel is based on an integral scheme consisting of 4 operational amplifiers with parameters similar to K 140 EL 16. The information accumulated in selection and memory schemes is transmitted to the dispatcher. The latter is communicated to the detectors so that to make them ready for receiving a new information.

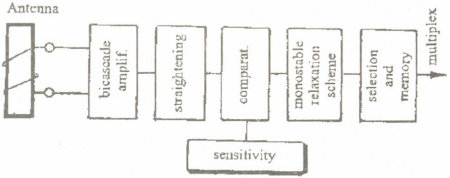

Fig.4. Scheme of the only channel

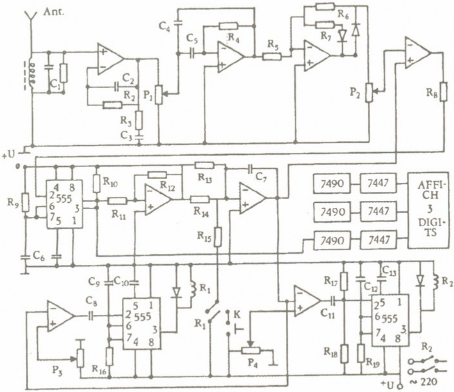

Fig.5

Precision of watching for the movement of the front of the storm depends on different factors. Among them errors caused by straight orientation of antenna, measuring technics and dissimilar group around antenna should be pointed out.

What differs proposed system of localization from other similar system is the fact that our system is able to react to electrical discharges inside stormclouds and detect all the necessary information long before the main lightning strikes the ground.

The errors point out above can be decreased by the proper choice of the places for antennae and their orientation and by excluding influence of external factors on measuring-transmitting circuits.

By increasing the amount of detectors and sensitivity of each channel we can watch movement of stormclouds for considerable distance.

In spite of its seeming immensity the system demands very simple treatment and enables us to change its separate parts without stopping its continuous work.

Institute of Electronics
Academy of Sciences of Bulgaria

Georgian Technical University

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BIOCHEMISTRY

T.Buachidze, L.Topuria

Purification and Partial Characterization of Invertase from Fungal Fusant

Presented by Academician G.Kvesitadze, July 21, 1996

ABSTRACT. **Invertase from fungal fusant was purified by ammonium sulfate fractionation, DEAE-cellulose chromatography and Sephadex G-100 gel filtration. The purified enzyme appeared homogeneous on disc electrophoresis. Its molecular weight was estimated to be 76000 by sodium dodecyl sulfate-polyacrylamide gel electrophoresis and 75000 - by gel filtration. The carbohydrate content of the enzyme equals to 25%. The optimum activity of enzyme was observed at about 50°C and it was stable up to 55°C. Although the enzyme was stable between the pH 4.0 and 5.5 the optimum pH for its activity was about 4.6.**

Invertase (β-fructofuranosidase, EC 3.2.1.26) is widely studied in yeasts and micromycetes [1-4], and comparatively less studied in bacteria [5,6]. As far as we know there have been no reports on the purification of invertase from fungal fusant until now. This paper deals with the purification and some properties of invertase from fungal fusant.

Materials and Methods

Materials. The following materials were used in our work: DEAE-cellulose, DE-52 ("Whatman" U.K.), Sephadex G-100 ("Pharmacia Fine Chemicals" Sweden), Chemicals for electrophoresis ("Reanal" Hungary). Other chemicals used were of analytical grade and obtained from "Reakhim" firm (Russia).

Cell extract. Invertase was extracted from fusant obtained by means of fusion protoplasts from micromycetes - *Aspergillus niger* and *Allesheria terrestris* [7]. The disintegration of fusant biomass was carried out by means of plasmolyze, utilizing toluene according to the modified method of Yun [8]. The mixture was centrifuged to remove cell debris. The resultant solution was referred as the cell extract. This extract was used as crude enzyme solution.

Enzyme assay. The activity of invertase was measured by following release of reducing sugar from sucrose. The reaction mixture consisted of 0.5 ml of 6% sucrose solution, 0.4 ml of 0.05 M acetate buffer solution (pH 4.6) and 0.1 ml of diluted enzyme solution. The reaction was carried out at 50°C for 15 min. The amount of reducing sugar released was determined by the method of Šomogyi [9]. One unit of enzyme activity was defined as the amount of enzyme capable of liberating 1 μM equivalent of glucose in 1 min. Specific activity is expressed as units per mg of protein.

Protein measurement. Protein concentration was measured by the method of Lowry et al. [10] with bovine serum albumin as the standard protein. The protein in eluates column chromatography was monitored by following absorbance at 280 nm.

Purification procedure

Stage 1: Ammonium sulfate. To the cell extract, solid ammonium sulfate was added to give 45% saturation and the pH was adjusted to 4.9. After 3h the resulting precipitate was centrifuged and additional ammonium sulfate was added to the resultant supernatant to give 70% saturation. After 6 h the resultant precipitate was collected, dissolved in a small volume of buffer and ultrafiltrated ("Amicon", PM 30 membrane, USA).

Stage 2: DEAE-cellulose. The ultrafiltrated and concentrated enzyme solution was applied to a column (2.0×29 cm) of DEAE-cellulose equilibrated with the 0.05 M acetic buffer pH 4.6. The elution was carried out with a linear gradient of NaCl (0 to 0.3 M) in 0.5 L of the same buffer. The active fractions were pooled and concentrated by ultrafiltration.

Stage 3: Sephadex G-100. The concentrated enzyme solution was put onto a Sephadex G-100 column (2.0×65) equilibrated with 0.05M acetic buffer, pH 4.6 and eluted with the same buffer. The active fractions were combined and concentrated by ultrafiltration.

Gel electrophoresis. Polyacrylamide disc gel electrophoresis (PAGE) was done in a 12.5% acrylamide gel with 0.05 M Tris - 0.38 M glycine buffer solution at pH 8.3 [11].

Molecular weight determination. The molecular weight of the purified enzyme was estimated by sodium dodecyl sulfate gel electrophoresis [12] and Sephadex G-100 gel chromatography [13].

Estimation of carbohydrate. The carbohydrate content of a purified invertase was determined by the phenol-sulfuric acid method of Dubious et al. [14] using glucose as standard.

Results and Discussion

We have chosen the traditional methods of protein purification, namely, ammonium sulfate fractionation, ion-exchange chromatography and gel filtration. After each stage the method of ultrafiltration was used to concentrate protein and attain the pH in the solution.

We avoided the widely used method of organic solvents (ethanol, acetone, isobutanol) because enzymatic activity is partially lost. That's why we prefered the method of precipitation of protein by ammonium sulfate.

During the DEAE-cellulose column chromatography stage, invertase was recovered in one peak used as the sample for further purification. (Fig.1) After the second gel chromatography with Sephadex G-100, the specific activity of the

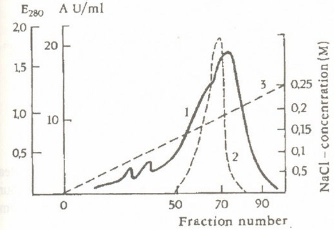

Fig 1. Eluation pattern of invertase from DEAE - cellulose column:
1 – Absorbance at 280 nm.
2 – Invertase activity.
3 – NaCl concentration.

enzyme was about 62-fold that of the crude enzyme extract.

The results of the enzyme purification are summarized in the Table.

Table

Summary of purification of invertase from fusant

Purification stage	protein (mg)	Total activity (unit)	Specific activity (u/mg)	Degree of purification	yield (%)
Cell extract	875.0	4200	4.8	1.0	100
Ultrafiltration	634.0	3950	6.24	1.3	94
$(NH_4)_2SO_4$ fractionation	82.4	2925	35.5	7.4	70
DEAE-cellulose column	7.4	2014	272.2	56.7	48
Gel-filtration G-100	4.2	1250	297.6	62.0	30

The obtained preparation gave a single band on polyacrylamide disc gel electrophoresis (photograph not shown).The molecular weight of the enzyme was estimated to be 75000 by gel chromatography and 76000 by sodium dodecyl sulfate polyacrylamide slab gel electrophoresis (Fig.2). According to literary data, the molecular weight of invertase from *Aspergillus ficuum* invertase equals 84000 [15].

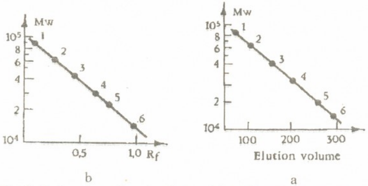

Fig.2. Estimation of molecular weight of invertase by gel filtration on Sephadex G-100 (a) and SDS-polyacrylamide gel electrophoresis (b). The molecular weight of standard proteins are: 1 – Phosphorylase b -94000, 2 – Bovine serum albumin -68000, 3 – Ovalbumin -43000, 4 – DNA ase -31000, 5 – Mioglobin -17000, 6 – Lizocim -14000.

It's generally known, that invertases produced by almost each type of micromycetes are glycoproteins and the carbohydrate content in their molecules is 10-50%. In our case invertase molecule contained 25% of carbohydrate, while invertase from *Aspergillus awamori* contains about 50% of carbohydrates [16].

The effects of pH and temperature on invertase activity were examined. The enzyme activity was measured at various pHs (from 3.5 to 5.5) and temperatures (from 45°C to 70°C). The highest activity was observed between pH 4.0-5.0 and the temperature optimum for enzyme activity was about 50°C.

The enzyme was stable over the pH range 3.5-5.5 and up to 55°C. In addition invertase from fusant was stable at 60°C for 80 min and the remaining activity levels after 30 min at 65° was about 50%.

Thus, invertase isolated and purified from a fungal fusant has molecular weight 76000, carbohydrate content in molecule is 25%, the enzyme is stable for 80 min at 60°C, pH-stability is 3.5-5.5 region.

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Erratum:

"Selection of Optimum Nutrient Medium for β-Galactosidaze Biosynthesis by Bacterium Thermoanaerobacter Sp.2905"

Bull. Georg. Acad. Sci., v. 153, N 3

M.Tsereteli, M.Baramidze, A.Tsereteli

The Institute of Plant Biochemistry of the Georgian Academy of Sciences, Tbilisi, 380059

1. β instead of **B**

2. On page 285, the ninth paragraph contains errors that were introduced during the production of the article. The correct paragraph is reproduced here:

"Protein concentration in culture liquid and in adjustable cell extract was determined by Lowry et al [5]. β-galactosidase activity in culture liquid as well as in cell extract was determined by Kuby and Lardy [6]. The amount of enzyme that hydrolyses 1μ mole ortho-nitrophenil-β-D-galactopyranoside at 70°C, pH-7.0 is adopted as a unit of β-galactosidase activity".

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R.Akhalkatsi, T.Bolotashvili, R.Solomonia, Corr. Member of the Academy N.Aleksidze

Identification of Concanavalin A Binding Proteins of Rat Brain Cellular Nuclei By Gel Electrophoresis and Blotting

Presented March 5, 1996

ABSTRACT. **The existence of concanavalin A-binding proteins has been discovered in rat brain cellular nuclei PBS soluble (180, 150, 125 and 63 kD) and triton X-100 extractable fractions (180, 175, 150, 125, 120, 107, 95, 70, 63, 57 kD) by the method of gel electrophoresis and blotting. A hypothesis is presented, that rat brain nuclear membrane pores modulator is concanavalin A lectin like protein.**

The existence of concanavalin A (Cen A)-binding glycoproteins in the rat brain cell nuclei outer membrane has been supposed [1,2]. This conclusion has been founded on the results of rat brain cell nuclei agglutination by concanavalin A [2]. Our report deals with study of identification and separation of PBS-soluble Con A-binding proteins of the rat brain cell nuclei by gel electrophoresis and blotting.

White rats of both sexes weighing 100-120 g were used as experimental objects. Rat brain cell nuclei were isolated by the method of Chauveau [3]. The purity of nuclear fractions was controlled by microscope. The nuclear suspension was centrifuged and to the nuclear pellet 20 mM potassium-phosphate buffer (pH 5.0) + 0.9% NaCl (PBS) with 5 mM phenylmethylsulphonyl fluoride in the ratio of 1 to 3 was added and homogenized (±4°C). After centrifugation (16000 g/20 min) the PBS soluble protein fraction (supernatant) was dialyzed against PBS (pH 7.4). The pellet, a crude membrane fraction was washed by PBS (pH 5.0) in order to remove fully PBS soluble proteins. Then 0.1% triton X-100 solution, prepared on 40 mM PBS (pH 7.4) was added, homogenized and the suspension was left for extraction (30 min ±4°C). After centrifugation at 6000 g/20 min the supernatant, the membrane protein fraction, was stored in vials at -70°C until use.

To identify the rat brain cell nuclear glycoconjugates, the concanavalin A was used as a lectin, which spesifically interacts with terminal glucose and/or mannose of glycoproteins. The membrane and PBS soluble proteins were dialized against bidistillate and dissolved in 5% SDS. The amount of the protein was determined by the method of Peterson [4]. The electrophoresis was carried out in the 5-12% gradient of polyacrylamide linear gel as described by Laemli [5]. In all experiments 100 μg of proteins were used. The electrophoretic transfer of proteins from the polyacrylamide gel to nitrocellulose filters (the size of pores equals 0.22 microns) was carried out according to Burnette [6]. The filters were incubated in the 10 % solution of bovine serum albumin (BSA) for 60 min. Then in a 50 mM Na-phosphate buffer (pH 7.5), containing ^{125}I -labelled Con A (100 μg/ml, 1×10^6 cpm/min), 200 mM NaCl, 1% BSA, for 2 hours at ±4°C. The filters were washed 5 times, dried and autoradiographed. The X-ray film PM-B was used for autoradiography. Parallel the specificify of the developed strips was controlled by the filter, incubated in the buffer with the same amount of proteins, containing the iodinized lectin and α-methyl-D-mannoside and α-methyl-D-glucoside in the final concentration of 0.2 M, as haptens, competitively blocking the Con A binding centers. Filters were counted in a Packard gamma counter and the specific binding defined as binding in the absence minus binding in the presence of excess (0.2 M) unlabelled haptens.

The calibrating curve was constructed by the set of standard proteins (Sigma). The standart proteins were also subjected to blotting and the electrophoregrams were coloured by Coomassie blue R-250 [6]. All other chemicals used in this study were of the highest purity available.

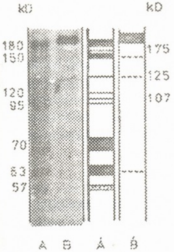

Fig. 1. The radioautograms of PBS-soluble (B) and triton X-100 extractable (A) concanavalin A binding protein and their scheme (B^1, A^1).

As a relust of the carried out experiments Con A-binding 4 glycoproteins (180,150, 125 and 63 kD) were revealed in the rat brain cell nuclei PBS soluble protein fraction (Fig. 1, B, B^1). The results of blotting experiments as a photo of autoradiography (A, B) and their schemes (A^1 , B^1) are represented in Fig. 1. One can see that among those only 180 kD glycoprotein had the highest concentration, the rest of the glycoprotein were revealed as tracks on radioautograms.

The rat brain cell nuclei membrane proteins, extracted by triton X - 100 , unlike the PBS soluble protein fractions, revealed a great capacity in ^{125}I-concanavalin A binding (Fig. 1 A, A^1). The molecular weights of Con A - positive glycoproteins varied from 57 to 180 kD, their common amount equals to 10. Three proteins with the molecular weight of 180, 70 and 63 kD are more clearly seen on the autoradiogram, seven of them belong to minor glycoconjugates (175,150, 125 107, 95 and 57 kD). It must be mentioned that all the four PBS soluble glycoconjugates were discovered in the membrane protein fraction extracted by triton X - 100. According to the literature Con A-binding glycoprotein with the molecular weight of 180 kD is discovered in the rat nuclei of liver [7]. It was proved that 180 kD glycoprotein is nuclear membrane pores protein, whose carbohydrate part is oriented inside cisternal space.

Con A -positive glycoprotein with the molecular weight of 63 kD was identified in the nuclear membrane pores by Davis and Blobel [8]., and later in the rat liver nuclei (63 - 65 kD) [9]. It was established that this 63 -65 kD protein is also a part of the nuclear membrane pores protein and plays an essential role in active transport mechanisms particularly the blocking of the nuclear pores 63 -65 kD glycoprotein with wheat germ agglutinin (WGA) inhibits the nuclear protein transport [9]. It is necessery to emphasize that the rats; intact liver nuclei are stained with FITC-WGA conjugates, while the damaged nuclei - with FITC-Con-A. These results suggest that the WGA-binding GlcNac glycoconjugate residues are localized on the external part of the nucleus membrane. This gives us possibility to employ a lectin column chromatography and preparative electrophoresis for the separation of rat brain cell nuclei membrane 63 kD protein to get further support for defining their functional role in the brain cell nuclei membrane activity. Fortunately our results represented in Fig. 1, assure us of perspectivity of the current research. In contrast with these findings, neither membrane bound, nor PBS soluble protein fractions of rat brain cell nuclei were able to bind WGA [1, 2, 10-15]. Only the slight binding trace of WGA has been discovered by us in the nuclei membrane protein fractions extracted by triton X-100. It must be noted that this ability was lost at the level of separate proteins obtained after triton X-100 extract fractionation on the column Protein PAK-300-SW [11,15].

With respect to the above-mentioned it is probably possible to conclude that Con A, but not WGA, may play a critical role in brain nuclear membrane pores modulation. The research is in progress to identify Con A like lectin as rat brain cell nuclear membrane pores modulator.

I.Javakhishvili Tbilisi State University

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BIOCHEMISTRY

I.Chkhikvishvili, A.Shalashvili, G.Papunidze, I.Targamadze, T.Meladze

Polymethoxylated Flavones and Flavanones of Citrus Flowers

Presented by Corr. Member of the Academy N.Nutsubidze, June 13, 1996

ABSTRACT. **Polymethoxylated flavones and flavanones have been investigated in Citrus flowers. It was revealed that the flowers of orange, mandarin, lemon and grapefruit contain polymethoxylated flavones: 5-OH-6, 7, 8, 3', 4' - pentamethoxyflavone, tangeretin, tetra-O-methylscutelarein, 3, 5, 6, 7, 8, 3', 4' - heptamethoxyflavone, nobiletin, sinensetin and flavanones: didimin, poncirin, hesperedin, neohesperedin, narirutin, naringin, eriocitrin and neoeriocitrin.**

One of the main peculiarities of citrus plants is high content of flavonoid compounds and their qualitative diversity [1]. At the same time it should be mentioned that chemical composition of flavonoids in citrus fruits and leaves was investigated, but in other parts the subject was less studied. Citrus flowers are of great interest as they can be used for production of nonalcoholic soft drinks [2]. It is known that citrus plants intensively blossom and a great number of flowers fell down [3]. Thus the goal of our work is to study chemical composition of polymethoxylated flavones and flavanones of some citrus flower flavonoid compounds of the varieties cultivated in Georgia. According to the existing data polymethoxylated flavones are characterised by fungistatic activity [4], but some of them (tangeretin, nobiletin) activise monooxigenase system of human liver at detoxication of carcinogenic compounds [5]. Citrus flavanones play a great role in technology of citrus processing and possess pharmacological activity [1].

Flowers of lemon "Mayer" (Citrus limonia Osbeck), lemon "Kartuli" (Citrus limon Burm), mandarin "Unshiu" (c. unshiu Marc.), orange of the variety Adgilobrivi (c. sinensis Osbeck) and grapefruit, variety Dunkan (c. paradist Mcf.) were gathered in Batumi at the experimental field of Georgian Canning industry of Research and Educational Institute.

In order to study polymethoxylated flavones 50 g of flowers of lemon "Mayer", mandarin and orange fixed by 150 ml ethanol on boiling bath, were taken. Ethanol was evaporated after filtration in a rotating evaporator. Polymethaxylated flavon extraction was carried out from resulted water solution through separating funnel 5 times with 20 - 20 ml of benzol. After ethanol extraction the remained flowers were dried in absorbing chamber. The extraction of polymethoxylated flavones from the residue was conducted by benzene on a boiling bath 3 times (150 - 150 ml). Benzene extracts were collected and evaporated to dry redisue that was dissolved in the system: hexan-isopropanol (70 : 30). Qualitative composition of polymethoxylated flavones was stated by the method of two dimensional thin-layer chromatography ("silufol" plate, 1 direction: chloroform - methanol 99 : 1, 2 direction: hexan - isopropanol (70 : 30). As an authentic samples we used polymethoxylated flavones, from mandarin Unshiu fruit skin (6). Quantitative content of flower polymethoxylated flavones was determined by high pressure liquid chromatographer (Laboratorni pristoje, Prague, Column SGX-5 m, 3×150 mm); elution was carried in a solvent system, at a rate 400 l/min, sensitivity 0.2, pressure 180mpa, detection 280 nm. Calibration curve was constructed against nobiletine [7,8] (Fig.1 a, b).

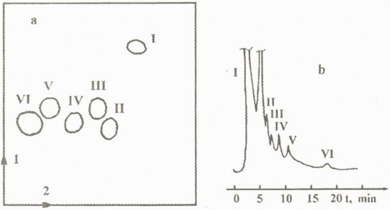

Fig.1. Separation of polymethoxylated flavones by thin layer (a) and high pressure liquid (b) chromatography methods in orange flowers.
I. 5-oxy-6, 7, 8, 3', 4' - pentamethoxyflavone, II. 5, 6, 7, 8, 4', - pentamethoxyflavone (tangeretin), III. 5, 6, 7, 4' - tetramethoxyflavone, IV. 3, 5, 6, 7, 8, 3', 4' - peptamethoxyflavone, V. 5, 6, 7, 8, 3', 4' - methoxyflavone (nobiletin), VI. 5, 6, 7, 3', 4' - pentamethoxyflavone (sinensetin).

According to the received data (Fig.1) it was found that the qualitative composition of polymethoxylated flavones of lemon "Mayer", mandarin "Unshiu" and orange "Adgilobrivi"flowers was analogous and they contained: 5-oxy-6, 7, 8, 3', 4' - pentamethooxyflavone [1], 5, 6, 7, 8, 4' - pentamethooxyflavone (tangeretin, II), 5, 6, 7, 4' - tetramethoxyflavone (tetra - O - methylscutelatein III), 3, 5, 6, 7, 8, 3', 4' - heptamethoxyflavone (IV), 5, 6, 7, 8, 3, 4 - hexamethoxyflavone (nobilitin V) and 5, 6, 7, 8, 3', 4' - pentamethoxyflavone (sinensetin VI). Proceeding from quantitative composition of polymethoxylated flavones (Tabl. 1) it should be stated that the flowers of orange "Adgilobrivi" contain twice more of these compounds, than lemon "Mayer" and mandarin "Unshiu". From separate polymethaxylated flavones in all these three plant flowers predominates 5-oxy-6, 7, 8, 3', 4' - pentamethoxyflavone, then comes tangeretin and 3, 5, 6, 7, 8, 3', 4' - hentamethoxyflavone and the rest three compounds are presented in comperatively less quantities.

To study flavanones from flowers of lemon "Mayer" lemon "Kartuli", mandarin, orange and grapefruit the analysed flower samples were fixed at 60°C by drying.

Table 1

Quantitative composition of polymethoxylated flavones in citrus flowers (mkg/g of fresh weight)

№	Polymethoxylated flavones	Orange "Adgilobrivi"	Mandarin "Unshiu"	Lemon "Mayer"
1.	5-oxy-6, 7, 8, 3', 4' - pentamethoxyflavone	299	185	190
2.	5, 6, 7, 8, 4' - pentamethoxyflavone	66	33	51
3.	5, 6, 7, 4' - tetramethoxyflavone	29	3	2
4.	3, 5, 6, 7, 8, 3', 4' - heptamethoxyflavone	45	16	4
5.	5, 6, 7, 8, 3', 4' - hexamethoxyflavone	32	14	17
6.	5, 6, 7, 3', 4' - pentamethoxyflavone	18	2	2

Flavanone extraction was carried out from 5 g of dissected material by 80% methanol on boiling bath. Extracts were collected, filtrated, evaporated and diluted with methanol. Qualitative composition extracted from flowers flavanones was determined by the method of high effective thin-layer chromatography; solvent system: nitromethan - methanol (5 : 2), 5 - 7 minutes. Chromatogram exposition was carried by 2% borhydride methanol solution in hydro-chloric acid medium. As flavone authentic samples were used hesperedin (Gee Lauson, Chemicals LTD, England), neohesperedin (Hoffman - La - Roche, Switserland), narirutin, isolated from fruit pulp of mandarin Unshui [10], naringin (Loba - Chemie, Austria) and eryocitrin, received from lemon "Dioscuria" fruit skin [11]. It should be stated that citrus plants contain two types of flavones: flavanonrutinosides, that have no bitter taste and flavanonneohesperidosides with rather bitter taste.

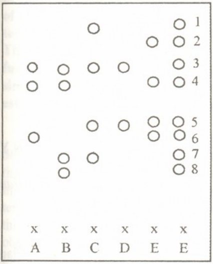

Fig.2. Flavanone separation by the method of polyamid high effective thin-layer chromatography in citrus flowers.
A - lemon "Mayer", B - lemon "Kartuli", C - mandarin "Unshiu", D - orange "Adgilobrivi", E - grapefruit "Dunkan", F - authentic samples: 1 - didimin, 2 - porcirin, 3 - hesperidin, 4 - neohesperidin, 5 - narirutin, 6 - naringin, 7 - eriocitrin, 8 - neoeriocitrin

Proceedings from the received data (Fig.2, Tabl.2) lemon "Mayer", Georgian variety of lemon, grapefruit, "Duncan" flowers contain rutinosides as well as neohesperidosides. Two neohesperidosides: neohesperidin (hesperetin - 7 - O - neohesperidoside) and naringin (naringenin - 7 - O - neohesperidoside) and one rutinoside hesperedin (hesperetin - 7 - O - rutinoside) were found in lemon "Mayer". In lemon "Kartuli" rutinosides were represented by hesperedin and eriocitrin (eriodictiol - 7 - O - rutinoside), but neohesperidosides were represented by neohesperidin and neoeriocitrin (eriodictiol - 7 - O - neohesperidoside), grapefruit flowers contain neohesperidosides: neohesperidosides:neohesperidin, naringin poncitin (isosacuranetin - 7 - O - neohesperidoside) and one rutinoside - naritutin (naringenin - 7 - O - rutinoside). In contrast to three abovementioned plants the flowers of mandarin Unshiu and orange

"Adgilobrivi" contain only flavanontrutinosides and the received soft drink has no bitter taste. Flowers of mandarine "Unshiu" contain hesperedin, narirutin, eryocitrin and didimin (isosacuranetin - 7 - O - rutinoside) and orange flowers of "Adgilobrivi" - hesperedin and narirutin. Thus citrus flowers contain biologically active polymethoxylated flavones and flavanones and can be used as the source of these compounds.

Table 2

Flavanone qualitative composition of citrus flowers

Flavanones	Lemon "Mayer"	Lemon "Kartuli"	Mandarin "Unshiu"	Orange "Adgilobrivi"	Grapefruit "Duncan"
1. Didimin			+		
2. Poncirin					+
3. Hesperidin	+	+	+	+	
4. Neohesperidin	+	+			+
5. Narirutin			+	+	
6. Naringin	+				+
7. Eriocitrin		+	+		
8. Neoeriocitrin		+			

S.Durmishidze Institute of Plant Biochemistry
Georgian Academy of Sciences

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ECOLOGY

Academician K.Nadareishvili, H.Grebenchuk

Questions of Linear Forecasting in Ecology and their Decision by Methods of Decomposition on Double Orthogonal Systems of Functions

Presented October 1, 1996

ABSTRACT. **The paper furnishes for linear forecasting and retrospective analysis of the state of technogenic fields at the decision of problems of ecodynamics.**

Shown, that, an unsoluble situation, as it were, when the forecast of parameters of technogenic fields cannot give reliable results, the opportunity of construction of the mathematical theory, allowing at certain restrictions of the character of temporary series to build the correct forecasts and to make retrospective analysis. This opportunity is associated with the methods of decomposition of finite functions on double orthogonal basis. Thus, if the basis is orthogonal simultaneously on a final piece and on the whole axis, it is possible to build estimations in past and in future with any beforehand given error.

Linear forecasting and retrospective analysis of the state of technogenic fields at the decision of problems of ecodynamics are of utmost interest, since they allow to judge the tendencies of their development and to estimate their possible consequences. Thus initial material for the construction of such estimations is only small on the extent of temporary series of the observed information, which, cannot probably allow to solve this problem with a sufficient degree of reliability. Insignificant volume of the observed information, which can be used for the decision of these problems, is due to the fact that theoretical ecology is still in a stage of formation, and also to the circumstance, that the mankind has only recently realised danger to the nature, resulting from tecnogenic activity.

Nevertheless, an unsoluble situation, as it were, when the forecast of parameters of technogenic fields cannot give reliable results, the opportunity of construction of the mathematical theory, allowing at certain restrictions of the character of temporary series to build the correct forecasts and to make retrospective analysis. This opportunity is associated with the methods of decomposition of finite functions on double orthogonal basis. Thus, if the basis is orthogonal simultaneously on a final piece and on the whole axis, it is possible to build estimations in past and in future with any beforehand given error.

Considering the observations on the parameters of technogenic fields as the measuring process, it is possible to write down for the equation of convolution

$$f(y) = \int_{-\infty}^{+\infty} F(x)\, h(y-x)dx \qquad (1)$$

Where $F(x)$ is true course of parameters, $f(y)$ is the observed course of the same parameters, and $h(y-x)$ is pulsing characteristic of the measuring system.

Studying the equation (1), it is necessary to consider an opportunity of a choice of such decisions, which are correct. If one limits a class of inputs to some conditions of uniformity, it is possible to define value η^*, not dependent on individual properties of each of plausible inputs and satisfying the ratio

$$h^*(\varepsilon, h) \geq h(\varepsilon, h, F),\ \eta^* \to 0 \text{ where } \varepsilon \to 0. \tag{2}$$

In this case it is natural to speak about uniform correctness of the equal decision (1) in a considered class of inputs. It is important to note, that it is directly due to differential properties of inputs: the more abruptly function $F(x)$ can change, i.e. the thinner the structure of an input, the greater is the error.

Proceeding from this fact, we shall hereinafter consider the forecast for a class of inputs, at which complexity changes. Convenient from both practical, and computing point of view are the classes with final number of degrees of freedom. A natural way here will be consideration of more and more complicated classes, where the complexity is understood as a subtlety of their differential structure. Such classes are built on the basis of the widely used approach to the presentation of functions as series of the orthogonal system and consideration of their partial sums. It means, that as space, on which integral operator is set, finite dimension space is chosen and then the chain of extending spaces of increasing dimension is considered. Practically, certainly, suffice it to consider finite dimensionality of spaces, whose dimensions are to be defined by desirable complexity of allowable inputs and the more the complexity we are interested in, the more is their dimension.

Let $\{Y_k(x)\}$ is some fixed system of functions, complete on an interval $(-\alpha, \alpha)$, and let $\Phi_N = \{\Phi_N(x)\}$ class of functions, represented as

$$F_N(x) = \sum_{0}^{N} \alpha_k \Psi_k(x) \tag{3}$$

The functions $Y_k(x)$ are numbered so, that with the growth of k they become more and more complex. Therefore the highest possible value of variable $F_N^I(x)$ grows with the growth of N so, that there are the more and more complex inputs. For the considered class we shall define the maxi-mum error of deviation $\eta(\varepsilon, N, h)$, achievable for the "worst" function $F_N(x)$. This error will tend to zero together with ε for any fixed N.

Let us consider an elementary, but nevertheless practically the most interesting situation, when the input $F(x)$ is measured at a finite interval $(-\alpha,\alpha)$ with an error, characterized by mean square value, i.e. instead of $F(x)$ as a result of measurement we receive the function $F_\varepsilon(x)$, and it is known, that the error is limited by the value σ_α:

$$\int_{-\alpha}^{\alpha} \left|F_\varepsilon(x) - F(x)\right|^2 dx \leq \sigma_\alpha^2 . \tag{4}$$

According to the general statement of the problem, mentioned above, we shall take as possible inputs Φ_N functions of a kind

$$F_N(x) = \sum_{0}^{N} \alpha_k \Psi_k(x/\alpha, c) \quad (-\alpha \leq x \leq \alpha) \tag{5}$$

where $\Psi_k(z, c)$ are functions, satisfying the integral equation

$$\lambda_k \Psi_k(z, c) = \int_{-1}^{1} \left\{ \sin c(x-z) \Big/ \pi(x-z) \right\} \Psi_k(x, c) dx, \qquad (6)$$

$\lambda_k = \lambda_k(c)$ are eigen values, and function $\Psi_k(x/\alpha, c)$ are orthogonal [1] in the interval [−1, 1], so Φ_N it is N-is n-time space, tense on $\Psi_1, \ldots \Psi_N$. The function $\underline{F}(\omega)$ - the Fourier-image $F_N(x)$ is

$$\underline{F}_N(\omega) = \sum_{0}^{N} \beta_k \Psi_k\ (\omega/\Omega, c) \text{ where } \beta_k = \lambda_k \alpha_k \sqrt{2\pi\lambda_k} \qquad (7)$$

As soon as a class of allowable inputs of space Φ_N is chosen, then the functions $F_\varepsilon(x)$, received as a result of measurements in view of errors, should also naturally be attributed to this class. Really, if $F_\varepsilon(x)$ does not belong to Φ_N, then "extra" components, which are not contained in representation of type (5), should be discarded, filtered, as obviously insignificantones. In the geometrical terms the whole described situation looks as follows. In a Hilbert space of functions, integrated in a square in the interval $-\alpha < x < \alpha$, is considered as finite subspace of functions, admitting representation of (7). If the function $F_N(x)$ does not lie in the subspace Φ_N, then it is necessary instead of it to consider its projection to the given subspace. Now a problem of definition of a an input $F_N(x)$, i.e., final calculation of optimum coefficients β_k, is solved by the best mean-square approximation of given function by a final sum (7). Numbers β_k are Fourer coefficients of $\underline{F}_\varepsilon(\omega)$ on system $\{\Psi_k(\omega/\Omega, c)\}$:

$$\beta_k = \int_{-\Omega}^{\Omega} \underline{F}_\varepsilon(\omega)\, \Psi_k(\omega/\Omega, c) d\omega = \Omega \int_{-1}^{1} \underline{F}_\varepsilon(z\Omega)\, \Psi_k(z, c) dz. \qquad (8)$$

In other words, β_k are values of components of the projection of measured function $\underline{F}_\varepsilon(\omega)$ on a basic vector Ψ_k in subspace Φ_N. After calculation of β_k and α_k the sought meanings of an input at $|x| > \alpha$ are from (5); thus analytical continuation of functions $\Psi_k(x/\alpha, c)$ is built on the whole real axis.

We shall consider now the question of accuracy, with which meanings of an input are defined at $|x| > \alpha$ with the help of the described procedure. We shall calculate greatest possible for the given class Φ_N meaning of mean square deviation of a continued input $F_\eta(x)$ from true function $F(x)$ at a given meaning of an allowable error σ_α^2 of measurement in the interval $[-\alpha, \alpha]$. For this purpose one should find among all $F_N(x) \in F_N$ such a function of $F_N^0(x)$, for which the value

$$\sigma_\alpha^2 = \int_{-\alpha}^{+\alpha} |F_N(x)|^2\, dx \text{ is given, and the value } \sigma^2 = \int_{-\infty}^{\infty} |F_N(x)|^2\, dx \text{ is maximum.}$$

Using double orthogonality of system $\Psi_k(z)$ and Parseval's equation, we shall receive from (5)

$$\sigma_\alpha^2 = \alpha \sum_0^N (\alpha_k)^2 \lambda_k,\ \sigma^2 = \alpha \sum_0^N \alpha_k \qquad (9)$$

As numbers λ_k decrease with the growth of k, the value of s is maximum at fixed σ_α^2, if

$$\alpha_N = \sigma_\alpha / \sqrt{\alpha}\ ,\ \alpha_0 = \alpha_1 = \dots\ \alpha_{N-1} = 0 \qquad (10)$$

So that

$$F_N^0\ (x/\alpha) = (\sigma_\alpha / \sqrt{\alpha}\ )\ \Psi_k(x/\alpha,\ c) \qquad (11)$$

$$\sigma^2 = \sigma_\alpha^2 / \lambda_N \qquad (12)$$

It is noticable, that mean square deviation of a true input from approximated, according to the Parseval's equation, coincides with σ^2:

$$\int_{-\alpha}^{\alpha} \left|F(x) - F_\eta(x)\right|^2 dx = \int_{-\infty}^{\infty} \left|\underline{F}_\varepsilon(\omega) - \underline{F}(\omega)\right|^2 d\omega = \sigma^2 \qquad (13)$$

So that the value σ^2 yields simultaneously the value of the maximum error of extrapolation. Numbers $\lambda_N(c)$, as well as the functions $\Psi_k(z,\ c)$, depend on a single parameter c. At small c numbers λ_N quickly decrease with an increase of N.

Thus, considering the fact that is far from trivial and profound: double orthogonality of basic functions in (5), it is possible rather readily and in an obvious way to calculate an error of extrapolation.

Georgian Academy of Sciences
Scientific Centre of Radiobiology
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EXPERIMENTAL MORPHOLOGY

G.Gogichadze, T.Dolidze, T.Gogichadze

Possible Reason for Tumour Cells Appearance in Hemolytic Anemias of Different Origin

Presented by Academician N.Javakhishvili, June 10, 1996

ABSTRACT. **At hemolytic anemias of different genesis side by side with hemolysis process of somatic cells fusion may take place with formation of tumour cell. Some agents may induce both fusion process and destructive effects in somatic cells by induction on plasma membranes pores of different size.**

There are hemolytic anemias of different origin. For instance, these states may be induced by some exogenic hemolytic factors: different organic and non-organic hemolytic toxins (phosphorus, phenylhydrazin, saponins, arsenicum, lead) and biotoxins (snaky venom, mushroom poisons, fungus toxins, etc.), by some medicinal preparations, radiation, by some infectious agents and by heavy burns. Besides in some cases hemolytic anemias are induced by antibodies against own tissues (autoimmune hemolytic anemia). Reason of immunization at autoimmune hemolytic anemias may be infection diseases (grippe, malaria, acute anaerobic or streptococcal sepsises, pneumonias) and some other physical and chemical factors and effects. A strong relationship exists between autoimmunity and B-cell oncogenesis. According to clinical researches malignant tumours in autoimmune hemolytic anemias in 45-47%. Observation of literature sources, as well as our experience permits to suggest that quite frequently tumour calls in autoimmune hemolytic anemias have lymphoid and macrophagal nature [1].

At the same time by the opinion of some scientists [2,3,4], some toxins, even different infectious viruses (for instance, viruses of the grippe, Rubella and human immunodeficiency virus) and carcinogenic agents may be induced both by cytolytic (destructive) effects and fusion process in somatic cells.

Such different effect of these agents on somatic cells possibly depend on size of plasma membranes pores induced by them. In case of big size of the pores irreversible changes and cytolysis take place. For instance, high dose of carcinogenic agents leads to partial increase of dikaryons and polynuclear cells, but further increase of this dose induce cellular lysis. In case of this agents tropism to immunocompetent cells may induce immunodeficite of different degree [4]. In low doses of carcinogens dikaryons are most frequently observed.

Our goal is to explain the cellular mechanism of malignant tumours development at hemolytic anemias. We have carried out electron microscopical research of 25 patients blood with autoimmune hemolytic anemias. Cells were fixed in 1% glutaraldehyde and 1% osmium tetroxide and infiltrated with epoxy resin. Thin sections were stained with uranyl acetate and lead citrate and searched visually with a TESLA BS-500 electron microscope at instrumental magnification 3000-25000.

In peripheral blood of all patients with hemolytic anemias side by side with erythrocytes of normal morphology different stages of these cells destruction were observed and ghosts of erythrocytes and membrane fragments of these cells were revealed. In 16 cases we have found bi- and multinuclear cellular formations which often contained at least one nucleus of lymphocytes and macrophages. Such calls are named heterokaryons. More often cellular structures with homogeneous nuclei were seen (homokaryons) (Fig.1). This testifies possibility of fusion of above-mentioned immunocompetent cells with other ones and each other as well. Due to karyogamic theory of carcinogenesis fusion of somatic cells in certain conditions malignant tumours may be developed.

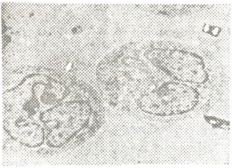

Fig.1. Peripheral blood of patient with autoimmune hemolytic anemia. Heterokaryon (→) and homokaryon (→). X 300

As leucocytes cellular membranes rigidity is more higher, it is possible that at destruction of erythrocytes (or immature cells of this line) by some agents (carcinogens, some toxins, infectious viruses, antibiotics, etc.), in leucocytes plasma membranes and pores of definite size are damaged promoting process of fusion of somatic cells. Perforations of more big size induce considerable destroy of cells plasma membranes and cytolyse (destruction) together with the perishing of this cells is followed.

Thus, at hemolytic anemias of different genesis side by side with hemolysis, process of somatic cells fusion with formation of tumour cells [5] may take place. For instance, after biting of snake with hemolytic action of venom(venom of *Vipera lebetina* and *Vipera russellii*) together with massive destruction of erythrocytes, fusion process of other cellular types with more rigid plasma membranes may be induced (for instance, leucocytes of different maturity, and probable cells of other type), with possible development of true malignant cell. Approximately such action may expect from fungus toxins (*Aspergillus flavus, Penicillum islandicum* and *Aspergillus ochraceus*) and so on. For instance, Aspergillus flavus toxin, aflatoxin induce malignant tumours of liver together with heavy toxic effect.

At autoimmune hemolytic anemias there are two possibilities of arising of malignant process: 1. After destructive action of autoimmune antibodies and immunocompetent cells on erythrocytes. It is also possible their fusogenic influence on this and other, cells, with possible development of tumour cell; 2. Some immunosupressive drugs used at autoimmune states, considerably increased risk of malignant tumours arising in particular, non-Hodgkin's lymphoma(B-cell malignancy), in resemblance of cancer of other kind. Besides, immunosupressive drugs such as cyclosporin A, chlorambucil and above all cyclophosphamide strongly increase occurence of chromosomal abnormalities in patients with connective tissue diseases. It is possible, that these substances together with cytolysis of immunocompetent cells, may induce process of fusion with formation of tumour cell.

Supposing that lymphocytes and macrophages are phenotypically dominant cells their fusion with each other and with another somatic cells may lead to tumour formation with lymphoid and macrophagal nature.

Consequently, fusion of immunocompetent cells with each other or with another ones may be regarded as possible cellular mechanism of malignant tumours formation in hemolytic anemias of different origin.

Research Center of Traumatology and Orthopedy

Research Institute of Hematology and Blood Transfusion

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EXPERIMENTAL MEDICINE

N.Okujava, Corr.Member of Academy V.Bakhutashvili, B.Korsantia

The Phagocytosis System in Patients with Destructive Pulmonary Tuberculosis and the Effectiveness of Plaferon in Stimulating of Phagocytic Activity

Presented July 19,1996

ABSTRACT. **In patients with destructive pulmonary tuberculosis the phagocytosis system was studied.Significant depression of activity of phagocytosis system in those patients was determined. The most pronounced shifts of phagocytic indices were noted in patients with fibro-cavitary tuberculosis. The rise of absorption and intracellular digestion capacity of neutrophils correlated with the severity of tuberculous process. The absence of dynamics of phagocytic indices was a significant practical value and pointed to the necessity of the use of immunomodulators along with antituberculous chemotherapy.**

Plaferon caused the positive dynamics of phagocytic indices.

Since the observation of Metchnikoff in 1891, it has been known that phagocytic cellular response is essential for competent host bacterial defence. The function of the phagocytic system depends not only on adequate number of cells and humoral factors but also on normal function of the phagocytic cells [1-7]. But till now many aspects of phagocytosis of this effective phase of struggle against M. tuberculosis are not well known.

The goal of our investigation was the to study of phagocytosis system in patients with various forms of destructive pulmonary tuberculosis and evaluation of the stimulating effect of immunomodulator plaferon on phagocytic activity.

264 patients with various forms of destructive pulmonary tuberculosis were studied. The patients were divided into two groups: 1st -130 patients receiving plaferon along with etiotropic therapy. Both groups were similar according to the age, sex, structure of clinical forms, incidences of newly detected patients and those who had been treated earlier.

Three main indices of phagocytosis system were studied: phagocyte count (phc.), phagocytic index (ph.ind.) and intracellular digestion of phagocytized bacteria (ph.dig.). Phagocytic activity of neutrophils in the peripheral blood of the patients according to Kost E.A. and Stepko M.I. [8] was determined. After the contact with staphylococcic standard cultures 100 neutrophils with engulfed mycobacteria in blood smears were counted (phagocyte count). The engulfed mycobacteria were counted and the average number of them per one neutrouphil was determined (phagocytic index). Neutrophil capacity for intracellular digestion of micobacteria tuberculosis was also studied.

The patients were examined in dynamics: on their admission, after two months of treatment and on their discharge.

Phagocytic indices in patients treated with tuberculostatics alone in regard to clinical form and results of treatment are presented in table 1. In *A* group were included patients with positive results of treatment and in *B* group - patients in whom efficiency of therapy was poor.

Table 1

The Phagocytosis system in patients with destructive pulmonary tuberculosis in dynamics

Clinical form of tuberculosis	number	Phagocytic indices	Admission	Treatment		Discharge	
				A	B	A	B
Fibro-cavitary	15	Ph count %	60,4±5,2 "	-	66,3±5,9	-	68,1±5,6
		Ph index	2,1±0,1"	-	3,0±0,1	-	3,1±0,2
		Ph dig.%	48,3±4,3"	-	51,4±4,7	--	53,3±4,1
Disseminated	30	Ph count %	68,4±5,4	69,3±5,4	70,1±5,9	70,9±6,5	68,0±5,8
		Ph index	2,7±0,1"	3,3±0,2	3,1±0,2	4,9±0,3	3,5±0,3
		Ph dig.%	53,3±4,7"	60,3±5,7	55,5±4,8	72,3±6,5	63,4±5,7
Infiltrative	45	Ph count %	69,1±5,5	71,1±6,8	68,2±6,0	70,3±7,0	69,5±6,5
		Ph index	3,0±0,1"	3,8±0,2	3,2±0,1	5,1±0,3""	3,4±0,3
		Ph dig.%	56,4±5,5"	68,3±6,0""	55,9±4,4	71,4±6,7""	58,6±5,1
Focal	40	Ph count %	68,8±5,7	70,5±6,6	67,8±5,7	70,5±6,3	68,2±6,6
		Ph index	2,9±0,1"	4,,1±0,2""	3,4±0,1	5,5±0,4"	3,4±0,3
		Ph dig.%	58,8±4,3"	69,3±5,8""	59,7±4,7	73,6±6,3"	60,7±5,9

Control group:Ph count-71,5%; Ph index-5,8; Ph dig. 75,7 %

(") The difference compared with control values is statistically significant.

("")The difference compared with initial values is statistically significant.

Table 2

Phagocytic indices in patients with destructive pulmonary tuberculosis treated with plaferon along with antituberculous therapy

Clinical form of tuberculosis	number	Phagocytic indices	Admission	Treatment		Discharge	
				A	B	A	B
Fibro-cavitary	19	Ph. count % Ph. index Ph. dig.%	60,4±5,2 " 2,1±0,1" 48,3±4,3"	- - -	63,8±5,7 2,8±0,1 52,1±4,9	- - --	65,4±6,1 3,4±0,2 55,3±4,9
Disseminated	30	Ph. count % Ph. index Ph. dig.%	68,4±5,4 2,7±0,1" 53,3±4,7"	68,9±5,2 4,5±0,2" 65,8±5,7"	69,5±6,0 3,4±0,1 56,7±4,4	70,1±5,8 5,1±0,3" 73,3±5,7"	69,5±6,6 3,8±0,1 65,1±6,9
Infiltrative	45	Ph. count % Ph. index Ph. dig.%	69,1±5,5 3,0±0,1" 56,4±5,5"	70,4±6,1 4,5±0,2" 72,2±5,9"	68,7±5,4 3,5±0,1 58,5±5,7	71,1±5,8 5,5±0,3" 74,4±5,6"	70,2±7,2 3,3±0,1 57,2±4,4
Focal	40	Ph. count % Ph. index Ph. dig.%	68,8±5,7 2,9±0,1 58,8±4,3	69,9±6,3 4,8±0,2" 75,2±6,5"	70,1±6,2 3,3±0,1 63,3±5,8	70,9±6,2 5,9±0,4 78,4±6,8	69,4±5,7 3,1±0,2 62,3±5,1

Control group: Ph.count-72,8%; Ph.index-6,8; Ph.dig.-79,7 %

Analysis of data showed that tuberculosis infection insignificantly influences the number of phagocytic neutrophils (exception has been noted only in the case of fibro-cavitary tuberculosis). Phagocytic count was reduced insignificantly (P<0,5). In contrast with that sharp reduction of phagocytic index and intracellular digestion of mycobacteria tuberculosis were revealed. Especially serious disturbances of phagocytosis were noted in patients with fibro-cavitary tuberculosis.

In cases with favouable course of the process, basicly referring to phagocytic index and digestion capacity of neutrophils, gradual activation of phagocytosis was observed. Pool of active cells remained on the same level. It is necessary to note that only in the case of focal pulmonary tuberculosis phagocytic index authentically rised - 5.5±0.4%, *P*<0.05 compared with initial indices - 2.9±0.1%. In cases with unfavourable course or inefficient treatment of the process significant dynamics of phagocytic indices has not been revealed.

Thus, on the basis of the obtained results it was possible to establish that phagocytic indices did not rapidly react on the changes of the patients clinical state. Long - term chemotherapy with the certain clinical effect contributes to the correction of phagocytosis defects. However, the correction of its indices were not marked in all cases, it was usually partial. Reduction of phagocytic activity as a matter of fact associated with the lack or absence of the clinical effect necessitates the use of the additional methods to influence the patients' organism, in particular with the help of immunomodulators. To restore the defects of phagocytosis mechanisms immunomodulator plaferon was used. The efficiency of this preparation in the treatment of tuberculous infection in combination with antituberculous chemotherapy was confirmed by our previous experimental investigations [8].

In patients with destructive pulmonary tuberculosis plaferon was used intramuscularly 3000 IUt in 2 ml of solution two times daily during 20 days. The results of investigation are presented in table 2.

Analysis of phagocytic indices demonstrated the rise of absorption and digestion capacity of active neutrophils, but authentically significant only in the patients of *A* group. Plaferon had no pronounced stimulating effect on the count of active phagocites. Percentage of the letters in all cases of different forms of destructive tuberculosis, in all periods of investigation regardless of the results of treatment and the use of plaferon remained at the same level.

In patients with focal and infiltrative pulmonary tuberculosis plaferon resulted in authentic rise of phagocytic index.

Thus, the results of our investigation showed significant depression of phagocytosis in patients with destructive pulmonary tuberculosis.

In patients with fibro-cavitary tuberculosis the most pronounced disturbances of phagocytosis system were revealed.

The rise of absorption and intracellular digestion capacity of neutrophils correlated with the severity of tuberculous process.

Plaferon aroused positive dynamics of phagocytic indices.

Institute of Medical Biotechnology
Georgian Academy of Sciences.

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N.Tsintsadze

HLA - Markers During Glaucoma and Cataract Diseases in Georgian Population

Presented by Corr. Member of the Academy N.Tatishvili, April 11, 1996

ABSTRACT. **Persons of Georgian nationality were examined: 69 patients having glaucoma disease and 73 patients with cataract.**

It was determined that the genetic markers of predisposition to the glaucoma disease are antigenes A1; A10; B12; gaplotipes A1-B35; A1-B5; A10-B35; B5-B12. Antigenes of cataract are HLA A3; B15; B40 and gaplotipes A11-B15; A2-B40.

Investigating genetics of various pathology great importance has the study of heterogeneity of this or that disease with antigeneities of HLA - system because the study of genetic-population of HLA antigeneities informs us about spreading of various HLA genes among different nationalities. Besides frequency prediction of this or that disease is possible (their sensibility being interchained with HLA-genes). In some cases HLA antigenes have the meaning of diagnosis.

It is known that wide spread of glaucoma and cataract in the whole world and their bad medical and social prognosis are main problems of medicine. The above mentioned has a great practical meaning for investigation of different risk factors, cure and prophilaxis of these pathologies [1,2].

The main part in formation of glaucoma and cataract has no doubt genetic determination, but its concrete mechanisms for today are not fully investigated (3,4,5). As for literature information about connection of HLA-system with these glaucoma pathologies are controversal and sometimes are not right [6-12].

From this respect we investigated HLA-antigenes "batary method" with the help of the standard two-stage microlimphotoxical test after Terasari and Mc.Clelland (1964) (coloured by trephan blue or airon). The typical system was made with HLA-antiserum of Leningrad Scientific Institute of Hematology and Blood Transfluse and commercial serum of German Firm "Behring". 28 antigenes of the I class were investigated.

In the result there were revealed persons with high risk of glaucoma and cataract (risk-groups) giving chance for prophilaxis of these deseases. Besides we paid attention to the existance population peculiarities association between HLA-complex and glaucoma and cataract in Georgian population as Georgian population is not dependent on the ethnic territory and is characterized by various features of spreading HLA-antigenes [13,14].

While studying antigenes spread during cataract in Georgia we consider that immunogenetic markers in Georgian population are antigenes: HLA A1, A-10, B[12]; gaplotipes A1-B5; A1-B35; A10-B5; A10-B35. Genetic-markers of glaucoma are HLA-antigenes A1, A10, B12 and gaplotipes A1-B5, A10-B35.

Tbilisi State Medical University

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E.Pataraia, Ch.Baumgartner

Clinical Significance of Ictal EEG Patterns in Patients with Mesial Temporal Lobe Epilepsy

Presented by Corr. Member of the Academy V.Mosidse, August 22, 1996

ABSTRACT. **We analyzed lateralization accuracy and reliability of EEG seizure patterns in 118 seizures in 24 patients with mesial temporal lobe epilepsy as documented by hippocampal atrophy on MRI and unitemporal spikes on interictal EEG. Two blinded electroencephalographers independently determined morphology, location and time course of ictal EEG changes. A lateralization was possible in 88.4 to 92.0% of seizures. Lateralization always corresponded to the side of the interictal spike focus and of hippocampal atrophy on MRL. Lateralization was never incorrect. Lateralization at seizure onset was possible in only 30.4 to 33.9% of seizures. A later significant pattern which allowed lateralization occured in 83.8 to 93.6% of seizures with a non-lateralized onset and started 12.8 to 13.2 seconds after EEG seizure onset. Interobserver reliability for lateralization was excellent with a kappa value of 0.85. Concerning the individual patients, in 19-20 patients all seizures were lateralized, in 4 patients a mixture of lateralized and non-lateralized seizures occured and in one patient no seizure was lateralized.**

Introduction

In recent years the syndrome of mesial temporal lobe epilepsy (MTLE) has been delineated and distinguished from other forms of temporal lobe epilepsy. MTLE can easily be diagnosed on the basis of clinical history, anterior temporal spikes on scalp-EEG and the appearance of hippocampal atrophy or sclerosis on MRI scan [1,2]. Patients with MTLE frequently have medically refractory seizures and are considered excellent candidates for epilepsy surgery [1,2].

Presurgical evaluation of epilepsy patients relies on independent and converging evidence from history, clinical seizure semiology, interictal and ictal EEG, neuropsychological testing, the intracarotid amobarbital procedure (IAP) as well as structural (MRI) and functional imaging studies (SPECT and PET) [1,3,4,5]. Prolonged video-EEG monitoring has been the comerstone of presurgical evaluation [6-8]. Although interictal EEG has proven to provide valuable lateralizing information [9,10], ictal EEG recordings are generally considered more reliable for the localization of the seizure onset zone centers [6,7,11,12]. Several studies have addressed the lateralizing and localizing value of various ictal EEG patterns [13-21]. Specifically, interobserver reliability [19,20], the clinical significance of ictal EEG changes in patients with uni- and bitemporal independent interictal spikes and the correlation with depth electrode recordings have been systematically investigated. However, these studies did not differentiate between the various subtypes of temporal lobe epilepsy and specifically did not separate out patients with MTLE.

In some recent articles, ictal EEG findings in patients with mesial temporal lobe epilepsy were reported [21-22]. However, these studies did not present a detailed analysis of ictal EEG patterns, they did not distinguish patients with uni- and bitemporal interictal discharges and finally did not assess interobserver reliability by blinded EEG analysis.

We analyzed systematically occurrence and lateralizing reliability of various EEG patterns in patients with unilateral mesial temporal lobe epilepsy, then we assessed the interobserver reliability of these EEG changes. And finally, we analyzed the ictal EEG changes for the individual patients which is of clinical relevance for the decision process in presurgical evaluation. Specifically, we investigated the clinical significance of non-lateralizing ictal EEG patterns.

Methods

Patients

We studied 24 consecutive patients (10 women, 14 men; mean age: 33,6 years, range 19 to 54 years) with medically refractory mesial temporal lobe epilepsy. Mesial temporal epilepsy was defined by typical seizure semiology, interictal spikes with a maximum over the anteromesial or midtemporal electrodes (FT 7-10, T 7-10, SP 1-2) and hippocampal atrophy or sclerosis on MRI scan.

EEG recording

All patients underwent prolonged video-EEG-monitoring for a definite localization of the epileptogenic zone for an average of 5 days (range: 4-14 days). Anticonvulsant medication was reduced or completely withdrawn to facilitate seizure occurrence [2,3] EEG was recorded from gold disk electrodes placed according to the extended International 10-20 System with additional temporal electrodes [24] and from bilaterally placed sphenoidal electrodes. EEG signals were recorded referentially against Pz, amplified, filtered with a bandpass of 0.3-70 Hz, analog-to-digital converted with a sampling rate of 256 Hz (12 bit) and stored digitally for off-line data analysis. A commercially available EEG monitoring system was used for data acquisition and analysis (Pegasus Monitoring System, EMS Company, Korneuburg, Austria) which allowed reformating of the data in any desired montage [25].

Interictal EEG analysis

The frequency and location of interictal spikes was assessed by visual analysis. At least two minutes of artifact-free EEG recording per hour were reviewed. A minimum of 100 spikes across the various stages of the sleep-wake cycle were analyzed in each individual patient . Only patients with unitemporal interictal spikes were included in the present study. According to previous studies unitemporal spikes were defined as a ratio of more than 90% of spikes occuring over the more affected temporal lobe [10].

Ictal EEG analysis

The following montages were used for ictal EEG analysis: 1.Longitudinal bipolar montage. 2.Transverse bipolar montage. 3.Referential montage where all electrodes were referenced against PZ.

Similar to previous studies [19] the following parameters were assessed for each EEG seizure pattern:

1. Time course of ictal discharge:

a) pattern at onset (PAO)--the first unequivocal ictal EEG change lasting for at least 3 seconds was defined as the pattern at onset.

b) later significant pattern (LSP)--a LSP was defined when a significant change in the morphology or location of PAO occurred.

2. Location of ictal discharge:
- temporal, left or right;
- hemispheric, left or right;
- bilateral, higher on the left or right
- bilateral, non-lateralized.

3. Lateralization of ictal EEG.

Only temporal (a) and hemispheric (b) ictal discharges were considered as sufficient for lateralization. Both bilateral patterns (c and d) were regarded as non-lateralized. The ictal EEG lateralization was classified as correct if it corresponded to the side of atrophy or sclerosis on MRI scan and to the lateralization of interictal spikes.

Interobserver reliability

All ictal EEG patterns were analyzed by two independent experienced EEGers (C.B. and E.P.). Seizures were reviewed by digital EEG (all options of reformating, digital filtering, gain adjustment etc.) in random order generated by a random generator across all patients and seizures; thus seizures from the same patient generally were not reviewed serially. Only the time of clinical onset was marked in each EEG file, whereas the EEGers were blinded for all other data (e.g. patient name, results of interictal EEG, side of atrophy on MRI). We used the kappa statistic in order to assess interobserver agreement [26].

Results

We reviewed 118 seizures in 24 patients, 6 seizures had to be excluded from further analysis because of artifacts. Therefore, 112 seizures were included in the final analysis. The number of seizures for each individual patient ranged from 2 to 11 (average: 5 seizures).

Pattern at onset (PAO)

Concerning location, the PAO was classified as regional over either temporal lobe in 36(32.1%, EEGer-1)resp. 31 seizures (27.7%, EEGer-2), as lateralized over one hemisphere in 2 (1.8%, EEGer-1) resp. 3 seizures (2.7%, EEGer-2), as non-lateralized with a maximum over one hemisphere in 9 (8.1 %, EEGer-1) resp. 11 seizures (9.8%, EEGer-2) and finally as non-lateralized in the remainig 65(58%, EEGer-1) resp. 67 seizures (59.8%, EEGer-2). Thus lateralization from the pattern of onset was possible for 38 (33.9%, EEGer-1) resp. 34 seizures (30.4%, EEGer-2). For the individual patients lateralization from the PAO was possible for all seizures in one patient (4.2%, EEGer-1) resp. two patients (8.3%, EEGer-2), for at least one seizure in 13 patients (54.2% EEGer-1 and 2) and for no seizure in 9 (37.5%, EEGer-1) resp. 10 patients (41.7%, EEGer-2).

Later significant pattern (LSP)

Per definition a later significant patters (LSP) was considered only for 74 (EEGer-1) resp. 78 (EEGer-2) seizures where the PAO was non-lateralized. A LSP was observed in 62 (83.8%, EEG-1) resp. 73(93.6%, EEGer-2) of these seizures. The LSP started at an average of 13.16±9.58 seconds (EEGer-1) resp. 12.81±7.60 seconds (EEGer-2) after EEG seizure onset (median: 10 seconds (EEGer-1) resp. 12 seconds (EEGer-2); range: 4-59 seconds (EEGer-1), 4-39 seconds (EEGer-2).

The LSP was regional temporal in 60 (96.2%, EEGer-1) resp. 64(87.7%, EEGer-2) seizures, lateralized over one hemisphere in 2(3.2%, EEGer-1) resp. 7(9.6%, EEGer-2) seizures and non-lateralized with a maximum over one hemisphere in 2 seizures (EEGer-2, 2.8%). In all 62 (EEGer-1) and 71 (EEGer-2) seizures with a lateralized LSP, this pattern was sustained lasting for >10sec.

Lateralisation from ictal EEG - correlation with interictal EEG and MRI-scan

A definite lateralization from ictal EEG was possible in 89 (88.4%, EEGer-1) resp. 93 (92.0%, EEGer-2) seizures. The side of lateralization was always correct, i.e. corresponded to the side of the interictal spike focus and of hippocampal atrophy or sclerosis on MRI scan. Conversely, in13 (11.6%, EEGer-1) resp. 9(8.0%, EEGer-2) seizures a lateralization was not possible from ictal EEG. Concerning lateralization for the individual patients we obtained the following results: In 19 (79.2%, EEGer-1) resp. 20(83.3%, EEGer-2) patients all seizures were correctly lateralized. In 4 (16.7%, EEGer-1 and EEGer-2) a mixture of lateralized and non-lateralized seizures was observed. In one patient (EEGer-1, 4.2%), no seizure could be lateralized.

Interobserver reliability

Interobserver reliability for lateralization considering the information derived from both the PAO and the LSP was excellent (kappa = 0.85). Accuracy for PAO lateralization was fair (kappa = 0.59) and for LSP - it was good (kappa = 0,69).

Conclusion

Our results indicate that in patients with unilateral mesial temporal lobe epilepsy a correct lateralization of ictal EEG patterns corresponding to the side of hippocampal atrophy is possible in most seizures and in patients with an excellent interobserver reliability, that lateralization - if defined by strict criteria - is never incorrect; finally in rare instances of patients with non-lateralized seizures reevaluation of MRI and interictal spikes for evidence of bilaterality should be performed. This is the first study a dealing with systematic analysis of ictal EEG patterns in patients with unilateral mesial temporal lobe epilepsy including assessment of interobserver reliability.

We used bippocampal atrophy and sclerosis on MRI scans and unitemporal interictal spikes for correct and incorrect lateralization of ictal EEG. Thus, in contrast to previous studies using successful epilepsy surgery as the criterion for identification of the epileptogenic zone our study cannot be used to derive any conclusions concerning postoperative outcome. However this should have no implications on the validity of our results because several reports have documented excellent prognosis in patients with unitemporal spikes and ipsilateral atrophy or sclerosis on MRI scan. Furthermore, 15 of our 24 patients (63.5%) were rendered seizure-free by a selective amygdala-hippocampectomy with a mean postsurgical follow-up of 18 months (range: 5 to 25 months).

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Tbilisi State Medical Univerisry

Universitätsklinik für Neurologie
Vienna, Austria

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PALAEOBIOLOGY

K.Chochieva, N.Mamatsashvili

Spore-Pollen Complexes of Chaudian Deposits Bearing the Decidua of Colchis Redwood Forest

Presented by Academecian L.Gabunia 19.03.1996

ABSTRACT. **Recent spectra of layers bearing the decidua of redwood forest of Colchida are presented. Data of palynological analysis as well as paleocarpological testify Colchida's antiquity as a refugium, its connection with tropical and subtropical regions in geological past and ultimate delayed rate of its organic world transformation through anthropogene.**

Conifers have been constantly observed in the vegetative cover of Georgia since late Triassic - Early Jurassic. Nevertheless the fossil fall off or decidua of the coniferous forests are rarely noticed except for pollen. Thus, the accumulation of small plant remains consisting of cones, cone scales, seeds, needle, leafy shoots of *Taxodiaceae* and *Cupressaceae* in compact fine sandy clays of the Chaudian basin (1.2 -0.7 m.y), is especially noteworthy [1]. The bedrock was so saturated with remains of conifers, especially with cones, needles and seeds of *Sequoia* and *Cupressus*, that it was impossible to clean any sample taken for analysis without its damage. *Sequoia* cones were usually found opened, scaled off, while *Cupressaceae* cones were closed. During the first summer expeditions, through a thin layer of transparent water of silted up brook one could still see big branches of these taxa deprived of any traces of transportation. However, any attempts to cut out or strike off a sample resulted in its crumbling. Faint trace of the branches on the rock vanished merging with the colour of the clay becoming bluish-grey in the air. There was no doubt that alocation of fossilization of coniferous forest decidua has been revealed. Thus the first and the only finding for today is coniferous forest of fossil decidua in the Caucasus. The decidua of the coniferous forest is compared to the type of the coast redwood of California [2]. According to the paleocarpological analysis, in the basin of accumulation (plant remain carrying layers) the stand of *Sequoia* and *Cupressus* trees predominated, whereas the palynological data have shown prevalence only of Sequoia (67-80%). In spite of the fact that there were plenty of cones, seeds and fragments of branches, *Cupressaceae* were represented only by single pollen grains. Though in the spectra of these deposits leafy stalks of moss were represented in every weight of the organic part of washed samples. Bryales were not found in spores at all. This might happen due to lack of unsufficient samples analysis. Time passed and palynological study of these deposits was renewed. In the literature available for us there was no information about a decidua *Taxodiaceae* existent out of their present-day habitat for such a long period of time, such a variety of family taxons at any late Pliocene and all the more in Anthropogene flora. For the sake of getting minutely data and checking the previous ones the sediments conventionally were subdivided into three layers according to the spreading roughly some 0.3 m dividing them from each other. A sample has been taken from each layer and from each sample a weight of some 40-60 gramms was taken for analysis. The received spectra were characterized by stable taxon composition for the Chaudian horizont spectra as well as for all paleo-palynocomplex of Colchida: various trees dominating, plenty of coniferous pollen, wide variety of Taxodiaceae, small amounts of pollen, while very wide combination of Angiospermae taxons. Somewhat unexpected was the absence of *Podocarpus* and *Abies* pollen which are constantly present in all pollen spectra of Cenozoic flora of Georgia and accordingly was presented in the same Khvarbeti florula [3]. Different appeared to be the data on presence in the roch pollen those of *Sequoia, Taxodium, Cryptomeria*. The first samples analyzed showed *Sequoia* somewhat from 26- to 80% of the spectra than the newly derived picture showed *Taxodium* as the leading taxon. All the data obtained witnessed that considerable consistence of *Taxodium* and *Cryptomeria* pollen are characteristic not only of old Euxine deposits of the West Georgia [4].

The results of palynological analysis appeared to be so much significant that it was decided to show not only the taxons variety composition but quantity and correlation of pollen in spectra as well.

Table

List of Taxons	Sample numb.	No 1 %	Sample numb.	No 2 %	Sample numb.	No 3 %
Tsuga diversifolia	17	3.6	7	4.0		
T.canadensis	18	3.8	11	6.2	19	7.1
T.sp.	19	4.0	8	4.5	6	2.2
Picea			2	1.1	1	0.3
Pinus	6	1.2	3	1.7	7	2.6
Taxodiaceae	37	7.8	22	12.5	22	8.2
Sequoiadendron	4	0.8				
Sequoia	21	4.4	6	3.4	18	6.7
Metosequoia	4	0.8	6	3.4	1	0.3
Taxodium	215	45.5	59	33.7	116	43.4
Cryptomeria	40	8.4	9	5.1	18	6.7
Cunninghamia	4	0.8				
Glyptostrobus	7	1.4	6	3.4	10	3.7
Cupressaceae			1	0.5		
Cupressus	7	1.4	1	0.5	2	0.7
Libocedrus	4	0.8	2	1.1	1	0.3
Juniperus	3	0.6	1	0.5	1	0.3
Salix					1	0.3
Populus	1	0.2				
Pterocarya	3	0.6	4	2.2	2	8.7
Juglans	9	1.9	2	1.1		
Engelhardtia					1	0.3
Platycarya	1	0.2	1	0.5		
Carpinus caycasica	1	0.2			2	0.7
Corylus	6	1.2			1	0.3
Betula					1	0.3
Alnus	2	0.4	2	1.1	4	1.4

(Continued)

Castanea	2	0.4	1	0.5	9	3.3
Quercus			1	0.5	4	1.4
Ulmus	5	1.0			1	0.3
Zelkova					1	0.3
Liquidambar			1	0.5		
Rosaceae					1	0.3
Phellodendron					3	1.1
Aesculus	6	1.2	1	0.5	5	1.8
Nyssa	7	1.4				
Fatsia	3	0.6	2	1.1		
Ericaceae	4	0.8	1	0.5		
Undetermined	16	3.3	15	0.5	9	3.3
Gramineae	1		4		3	
Chenopodiaceae						2
Nyphar						1
Umbelliferae	2		1			
Bifora	2					
Plantago	1					1
Compositae	3		5		1	
Artemisia	2		1		4	
Undetermined						
Sphagnum	1	0.3				
Polypodiceae	260	92.5	62		159	90.8
Onaclea	5	1.7	6		8	4.5
Pteris					1	0.5
Polypodium	8	2.8			4	2.2
Osmunda	3	1.0	1		3	1.7
Cyatheae	2	0.7				
Undetermined	2	0.7	1			

The composition of *Taxodiaceae* and *Cupressaceae*, presence of *Theaceae, Araliaceae, Ruraceae* and *Symplocaceae* were somewhat separating the Khvarbeti florula. But as soon as in the Chaudian deposits (the left bank of the river Tchakhvata and the vicinity of Japhareuli village) the presence of Sequoia and Athrotaxis cones, seeds and fruits of *Eurya, Fatsia, Phellodendron* and *Symplocos* were revealed the bordering lines of the florula have become more unstable and misty. It has appeared an integral whole, natural of the Chaudian flora of Guria- rich, woodsy, different from its contemporaries of Europeans, Japan's, North America's flora considering many "exotics" - the relicts of the ancient flora of the same territory. Echoes of some early connections, geologically deep roots were quite close to tropical and subtropical vegetations. We have noted before the "noncorrespondency" [5] of the Chaudian flora compound according to the layers bearing the leftovers. In any case they comperatively remote in time, the Khvarbeti florula, is one of the brightest, ever possible and sure witness of such a combination and it testifies antiquity of Colchida as a refugium.

L.Davitashvili Institute of Palaeobiology
Georgian Academy of Sciences

V.Bagrationi Institute of Geography

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HISTORY OF SCIENCE

L.Razmadze, Corr. Member of the Academy D.Jokhadze

Darwin's Evolutionary Theory and "Kvali"

Presented August 29, 1996

ABSTRACT. **The Georgian weakly political, scientific and literary newspaper "Kvali" (1893 - 1904) was one of the printed organs in Georgia publishing the most complete scientific articles on Darvin's evolutionary theory. The material described in this paper gives chance to the wide sections of population of Georgia to know about new achievements in different sciences as well as those of biology through newspapers.**

Charles Darwin's evolutionary theory that reliably explained existing living nature's variety-animals and plants and established the order of organic world's historical development is considered one of the biggest achievements in the intellectual life of the mankind. It had to deny previously believed ideas and instill new ones. The whole period after Darwin biology particularly the achievements of the last several decades certify the truthfulness of the principles expressed by this remarkable investigator and thinker. This is also proved by the results of investigations of life phenomenon on the molecular level [1, 2].

The above mentioned theory which was first published in 1859 drew the civilized world's attention and very soon it became the subject of discussions not only for specialist-biologists but of the whole society. Prominent Georgian publishmen tried to give chance to the bulk of the population of Georgia to get to know about fresh achievements in different sciences and biology through newspapers. They had to do this because at that period (until 1918) there were no high educational institutions in Georgia, where it would be possible to study sciences thoroughly. When we began to study the history of spreading the knowledge of different sciences and biology among them and especially the spreading of information about Darwin's evolutionary theory, we found out that one of the newspapers very active in this aspect was "Kvali"- a political, scientific and literary weakly newspaper, which had been published from 1893 to 1904. It was in this newspaper, where in 1896 (N 43) an information about the edition of a French author E. Feriera's new book "Darwinism" was published in Georgian translation by I.Pantskhava. "Kvali" considers the eddition of this book to be a very pleasant phenomenon and points out that "this book is widely known in Europe and is translated into all the languages of cultural countries. How astonishingly deep and interesting this theory is, the reader will understand from this book".

We must mark off the series of articles that was published in "Kvali" in several issues of 1897 (N N 10-15) with the same title "The development of animals' embryo" [4]. These articles belong to well known Georgian publicist I.Pantskhava. At first the author characterizes the achievements of science of that period and notes that the science developed mainly in XVIII-XIX centuries because of finding true facts "that defeats all different false believes created by ignorant and foolish mind, those false believes which are supported, defended and cared by religion". The achievements of science "...widened and recognized the whole world, nature and everything what it consists of as a true fact, develops and changes".

Everything grows, changes and develops. This fact is expressed by the author with the word "evolution". He thinks that life is connected with the proteins and marks that "in the eighties protein was received in chemical laboratory". It seems that the author points to the protein (polypeptide) syntheses experiments by the German scientist E.Fisher. The scientist thought it was possible to join 18 different aminoacids through peptide joints. Afterwards Fisher's follower E.Abdelharden made the number of aminoacids up to 19. At the same time the author tries to connect organic substances with inorganic because he believes that it's quite possible to create organic beings from inorganic substances.

In several issues of "Kvali" of 1899 (N N 42, 44, 45, 49 and 52) there are series of articles with general title: "From Copernicus till Darwin" adapted by P.Surguladze from different foreign texts [5]. These articles revise general achievements of different sciences for that period, they characterize the meaning of great natural scientific discoveries. The last two articles were completely dedicated to the description of main features of Darwin's evolutionary theory. These articles were published as a book with the same title in 1900.

Three rather large articles published by "Kvali" in 1902 are very interesting from the point of describing Darwin's theory [7]. Each article occupies half a page. We can see that the author is highly informed in this matter. The articles are signed with symbols XX.

In the first article which is preceded by Darwin's rather big portrait, the author notes that "beginning from the second half of the last century dominate two very strong theories from the scientific point of view and its practical importance. Both these theories became the stars for the most vivid and enthusiastic part of mankind. One theory concerns the society events and the second is called Darwinism (In the first theory Surguladze means Marxism. - the authors). Darwin's theory or Darwinism is a method, research mode, that is necessary for all: for nature scientist, for a historian, for a law specialist and for a moralist. None of human mental activities is left outside the influence of Darwinism". The author tells us Darwin's biography in a very exciting way. He describes Darwin's family genealogy. It is said that Darwin's grand father Erasmus Darwin wrote and edited a composition with the title "Zoonomy", where ideas about the ways of developing nature are given. The author tells us the story about Darwin and the other well known nature scientist Alfred Walles. He tells how Alfred Walles came to an idea about the law of natural selection independently from Darwin. At the end Walles himself recognized Darwin's priority in discovering the law of natural selection.

In the second article the author discusses the question of natural selection and its main attributing factor - that is fighting for existence as the matter of complex intercourses among living organisms. At the same time he gives his arguments. Particularly he marks off that the organisms are greatly inclined towards multiplying and together with this they multiply far more than the existing conditions can allow. This is the reason, why the fighting for existence arises, when a great number of creatures perish. Only the best adapted, those who have some advantage against the others in the given concrete conditions may preserve. In this article the author reminds us of Darwin's idea about the fact that struggle between the individuals of the same kind is more severe than that of the other species. It is known that for this idea Darwin was criticized not very long ago, because of official outlook in the former Soviet Union. Darwin was accused to be under the influence of English economist and theologian Malthus and this was considered Darwin's great mistake. We should say here that neither Maltus nor Darwin's theory of evolution has anything to do with the false official Soviet commentaries. Soviet commentaries always were in accordance with the points of view of representatives of the establishments.

The author of the article thinks that the principle of struggle for existence is true for the human society too, because in the society population increases much quicker than the means for surviving.

In the third article the author analyzes the problem of natural and artificial selection of animals and plants and practical agriculture which helped Darwin to discover the law of natural selection. The author of the article underlines Darwin's thesis that in artificial selection the changing is directed by a man in his own benefit. In natural selection the changing is directed by nature itself and it brings benefit to the organism itself: in struggle for existence only those features are left which are beneficial for organism. He recites Darwin himself: "A man selects in his own benefit, but nature for the benefit of organism. The author resumes: "Fixing of all useful changes, perpetual struggle, changing dying off of weak and defending of adapted- that is the way that nature follows. The way organisms are changing and developing little by little.

As already was said above, all three articles in "Kvali" are not signed or better to say are signed by XX. It seems that they belong to famous Georgian writer and publicist George Tsereteli, who usually signed his writings in this manner [8]. He was well acquainted with the problems of natural science. The first information about Darwin and his theory belongs to him [9, 10].

All said above certifies that newspaper "Kvali" is one of those Georgian periodicals which was eager to inform Georgian society about civilized worlds scientific knowledge (in our case - biology). There is no doubt that by this the soil was being fertilized for nurishing the high-level educational institution, for founding the University of Georgia.

S.Durmishidze Institute of Plant Biochemistry
Georgian Academy of Sciences

I.Javakhishvili Tbilisi State University

Gurjaani Agricultural Institute

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Corr. Member of the Academy T.Sanadze

To the 30th Anniversary of Discrete Saturation Pulsed EPR Spectroscopy

Presented August 6, 1996

ABSTRACT. **History of discovery and development of the discrete saturation pulsed spectroscopy and examples of its application are presented. Invited lecture on the XXVII Anniversary Congress AMPERE, Kazan, 1994 is offered.**

I'll tell you about pulsed spectroscopy based on burning of multihole spectra in EPR lines. This method was discovered and developed in Tbilisi State University and this year it is exactly 30 years of age. Creation of this spectroscopy, as you will see, is tightly connected with E.K.Zavoiski. Recently hole burning spectroscopy was modified by Prof. A.Schweiger who performed Fourier transform of such spectra and now it finds ever-widening application. In connection with this date I would like to present you the historical development of multihole burning spectroscopy. I think, it may be of interest.

In 1954, when EPR was only 10 years of age and we all were forty years younger, I came to the laboratory of Prof. A.M.Prokhorov in the Lebedev Institute as a post-graduate being sent for some months to study the EPR technique in order to continue experiments at low temperatures in Tbilisi. At that time a cryogenic laboratory at our University founded by Prof. E.Andronikasvili has been created. In Moscow I received a scheme of a superheterodyne EPR spectrometer worked out by A.Manenkov and A.Prokhorov and as a present of main components of my future hand-made spectrometer. I also got from them fluorite single crystals with the uranium impurity. In 1956, when my EPR spectrometer began to work at low temperatures and I obtained the first spectra of uranium in fluorite, I immediately wrote about it to Prof. A.Prokhorov. At once I received from him three sheets preprint by B.Bleany and his co-workers [2], in which this spectrum was investigated in detail. In 1955 Prof. A.Prokhorov and Prof. B.Kozirev visited Prof. B.Bleany in Oxford and agreed to send each other new works. So at that time I had to stop my work at the spectrum. In two years I returned to it to investigate relaxation processes and a fluorine hyperfine structure which was recognized complex by B.Bleany himself and remained unexplained. You can see this spectrum in main orientations of the magnetic field in relation to fluorite crystalline axes (Fig.1). In other orientations the structure is not resolved. For many years I tried to describe this

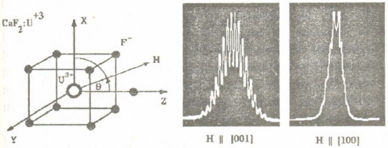

Fig.1. The magnetic center tetragonal Symmetry U^{3+} in CaF_2 according to B.Bleany model and oscillograms of the EPR lines in orientation H|| [001] and H|| [100]. Oscillograms are taken from article [5].

structure to understand what kind of splitting level took part in its formation, but without result.

Some of my colleagues dissuaded me against examining this spectrum, as they thought it was a mere waste of time, but to do so was beyond my power. This simple and at the first sight unimportant problem (explanation of a particular spectrum), became very important for me and from time to time it compelled all my attention.

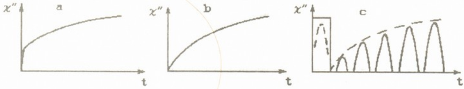

Fig.2. A typical shape of restoration of the absorption curve in EPR line at the pulse saturation: a) pulse duration is shorter than crossrelaxation time in EPR line; b) Pulse duration is longer than crossrelaxation time; c) the same at the saturation of the whole line at switched on magnetic field modulation (modulation frequency 50 Hz).

In investigation of relaxation processes in these lines by a pulse saturation technique the absorption restoration curve consisted, as usual, of two parts: a short-period and a long-period (Fig.2 a). They corresponded to cross-relaxation inside the line and to spin-lattice relaxation. It was easy to obtain a pure long-period process by lengthening the saturating pulse (Fig.2 b) or saturating the whole line at switched on magnetic field modulation. We could not obtain pure cross-relaxation process by pulse shortening and to find out the hindrance. We applied a short pulse at switched on magnetic field modulation. The picture obtained was rather impressive. The pulse was nonsynchronized with the magnetic field sweep in that moment and slowly floated along the line. At the same time the EPR line "breathed" as if it were alive.

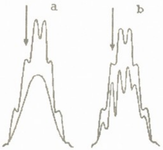

Fig.3. Schematical plot of pulse saturation of the EPR line of U^{3+} in CaF_2 at the orientation of the magnetic field H||[100]: a) pulse acting on the top fluorine HFS; b) pulse acting on the bottom.

When the pulse acted on the peak the whole structure disappeared and when it was between the peaks the whole structure was still better resolved (Fig.3). It happened in spring 1964 just before the conference on magnetic resonance in Krasnoyarsk. I hesitated before my report whether to tell or not about this effect and then decided to continue the investigation and first make it clear for myself. The fact that the effect observed is connected with hyperfine interaction was clear to me before.

In autumn of the same year I had distinct patterns of the burnt out spectrum of uranium in fluorite in main orientations of the magnetic field. At rotation of the magnetic field a strong angular dependence of the holes spectrum was observed. In the EPR line the structure disappeared but the pulse resolved it again. Numerous holes in EPR lines were also observed in other samples as well as in nonsinglecrystalline samples, for instance in the lines of a free radicals in the polyethylene irradiated in the nuclear reactor (Fig.4).

Going ahead, I should say that the observed hole spectrum or discrete saturation spectrum, as we call it, is a kind of a spin packet in the inhomogeneously broadened EPR lines. In this case it is impossible to burn out a single hole in the line. The entire hole spectrum appears together with the central hole and increases intensively with the pulse power or duration (Fig.5). The discrete saturation effect made everybody surprised but still remained an enigma - something strange.

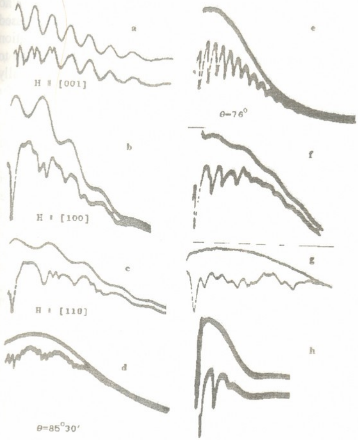

Fig.4. Oscillograms of DS spectra and the shape of the same portions of the line without saturating for tetragonal centers: a) - e) U^{3+} in CaF_2; f) Nd^{3+} in CaF_2; g) Yb^{3+} in CaF_2 H||[001]; h) free radicals in the polyethylene irradiated in the nuclear reactor.

Fig.5. Unperturbed shape of the EPR line (above) and DS spectra at two different levels of energy of the saturation pulse.

In 1966 in Tbilisi at the Institute of Physics a big conference on Plasma Physics was held. There I met Academician E.K.Zavoiski. He asked me to take him to my laboratory. "EPR is of more interest for me than plasma", - he said. As it is known, he had to stop his work on magnetic resonance. When he saw oscillograms of the spectra of the burnt-out holes he was very much surprised and insisted on publishing my work immediately even without explaining the cause of the effect.

I myself did not think of publishing the work in this way. The article on discrete saturation appeared in early 1967 [3]. In this article I intuitively guessed the main cause of DS - the change of the direction of the effective magnetic field acting as a nucleus surrounded by the magnetic center at an electron transition (Fig.6). In this case electron transitions with or without nuclear spin reorientation become of the same order of magnitude. There was only one step, perhaps half a step, to understand the mechanism of burning out of a hole spectrum, but it took me two years more. It is so clear and evident today, but that time nobody could take this step.

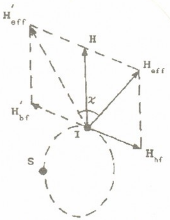

Fig.6. Schematical plot of the change of the direc-tion of the magnetic field on the nucleus at the electron transition.

For these two years I discussed this problem with such prominent scientists as Nicholas Bloembergen, who visited Tbilisi in 1967, and for a few days we discussed the causes of DS formation. In the same year a delegation of French physicists headed by Anatoli Abragam came to Tbilisi, they also discussed DS spectrum at a specially organized seminar. Discrete saturation was reported at the international symposium on free radical in Novosibirsk (1967) with an appeal to theorists to make an attempt to explain this phenomenon as well as in Grenoble in 1968 at the colloquies AMPERE.

During two years after the work had been published many scientists tried to explain this effect. And only after Grenoble I was suddenly enlightened and saw what should have been seen from the very beginning [4]. It is the most surprising in the history of DS technique.

Even today I cannot explain it. Why could nobody see what was so evident? Look how simple it is!

Let us consider the level scheme for eight equivalent

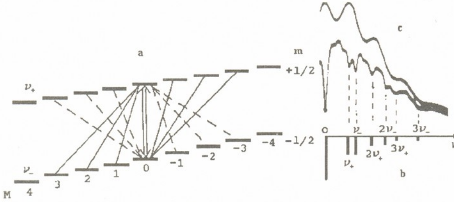

Fig.7. a) level scheme for the eight equivalent nuclei with I=1/2 surrounding the magnetic center with S=1/2; b) expected DS spectrum; c) experimentally observed spectrum for U^{3+} in CaF_2 H||[100].

nuclei with the spin I=1/2 surrounded by the magnetic center with effective spin S=1/2 (cubic centers in the fluorite type crystals at H||[100]* . In this scheme all kinds of electron transitions are allowed and that is important (Fig.7). Pulse saturation of the transition, indicated by an arrow, corresponds to the central hole. In this case the transitions indicated by solid lines correspond to the right hand side holes which at the same time are found in the state of partial saturation. Dashed lines indicate the transitions or holes left to the central hole.

And now look, how elegantly DS spectroscopy solves our initial task on explanation of the fluorine hyperfine structure of uranium in fluorite [5]. A pulse stroke

* For the magnetic centers of tetragonal symmetry in main orientations of the magnetic fields eight nuclei of the nearest neighbors are divided into two groups of four nuclei, but for the case under consideration - U^{3+} in CaF_2 approximation of the eight equivalent nuclei is quite acceptable.

like a stroke of a magic stick gives full information required for spectrum construction. It remains only to data enter into a computer. For this reason it is necessary to use the simple expression

$$2^{-9}[p(x^{s}+x^{-s}) + q(x^{r}+x^{-r})]^{8}(x^{\delta/2}+x^{-\delta/2}),$$

where

$$p = (4\nu_+ \nu_-)^{-1}[4\nu_F^{2} - (\nu_+-\nu_-)^{2}];\ q = (4\nu_+ \nu_-)^{-1}[(\nu_+ + \nu_-)^{2} - 4\nu_F^{2}];$$
$$s = 1/2(\nu_- - \nu_+);\ r = 1/2(\nu_- + \nu_+);$$

ν_+ and ν_- are splitting of upper and lower levels correspondently, δ -is splitting of levels by the ninth fluorine nucleus (Fig.1). This expression can be rewritten as a symmetrical polynom. The position of the spectral components is defined by degree of polynom's member and the intensity by the numerical coefficient in front of it. Each component is described by Gaussian form of line shape and its width is determined by the width of burning holes. This line is unusual. Each of its components has its own selection rules in order to change the total spin of eight fluorine nuclei surrounding the magnetic center: $\Delta M = 0, \pm 1, ..., \pm 8$, which gives 17 lines. 20 HFS lines (orientation [001]), are obtained by shift of this spectrum into three intervals, which is caused by the presence of an extra fluorine ion nucleus compensating an impurity charge according to B.Bleaney model of tetragonal center of uranium in fluorite (Fig. 8).

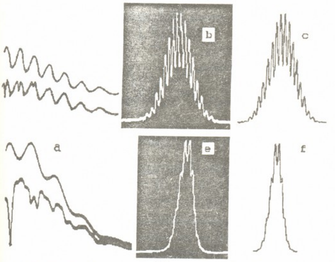

Fig.8. Tetragonal center U^{3+} in CaF_2: a) DS-spectrum; b) oscillogram of the fluorine HFS in orientation H||[001]; c) calculated spectrum (computer graphic); e)-g) the same for orientation H||[100].

Let us give another example of DS spectroscopy application. Recently, in studies of the EPR hole center or V_k - center in fluorite the DS method allowed to reveal a sodium ion near the V_k - center, whose presence had not been expected in the initial model [6]. DS spectroscopy allows to observe the sodium ion surrounded by the magnetic center directly on the oscillograph screen (Fig.9). Sample rotation allows to determine easily the direction and distance from it. Direction is determined by means of vanishing of DS holes and the distance by means of splitting in the holes spectrum. It looks like echoscopy, but it is a tiny magnetic nucleus near the magnetic center and not a calculus in the kidney.

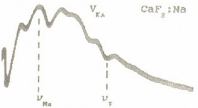

Fig.9. Oscillogram of DS-spectrum of V_k-centers in CaF_2:Na. Holes spectra from Na ions and F ions can be seen near the corresponding Larmor frequencies.

Radiofrequency discrete saturation (RFDS), as we call it, or DS-ENDOR represents the natural development of DS spectroscopy. It was suggested by Zevin and Brik in 1971 [7] and experimentally was discovered in 1972 by us [8,9].

When RF pulse follows the microwave pulse and radio frequency corresponds to splitting of electron levels by nuclei neighboring the magnetic center the resonance perturbation of DS spectrum was observed. As a result some DS holes must weaken and some new holes may appear. The application of RF field with the frequency ν_+ evokes redistribution of populations in the upper group of levels (Fig.10). Therefore it must cause the weakening of the system

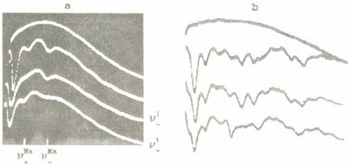

Fig.10. The signal of REDS: a) V_{KA}-center in the CaF_2:Na. The top oscillograme corresponds to the unperturbed shape of the EPR line. Lower - DS spectrum on the same line and action on resonance frequencies of the i-th nuclear surrounded by the magnetic center with are determind by weakening of the intensity of the first or second holes of Na ions; b) the same for more complicated DS-spectrum of Yb^{3+} in CaF_2.

of DS holes with the parameter ν_- and vice versa. So the DS-ENDOR method differs from the other ones with advantage of direct determination electron states. Saying figuratively in our case we get colour resonance frequencies whereas in the other ENDOR methods the frequencies are undistinguishable. Moreover, when different colour lines overlap because of separate receiving the frequencies they do not merge all together. Therefore resolution increases.

Let us consider application of DS-ENDOR method for V_k-center in CaF_2:Na. In Fig. 11 a is given angular dependence of sodium ion DS-ENDOR spectrum in the plane normal to the V_k-center axis. The agreement of the theory allowing for the low-symmetry quadrupole interaction and HFI with the experiment is illustrated by the plot.

In the second Fig.11 b the angular dependence for eight neighboring F nuclei divided into two nonequivalent groups surrounding Ca and Na ions is given. Another eight F nuclei are divided into three subgroups of nonequivalent nuclei (Fig.11c). The full information on HFI for all neighboring F ions is obtained, i.e. the HFI tensor components A are determined.

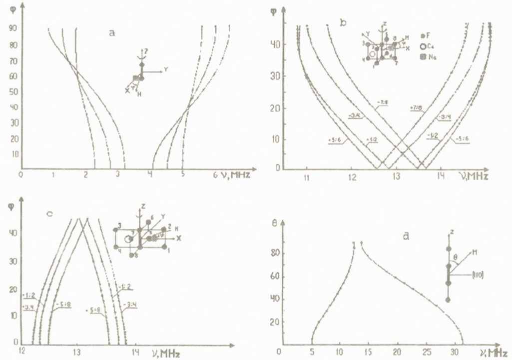

Fig.11. Angular dependence of RFDS spectra for V_k-center in CaF_2:Na: a) for Na^+ ion surrounded by V_k-center; b) for eight nearest ions of F^-, located by the ion of Ca^{2+} and ion of Na^+; c) for the next nearest eight ions of F^-; d) for the two F^--ions on the axis of molecule F_2^-.

In conclusion it should be noted that DS and DS-ENDOR pulse methods were suggested by us exactly 20 years ago at the 18-th Congress AMPERE in Nottingham, where Prof. Givi Khutsishvili was invited and made a review lecture [11]. These works formed the basis for the development of a new direction of EPR spectroscopy which now finds ever-widening application in the study of complex HFI in solids. Due to the development of this spectroscopy by Prof. A.Schweiger it becomes time domain and offers considerable promise for physics, chemistry and other fields.

I.Javakhishvili Tbilisi State University

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HISTORY OF SCIENCE

R.Shengelia

Treatment of Wounds in Georgian Traditional Medicine and New Aspects of Research in History of Medicine

Presented by Corr. Member of the Academy V.Mosidze, October 10, 1996

ABSTRACT. **Treatment of wounds in Georgian traditional medicine are discussed in the work. The material of about 500 manuscripts and more than 40 expeditions was analysed.**

The material is processed by computer. A special computer program was created and criteria and coefficient of reliability were determined. The latter is the basis for utilization/prohibition of traditional remedies. Strategy and policy of utilization of traditional remedies have been determined.

When analyzing all stages of the developmenl of medico-biological thought, when establishing every single stage, its regularity, cultural, political, economic phenomena, accompanying its development we come to the ideas changeable within time on correctness and universality of the way chosen by mankind. As a rule, revision of historical processes following a definite period of time is of speculative character, though it has a great importance when prognosing and planning future. While considering ancient medical manuscripts (anatomical and physiological treaties, medical books "karabadinis") and medical folklore (samples of folk medicine) we can distinguish two essential sides: 1) national, common to all mankind values - religion, philosophy, mythology, language, artistic thinking, which revealed in written monuments through mentality and views and in medical folklore they become evident in various rituals, legends, etc.; 2) the majority of remedies are produced from natural resources and thus their value and usage are ecologically justified. Their revival into everyday practice is inevitable which implies: assessment of efficiency, safety, quality, adaptation and application.

We have mainly investigated the second part of this two-sided problem, particularly, in nosology (a wound). The study of wound seems to be conditioned by the problems of secondary priority but in fact it is caused by recognition of problematic stagnation - a very characteristic trend of modern medicine. The wound treatment has not been improved during the past centuries and neither it has nowadays: a) the period of primary healing has not been reduced even in the world first class hospitals; b) the treatment of complicated wounds remains to be urgent. Any novelty in this direction is of great interest for surgeons all over the world.

When working on individual cases a number of general problems appeared calling for a response. Primarily we came across the fact that medical written monuments were not studied deeply. L.Kotetishvili, I.Abuladze, M.Saakashvili, M.Shengelia, A.Gelashvili, B.Rachvelishvili were pioneers and contributed a lot to the history of science. And still they could have some mistakes. Our work in this direction is to be regarded as appreciation and recognition of their contribution. This work includes: a) correct textual analyses of medical written monuments (historical, linguistic etc.); b) medical, philosophical analysis which are free of vulgar materialism; c) specificity of studies when investigating the epoch in relationship to foreign traditional medicine of the same period. Having compared historical facts we have established a fact of a considerable importance: the erroneous date of "Karabadini" by Kananeli (Book of Medicine) was shifted from XI century to the end of X century. And yet, our research focused on the question of specific practical importance of history of medicine. We believe this approach of science will take it's place granted to it since its origin. Due to some reasons it was lost in the course of time. The methods of Georgian traditional medicines, efficiency of remedies, their safety and quality that represent the coefficient of reliability - all this is a prerogative of a historian of medicine. A physician equipped with general historical methods, the one who has an experience and skills must do the following:

a) work on ancient medical written monuments;
b) identify natural resources and remedies;
c) establish the region of their spread;
d) find out parallels with foreign traditional medicine;
e) compare the results with the findings of modern medicine.

Then he has the right to give a primary evaluation of the methods of traditional medicine and remedies. Taking into account a number of parameters he can give recommendations for: a) practical use after minor tests; b) laboratory experimental studies at various levels; c) stating theoretical values; d) banning.

We posess WHO guidelines received in 1995 when the main conceptual system has been already formulated. We were pleased to find out a complete agreement with our positions though it's worth mentioning their free approach of WHO "Program of Traditional Medicine" on the method of traditional medicine and remedies. The authors of assessment of herbs give us a model of "Pharmokopea" where coefficient of reliability is limited by reference to ancient medical manuscripts. The confidence of these manuscripts is too high. We should notice that they contain all kinds of information: an exact identification of a plant, its spread, organoleptics, biochemical composition, physical and chemical tests. Special document is drawn on utilization of remedies, which cover the experience of developed and developing countries on production of remedies and on legislation and economic basis of their utilization.

The present work deals with wound treatment in Georgian traditional medicine and with *a priori* assessment of corresponding remedies, criteria of assessment and its system. First of all we should sum up the results of studies according to their quantitative index and determine the purposefulness and trends of tactics using remedies for wound treatment in ancient Georgian written monuments and folk medicine.

The propeties of the remedies were divided into two groups - basic and additional, which helped in case of radical treatment.The first group covers blood coagulation, antibacterial, antiinflamatory, wound closing, regerenation stimulating and analgetic remedies. The second group covers: biostimulators, tranquilizers-fever, sudorific, diuretics and swelling resolve remedies. This approach was used both towards ancient classical and folk medicine and it has been improved by us when assessing remedies of respiratory system.

On the basis of information analyses being at our disposal, every remedy is indexed by its characteristic in literature: basic or additional. Proceeding from the above we can judge about wound treatment, tactics of treatment, about the significance of the source in general and the epoch it belonged to.

We have studied 4 written manuscripts (in total they exceed 500). Kananeli's "Ustsoro Karabadini", "Tsigni Saakimo" (Medical Book) by Khojasvili - XII c; Zaza Panaskerteli-Tsitsishvili's "Samkurnalo Tsigni Karabadini" XV c. and David Batonishvili's "Iadigar Daudi" XVI c. They all are original high-level scientific works for their period.

Wound treatment occupies a significant place in Georgian traditional medicine but proceeding from the extremely high-level surgery in Georgian written medical monuments we believe in a possible existence of a special surgical monograph which has not come to our days.

Ancient Georgian surgeons distinguished wounds by their shapes (incised, stab, arrow induced, etc.). intensity of infections and bleeding, lingering, topographic location threat for life. They described a clinical picture, a patient's general state.

The treatment which was based on humoral pathological theory in addition to physicomechanical one (suture, draining tamponi, removal of foreign bodies by special instrument, excision of necrotic tissues) was strictly purposeful. The main purpose of treatment was to achieve blood coagulation, wound closure, regeneration and was also directed to promotion of antibacterial, antiinflamatory and analgetic effects.

Georgian surgeons used symptomatic-biostimulating, tranquilizing, antifever, sudorific, diuretic and swelling resolve means. These means are mainly combined remedies.

In the Georgian written monuments of medieval age the issues of diet which was also based on humoral theory, played a significant role in wound treatment and was directed to the health improvement.

The Georgian written monuments touch upon issues of ethic, based on a surgeon's high professionalism and honesty.

Georgian folk medicine is closely linked with ancient Georgian classical medicine, revealed in the mutual enrichment within centuries. Georgian folk medicine makes use of 138 herbs, among them 122 are used in modern medicine. We failed to identify 11 herbs owing to inadequate entries made during expeditions. Georgian folk medicine also uses 6O remedies of animal and 33 remedies of mineral origin.The natural medicines include combined and monocomponent remedies.

Contrary to classical medicine, Georgian folk medicine makes use of herbs characterized by a number of properties. The question is how strong these properties are, how strong are active ingredients accumulated in the plant. We can *a priori* consider these facts proceeding from the intensity of specific use of the plant, their study and literature. Hence, two tendencies became exident: vertical and horizontal ones. The first was called extensive characteristic of properties while the second one - intensive characteristic of properties. The rating of remedies is the sum of the cardinal parameters computer-aided storing, systematization.

Advanced computer aided method of information storing, systematisation and evolution was used to achieve this goal.

A special original program with mathematical insurance was created. A great amount of information was put in computer: 74 reference books, atlas, vocabulary and the variation line of " coefficient of reliability " with functional limits was formulated. Algorithm was determined by 12 data system. Among them two are informational and ten are parametric.

Informational data are names of remedies in Georgian, Latin and in IO foreign languages. Parametric data: spread, utilization, form of remedies, perfectness of prescription, level of study, known pharmacological effects, their use in modern medicine.

The mentioned informational and parameter system gave us a strictly determined individual pattern of every single remedy rating of which was numerically expressed in "coefficient of reliability". The upper level of coefficient is theoretically infinite.

In fact, in individual cases of wound treatment the optimal limit of coefficient of reliability can be determined from 3 500 to 4 700. This range enables us not only permit producton of herbal medicine but also make recommendations on their expected high efficiency.

Remedies which are in the area from 2 200 to 3 300 can be also used but the producer himself bears a moral, professional and juridical responsibility for their efficiency.

To our mind if the coefficient of reliability is under 2 000 a laboratory and experimental tests and conform of their efficiency and safety we can proceed the usage of remedies. In case of combined remedies it will come to arithmetic summing up.

Some well known remedies, for example Valerian, have unusually a low coefficient. It should be taken into consideration that the mentioned remedies are assessed for the purpose of a specific wound treatment and the mechanism of the fact mentioned becomes evident.

Hence it becomes necessary to create two types of computer models: particular and general. The latter is to be a universal index of natural materials. For time being we have avoided the assessment of animal and mineral natural material owing to the fact that with several exceptions the material has not been studied. Though we have created a computer card. We have also created a scheme of use of traditional medicine, where we tried to establish the principal sources, stages and structure which provide an efficient application of tradidonal medicine.

We are far from claiming that computerized study of natural material gives us an exact picture. Just the opposite, the method must enhance an individual approach for each type of material.

Nothing can substitute an investigator's intuition based on his knowledge. In addition to planned methodical study of traditional medicine and its application, this field is fraught with unexpected discoveries. .

Tbilisi State Medical University.

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