აქტუარული მათემატიკა დაზღვევაში

დალი მაგრაქველიძე

2016

redaqtori: **fiqria Rurwkaia**

teqnikis mecnierebaTa kandidati
saqarTvelos teqnikuri universitetis
asocirebuli profesori

recenzenti: **daviT burWulaZe**

fizika-maTematis mecnierebaTa doqtori
saqarTvelos teqnikuri universitetis
profesori

zurab abaSiZe

maTematikis akademiuri doqtori

saqarTvelos teqnikuri universitetis
asocirebuli profesori

wignSi ganxilulia aqtuaruli maTematikis zogierTi sakiTxi sicocxlisa da arasicocxlis dazRvevis WrilSi.

aRniSnuli saxelmZRvanelo gankuTvnilia ekonomikuri da teqnikuri profilis umaRlesi saswavleblebis bakalavriatisa da magistraturis studentebisaTvis.

ISBN 978-9941-0-9197-1

შინაარსი

Tavi 1. Sesavali

1.1 ZiriTadi ganmartebebi

sanam uSualod aqtuaruli maTematikis sagnis da meTodebis Sesaxeb visaubrebT, aucilebelia SemoviRoT ZiriTadi ganmartebebi da cnebebi, romelic dagvexmareba gaverkveT im maTematikuri meTodebsa da modelebSi, romlebic gamoiyeneba dazRvevaSi.

sicocxlis dazRvevis sferoSi aqtuaruli muSaobisaTvis erT-erT aqtualur amocanas warmoadgens mokvdavobis nimuSis Sefaseba romelic warmodgenili iqneba pirTa jgufis saxiT. am amocanis Sesrulebis ZiriTad instrumentad cnobilia sicocxlis cxrili (is aseve cnobilia rogorc mokvdavobis cxrili – sainteresso magaliTia, sityva da misi sawinaaRmdego sityva gamoiyeneba rogorc urTierTSemcvleli).

vTqvaT, ℓ_0 nebismieri ricxvia, Cveulebriv aiReba mrgvali ricxvebi, rogorebicaa magaliTad 1000 000. vTqvaT viwyebT ℓ_0 axaldabadebuli sicocxleebis jgufiT. gvsurs viwinaswarmetyveloT am adamianebidan ramdeni iqneba cocxali momavli drois nebismier momentSi. ra Tqma unda ar veliT amis zutad gamoTvlas, magram vimedovnebT mivideT mis axlo Sefasebamde, Tu Cven gagvaCnia sakmarisad kargi statistika. mocemul TavSi Semogvaqvs mokvdavobis stoqasturi modeli, sadac gamovikvlevT imis SedarebiT realur daSvebas, rom sidideebi romlebic Cven gvsurs gamoviyenoT SemTxveviTi sidideebia. vTqvaT ℓ_x aris raodenoba im 0 wlis sawyisi sicocxleebisa, romlebic jer kidev cocxlebi iqnebian x wlis Semdeg da vTqvaT d_x aris im originaluri 0 wlis asakis sicocxleebis raodenoba, romlebic daixocebian x-dan $x+1$ asakis SualedSi am sidideebs Soris ZiriTadi damokidebuleba aseTia:

$$\ell_{x+1} = \ell_x - d_x. \qquad (1.1)$$

sicocxlis cxrili warmoadgens ℓ_x da d_x monacemTa Sejamebas sadac x arauaryofiTi mTeli ricxvia. Semdegi warmoadgens sicocxlis cxrilis nawilis magaliTs (es mxolod TvalsaCinoebaa da cifrebi ar warmoadgens realurs):

სიცოცხლის ცხრილი

x	ℓ_x	d_x
0	100 000	2000
1	98 000	1500
2	96 500	1000
3	95 500	900
⋮	⋮	⋮
ω	0	

cxrili dasruleba romeliRac asakze, tradiciulad aRiniSneba ω-iT, amdenad $\ell_\omega = 0$. es warmoadgens cxrilis zRvrul asaks da aRniSnavs im pirvel asaks romelSic sawyisi jgufidan yvela daixoceba. ω-is faqtobrivi mniSvneloba Seicvleba konkretuli sicocxlis cxrilze damokidebulebiT, magram es, rogorc wesi miiReba ara umetes 110 wlisa.

1.2 albaTobebi

Tumca varaudobT, rom Cven SegviZlia viwinaswarmetyveloT ℓ_x zustad, mainc iarsebebs kidev SemTxveviToba Cvens modelSi, ramdenadac jer kidev cnobili araa mocemuli pirebidan romelime iqneba Tu ara A wertilSi cocxlad darCenilebs Soris drois konkretul momentSi. mosaxerxebelia SemovitanoT elementeruli albaTuri cnebebi. vTqvaT, arauaryofiTi n da x ricxvebisaTvis

$$ {}_nP_x = \frac{\ell_{x+n}}{\ell_x} \qquad (1.2) $$

ras niSnavs es wevri? ganvixiloT x asakis ℓ_x cocxlad darCenilebi. am jgufidan ℓ_{x+n} raodenoba miaRwevs $x+n$ asaks. es Tanafardoba gvaZlevs imis albaTobas, rom x asakis pirovneba, romelsac SemdgomSi mxolod (x) simboloTi avRniSnavT, miaRwevs $x+n$ asaks. vTqvaT:

$$ {}_nq_x = \frac{\ell_x - \ell_{x+n}}{\ell_x} \qquad (1.3) $$

es gvaZlevs imis albaTobas, rom (x) daiRupeba x da $x+n$ asakebs Soris. cxadia, rom

$$ {}_nq_x = 1 - {}_np_x . \qquad (1.4) $$

rogorc magaliTad, zemoT moyvanil cxrilSi gveqneboda ${}_2p_0 = 965/1000$, ${}_2q_1 = 25/980$.

ramdenadac qveda marcxena indeqsi `1~ xSirad gvxvdeba is gamoitoveba Canaweris moxerxebulobisaTvis. p_x niSnavs ${}_1p_x$-s da q_x _ ${}_1q_x$-s. q_x sidides xSirad uwodeben X asakSi *sikvdilianobis siCqares*.

ra aris imis albaToba, rom (x) gardaicvleba $x+n$ da $x+n+k$ asakebs Soris? es is sididea romelsac xSirad gamoviyenebT. arsebobs misi gamosaxvis sami gza:

$$\frac{\ell_{x+n}-\ell_{x+n+k}}{\ell_x}, \qquad (1.5a)$$

an

$${}_{n}p_{x} - {}_{n+k}p_{x}, \qquad (1.5b)$$

an

$${}_{n}p_{x}\,{}_{k}q_{x+n}. \qquad (1.5g)$$

ℓ –is mniSvneloba CasmiT SeiZleba Semowmdes, rom es samive gamosaxuleba tolia. TiToeuli maTgani intuitiurad SeiZleba aixsnas. ganvixiloT pirveli. mricxvelSi aris im adamianTa ricxvi romlebic aRweven $x+n$ asaks, romelic naklebia $x+n+k$ asaks miRweulTa raodenobaze. es sxvaoba unda iyos im adamianTa raodenoba, romlebic gardaicvalnen or asaks Soris. am sxvaobis adamianTa sawyis saerTo raodenobaze ganayofi gvaZlevs moTxovnil albaTobas. meore gamosaxulebaSi am raodenobas gamovxatavT rogorc albaTobas imisa, rom (x) icocxlebs n wels magram ar icocxlebs $x+n$ wels. mesame gamosaxulebaSi ganvixilavT or etaps. imisaTvis rom (x) gardaicvalos miTiTebul asakebs Soris man unda icocxlos $x+n$ asakamde. individialurad, $x+n$ asaks miRweuli adamiani unda gardaicvalos uaxloesi k wlis ganmavlobaSi. Cven gveqneba saSualeba am sami gamosaxulebidan visargebloT imiT, romelic CvenTvis ufro mosaxerxebeli iqneba. kidev erTi identuroba romelsac gamoviyenebT rogorc *gamravlebis wess* warmoadgens

$_{n+k}p_x = {_n}p_x\, {_k}p_{x+n}$, (1.6)

yvela arauaryofiTi n,k da x-sTvis. es uSualod SeiZleba Semowmdes (1.2)-dan gamomdinare. intuitiurad gasagebia, rom imisaTvis rom (x) icxovros $n+k$ weli, adamianma unda icxovros jer pirveli n weli, xolo Semdeg $x+n$ asakidan dawyebuli kidev unda icxovros k weli.

1.3 q_x mniSvnelobebiT sicocxlis cxrilis ageba

praqtikaSi sicocxlis cxrilis ageba xdeba $x = 0,1,,...\omega-1$-sTvis q_x mniSvnelobebis miRebis gziT. am mniSvnelobebis miReba warmoadgens statistikur problemas romelsac Cven dawvrilebiT ganvixilavT. es ZiriTadad xerxdeba gamokvlevis Catarebis gziT, romlis drosac vakvirdebiT sxvadasxva asakis adamianebi ramdens xans icxovreben. magaliTad, Tu Cven vakvirdebiT zusti 50 wlis asakis 1000-s kacian jgufs da maTgan wlis ganmavlobaSi 10 kvdeba, maSin q_{50} martivad SegveZlo Segvefasebina rogorc 0,01. es ra Tqma unda Zalian gamartivebulia da procesi sagrZnoblad rTuls warmoadgens. arapraqtikulia, rom SevkriboT zustad 50 wlis asakis xalxi drois erT momentSi da Semdeg davakvirdeT mTeli wlis ganmavlobaSi. praqtikulad, xalxi gamokvlevaSi Sedis sxvadasxva dros da sikvdilis garda sxva mizezebiTac tovebs mas. garda amisa unda uzrunvelyoT sxvadasxva asakSi mniSvnelobebs Soris SeTanxmebuloba. statistikur sagans, romelic cnobilia rogorc *gadarCenis analizi* am problemebTan aqvs saqme.

am wignSi q_x-is mniSvnelobas ganvixilavT rogorc mocemuls. Semdegi formulebis gamoyenebiT, romlebic uSualod (1.1) da (1.3) formulebidan gamomdinareobs sicocxlis cxrili SesaZlebelia aigos individualurad ℓ_0-dan dawyebuli:

$$d_x = \ell_x q_x, \quad \ell_{x+1} = \ell_x - d_x \ . \qquad (1.7)$$

magram Cven vxedavT, rom es arc ise sworia, rogorc wesi, sinamdvileSi aucilebelia ℓ_x-is da d_x-is gamoTvla. praqtikaSi sicocxlis cxrili ganisazRvreba mxolod q_x-is mniSvnelobebiT, rac savsebiT sakmarisia saWiro gamoTvlebisaTvis. tradiciuli formis upiratesoba mdgomareobs mis intiuciur mimzidvelobaSi da ara misi rogorc gamoTvlis instrumentad gamoyenebaSi.

1.4 sicocxlis mosalodneli xangrZlivoba

sicocxlis mosalodneli xangrZlivoba warmoadgens erT-erT yvelaze ufro xSirad citirebul aqtuarul cnebas. Cveni ganmartebiT, amis mizezs warmoadgens is, rom mas xSirad araswored iyeneben. ZiriTadi kiTxva SemdegSi mdgomareobs. ramdeni xani SeiZleba x asakis pirovnebam icocxlos? ra Tqma unda erTidaimave asakis sxvadasxva piris sicocxlis xangrZlivoba momavalSi gansxvavebuli iqneba. zogi icxovrebs ramdenime wlis manZilze, zogi ki maSinaTve daiRupeba, magram Cven SevecdebiT mivideT garkveul saSualo maCveneblamde. amisTvis erT-erTi midgoma iqneboda is, rom gvewarmoebina dakvirveba didi raodenobis x asakis adamianebze iqamde sanam yvela ar daiRupeboda. maSin Cven SevZlebdiT gamogveTvala momavalSi yvela am adamianis sicocxlis saerTo dro. misi sawyis jgufSi Semaval adamianTa raodenobaze ganayofi mogvcemda saWiro saSualo mniSvnelobis Sefasebas. Zalian gamartivebuli magaliTisaTvis aviRoT zustad 60 wlis asakis sami adamiani. davuSvaT erT-erTi gardaicvala 62 wlis asakSi, meore $72\frac{1}{2}$ da mesame - $91\frac{1}{4}$ asakSi. sicocxlis momavali saerTo dro iqneba $2 + 12\frac{1}{2} + 31\frac{1}{4} = 45\frac{3}{4}$. gavyofT ra mas 3-ze, SegveZlia SevafasoT, rom 60 wlis asakis adamians SeuZlia hqondes molodini, rom is kidev icocxlebs saSualod $15\frac{1}{4}$ wels. ra Tqma unda statistikurad zusti rom iyos sam adamianze metis arCeva gvWirdeba. garda amisa, am gamokvlevisaTvis saWiro drois periodi mas absoluturad arapraqtikuls xdis. amave dros aRsaniSnavia, roca gvaqvs sicocxlis cxrili SegviZlia miviRoT suraTi uSualod, dakvirvebis warmoebis gareSe. amaSi dasarwmuneblad ganvixiloT sxva midgoma imisaTvis, rom miviRoT momavalSi sicocxlis sruli dro. davuSvaT viwyebT ℓ_x - x asakis adamianebidan. erTi wlis Semdeg gveqneba ℓ_{x+1} cocxlad darCenili, romelTagan TiToeuli am jamSi Cadebs sicocxlis erT wels. ori wlis bolos aq iqneba ℓ_{x+2} cocxlad darCenili, romelTagan TiToeuli saerTo jams uzrunvelyofs kidev erT wliT. Tu amgvarad gavagrZelebT, Cven SegviZlia SevafasoT yvela sicocxlis momavali saerTo sicocxlis xangrZlivoba rogorc:

$$\ell_{x+1} + \ell_{x+2} + \ell_{x+3} + \cdots + \ell_{\omega-1},$$

da Tu mas gavyofT ℓ_x-ze miviRebT Semdeg sidides

$$e_x = \sum_{k=1}^{\omega-x-1} \frac{\ell_{x+k}}{\ell_x} = \sum_{k=1}^{\omega-x-1} {}_k p_x . \qquad (1.8)$$

e_x sidide cnobilia rogorc Semoklebuli sicocxlis xangrZlivoba an Semoklebuli sicocxlis xangrZlivoba x asakSi. sityva Semoklebuli aRniSnavs dawevas an Sekvecas, asaxavs ra im faqts, rom es zustad is raodenoba araa romelic Cven gvindoda. gazomvis alternatiul sqemaSi Cven cotas vityuebiT, am sqemisTvis zomas warmoadgens mxolod sicocxlis drois Semdgomi mTeli wlebi da ignorirebas ukeTebs sikvdilis wels. zemoT moyvanil Cvens magaliTSi, gamoTvlis alternatiuli meTodis Tanaxmad, im 60 wlis adamians romelic gardaicvala $72\frac{1}{2}$ wlis asakSi daericxeba saerTo sicocxlidan mxolod 12 weli, faqtobrivi $12\frac{1}{2}$ wlis nacvlad. pirovnebas romelic gardaicvala $91\frac{1}{4}$ asakSi daericxeba 31 weli, faqtobrivi $31\frac{1}{4}$ wlis nacvlad. Cven vamcirebT 0-dan 1 wels Soris wels yvela adamianisaTvis, da gonivrulia wlis naxevari miviRoT saSualod. rogorc wesi, WeSmarit sicocxlis xangrZlivobas uwodeben *sicocxlis srul xangrZlivobas* x asakSi da aRniSnaven $\overset{0}{e}_x$-iT

$$\overset{0}{e}_x = e_x + \frac{1}{2}.$$

$\overset{0}{e}_x$-s ufro dawvrilebiT gamovikvlevT momdevno TavebSi.

x-is nebismieri mniSvnelobisaTvis arsebobs e_x-is gamoTvlis martivi rekurentuli formula. (1.8)-is meore formulidan:

$$e_x = p_x + {}_2p_x + {}_3p_x + \cdots + {}_{\omega-x-1}p_x =$$

$$= p_x(1 + p_{x+1} + {}_2p_{x+1} + \cdots + {}_{\omega-x-1}p_{x+1}) =$$

$$= p_x(1 + e_{x+1}). \qquad (1.9)$$

meore striqoni miiReba (1.6) gamravlebis wesis gamoyenebiT. miaqcieT yuradReba imas, rom es aris Sebrunebuli rekurentuli formula, ramdenadac gvaZlevs Semdgomi, ufro meti argumentis mixedviT funqciis miSvnelobas. rekurentuloba iwyeba sawyisi $e_\omega = 0$ mniSvnelobidan. sasargebloa (1.9)-s mivceT intiuciuri axsna. momavalSi nebismier mTel riცxvmade misaRwevad (x)-ma unda Tavidan icxovros $x + 1$ asakamde, rogorc es gamosaxulia p_x mamravliT. maSin individi daasrulebs sicicxlis erT wels, da amis garda, saSualod, momavali dro, am asakis adamianisaTvis momavalSi mosalodneli mTeli ricxvi, $x + 1$ asaki, am droisTvis dasrulebuli iqneba.

xazgasmiT unda aRiniSnos, rom sicocxlis xangrZlivoba warmoadgens asakis funqcias. TiToeuli x asakisaTvis am asakSi saSualo sicocxlis xangrZlivoba gvaZlevs momavali wlebis im saSualo ricxvs, rasac (x) icocxlebs. ZiriTadi mizezi imisa, rom vaxdenT mis araswor citirebas mdgomareobs imaSi, rom mas gamovsaxavT rogorc ricxvs da ara rogorc funqcias. gazeTebSi an msgavs wyaroebSi SeiZleba vnaxoT aseTi Sinaarsis gancxadeba `sicocxlis xanrgZlivoba gaizarda 75,3-dan 75,8 wlamde~. avtori yovelTvis gulisxmobs sicocxlis xangrZlivobas mxolod 0 asakidan. es ra sakvirvelia sainteresoa, magram gadmogvcems garkveulwilad SezRudul informacias. adamians, romelmac miaRwia 80 wels da surs daadginos, kidev ramden xans SeuZlia savaraudod icocoxlos, ar exmareba gancxadeba imis Sesaxeb, rom axalSobilebi saSualod 75,8 wlamde cocxloben.

aseve unda aRiniSnos, rom sicocxlis saSualo xangrZlivoba saSualo xangrZlivobaa da ara adamianis saSualo asaki, romlis miRwevasac SeiZleba is moelodes. magaliTad, Cven vambobT, rom sicocxlis xangrZlivoba 50 wlis asakSi warmoadgens 31,2 wels, es ki niSnavs, rom 50 wlis asakis adamiani SeiZleba moelodos, rom miaRwevs 81,2 wlamde. am TvalsazrisiT garkveuli dabneuloba arsebobs, romelic isev da isev im tendenciis Sedegs warmoadgens sadac mxolod 0 asakis Sesaxeb ityobinebian da xangrZlivoba da asaki erTidaigivea.

arsebobs mravali sidide, romelis miRebac SesaZlebelia sicocxlis cxrilidan, da romelic sainteresoa, magram mxolod erTis Sesaxeb gavamaxvilebT aq yuradRebas. zogjer gvainteresebs momdevno n wlis ganmavlobaSi (x)–is cxovrebis saSualo xangrZlivoba, sadac n garkveuli fiqsirebuli xangrZlivobaa. sidide

$$\sum_{k=1}^{n}\frac{\ell_{x+k}}{\ell_{x}}=\sum_{k=1}^{n}{}_{k}p_{x} \qquad (1.10)$$

cnobilia rogorc Semoklebuli n *wliani droiTi sicocxlis xangrZlivoba* x *asakSi*. es gvaZlevs im adamianis, romelmac Semdgomi n weli icocxla, wlebis mTel ricxvs, romelic amJamad x asakisaa. es aris is, rasac Cven gamoviTvlidiT, Tu gavimeorebdiT zemoT moyvanil alternatiul sazom sistemas, magram dakvirvebas davasrulebdiT n wlis Semdeg. imisaTvis rom es avawyoT sikvdilis wels arasruli aRricxvisaTvis saWiroa garkveuli sifrTxile. isini vinc cxovrobda $x+n$ asakamde Seitanen srul n wels, aqedan gamomdinare mxolod $(\ell_x - \ell_{x+n})$ adamiani. romlebic daiRupnen n welian vadaSi unda iqnen gaTvaliswinebuli koreqtirebis dros. rom miviRoT ufro zusti n wliani droiTi sicocxlis xangrZlivoba x asakSi (1.10) raodenobas unda davumatoT ara $\frac{1}{2}$, aramed $\frac{1}{2}(\ell_x - \ell_{x+n})/\ell_x$, imisaTvis, rom miviRoT sruli *n wliani droiTi sicocxlis xangrZlivoba x asakSi*

$$\sum_{k=1}^{n} {}_k p_x + \frac{1}{2} {}_n q_x .$$

1.5 sicocxlis cxrilebis arCeva

sicocxlis cxrili im faqts asaxavs, rom asaki warmoadgens momavali mokvdavobis gansazRvris ZiriTad faqtors. arsebobs, ra Tqma unda, ramdenime sxva faqtoric, romlebic gavlenas axdenen momaval cxovrebaze, iseTebi rogoricaa sqesi, janmrTelobis statusi, cxovrebis wesi da georafiuli mdgomareoba. paqtikaSi aseTi faqtorebis gavlenebis damuSavdeba xdeba sicocxlis sxvadasxva cxrilebiT. zogi maTgani zRudavs im gansazRvruli jgufis sikvdilianobis gamokvlevas romlis gamoyofac Tqven gsurT. qvemoT moyvanilia praqtikaSi ganxorcielebuli zogierTi yvelaze mniSvnelovani gansxvaveba.

SemCneulia, gaurkvevelі mizezebiT, romlis sruli asxna aravis ar SeuZlia, rom qalebi kacebze didxans cocxloben. asakebis saSualo diapazonisaTvis damaxasiaTebelia, rom erTidaimave asakis qalebis sicocxlis xangrZlivoba mamakacebis sicocxlis xangrzlivobaze 5-dan 7 wlamde ufro metia. imisaTvis, rom es asaxon xdeba qalebis da mamakacebis sicocxlis cxrilebis gancalkeveba.

bolo dros mowevis safrTxis saCveneblad xdeba mniSvnelovani statistikuri monacemebis Segroveba. aman iqamde miiyvana sadazRvevo kompaniebi, rom isini mwevelTa da aramwevelTaTvis ageben sxvadasxva sicocxlis cxrilebs.

sicocxlis cxrilis arCeva aseve mniSvnelovnad iqneba damokidebuli gasayidad arsebuli kontraqtis tipze. mosaxleobis aRweris monacemebis safuZvelze miRebuli sicocxlis cxrilebi ar gamodgeba sadazRvevo miznebisaTvis. adamianebi, romlebic miiRebian sadazRvevo polisebisaTvis, rogorc wesi, gamokvleulni arian janmrTelobaze sadazRvevo kompaniebis mier, raTa darwmunebuli iyvnen, rom maTi jamrTelobis mdgomareoba damakmayofilebelia. maT SeuZliaT imedi hqondeT, rom zogagad saerTo mosaxleobidan aRebul imave asakis adamianebze met xans icocxleben. dazRvevisaTvis sicocxlis cxrilebi mxolod sadazRvevo kompaniis monacemebze dayrdnobiT igeba.

kidev erTi gansxvaveba SeiZleba iyos sxvaoba calkeul kontraqtebsa da jgufur kontraqtebs Soris. pirvel SemTxevavSi myidvelebi gadawyvetilebas iReben sadazRvevo xelSekrulebaSi Sesvlis Taobaze sakuTari jamrTelobis da sakuTari interesebidan gamomdinare. ukanasknelis dros ki damsaqmebeli yidulobs kontraqts, rom moicvas TanamSromelTa didi jgufi. orive SemTxvevaSi mosalodnelia mokvdavobis sxvadasxva modeli.

arsebobs mravali sxva magaliTic, romelsac aq ar ganvixilavT, Tumca zogierT maTganze mokled miTiTebuli iqneba sxva TavebSi. mkiTxvelma unda icodes, rom aqtuaris mniSvnelovan amocanas warmoadgens konkretuli gamoyenebisaTvis Sesabamisi cxrilis arCeva.

1.6 standartuli aRniSvnebi da terminologia

ukve SemovitaneT standartuli simboloebi ${}_np_x$, ${}_nq_x$, e_x $\overset{0}{e}_x$ da ω.

simbolo ${}_{n|k}q_x$ niSnavs albaTobas imisa, rom (x) mokvdeba $x+n$ da $x+n+k$ asakebs Soris, es is raodenobaa, romelic Cven ukve sami gziT gamovsaxeT, rogorc es naCvenebi iyo (1.5)-Si. qveda indeqsi 1 gamoitoveba, ase rom ${}_{n|}q_x$ niSnavs ${}_{n|1}q_x$-s. am vertikaluri xazis gamoyeneba warmoadgens tipiur aqtuarul saSualebas `lodinis periodis~ aRsaniSnavad. am SemTxvevaSi simbolo gamoiyeneba imis aRsaniSnavad, rom adamiani daicdis n wels da daiRupeba Semdgomi k wlis ganmavlobaSi.

(1.10)-Si sidide aRiniSneba $e_{x:\overline{n|}}$ da sruli sicocxlis xangrZlivoba aRiniSneba $\overset{0}{e}_{x:\overline{n|}}$ -iT.

1.7 cxrilis nimuSi

Semdeg TavebSi eleqtronuli cxrilebis problemisaTvis, romlebic moiTxoven sicocxlis cxrilebs Semogvyavs cxrilis nimuSi is Semdegnairad moicema

$$q_x = \begin{cases} 1-e^{-0{,}00005(1{,}09)^x}, & x = 0,1,\dots,118, \\ 1, & x = 119. \end{cases} \qquad (1.11)$$

gvaqvs $\omega = 120$. formula advilad programirdeba cxrilad, swored esaa cxrilisTvis aseTi formis micemis mizezi. arsebuli cxrilis warmodgena saWiroebs individualurad cifrebis Seyvanas. am parametruli formis kidev erT upiratesobas warmoadgens is, rom ori mudmiva 0,000 05 da 1,09 SeiZleba

Seicvalos imisaTvis, rom uzrunvelyofili iyos Sedarebis mizniT sxvadasxva cxrilebis mravalferovneba. Cveni cxrili cxrilis nimuSia da ar unda iqnas miRebuli rogorc realuri Tanamedrove sikvdilianobis realisturi suraTi. es gansakuTrebiT namdvilia axalgazrda an Zalian moxucebuli asakisaTvis, rogorc es mogvianebiT teqstSi iqneba ganxiluli.

1.8 sakontrolo davalebebi

a tipis davalebebi

davaleba 1.1

mocemulia, rom $q_{60} = 0{,}20$, $q_{62} = 0{,}25$, $q_{63} = 0{,}30$, $q_{64} = 0{,}40$.

(a) ipoveT ℓ_x 60-dan 65 wlamde asakisaTvis, dawyebuli $\ell_{60} =$ 1000-dan

(b) ipoveT Semdegi qmedebebis albaToba:

(i) (61) mokvdeba 62 wlidan 64 wlamde asakSi.

(ii) (62) icocxlebs 65 wlis asakamde.

(g) mocemulia, rom $e_{65} = 0{,}8$, ipoveT e_x x = 60-dan 65-mde-sTvis.

davaleba 1.2

mocemulia, rom ${}_5p_{40} = 0{,}8$, ${}_{10}p_{45} = 0{,}6$, ${}_{10}p_{55} = 0{,}4$. ipoveT imis albaToba, rom (40) gardaicvleba 55 da 65 asakebs Soris.

davaleba 1.3

davuSvaT, rom tipiuri 70 wlis asakis 100 kaciani jgufidan pirveli wlis manZilze mokvdeba 10, meore wels 15, xolo mesame wels 20. gamoTvaleT q_{70}, q_{71}, q_{72}, da ${}_3p_{70}$.

b tipis davalebebi

davaleba 1.4

davuSvaT rom

$\ell_x = 100 - x, \quad x = 0,1,\dots,100.$

(a) ${}_np_x$ (b) ${}_nq_x$, (c)-sTvis moZebneT imis albaTobis gamosaxuleba, rom (x) mokvdeba $x+n$ da $x+n+k$ asakebs Soris.

davaleba 1.5

mocemulia, rom $e_{60} = 17$, ${}_{10}p_{50} = 0{,}8$ da rom 50 wlis asakSi mosalodneli 10 wliani Semoklebuli cxovrebis dro Seadgens 9,2-s ipoveT e_{50}

davaleba 1.6

teqstTan miaxloebiT, daamtkiceT Semdegi mtkicebuleba da mieciT intuiciuri axsna:

$$\overset{0}{e}_x = \frac{1}{2}q_x + p_x(1 + \overset{0}{e}_{x+1}).$$

davaleba 1.7

davuSvaT, rom q_x udris mudmiv q-s yvela x-Tvis. (avRniSnoT, rom amSemTxvevaSi $\omega = \infty$). vipovoT gamosaxuleba q da n terminebSi (a) ${}_np_x$, (b) e_x sTvis. rogor fiqrobT es gvaZlevs realistur sicocxlis cxrils? ratom an ratom ara?

savarjiSoebis eleqtronuli cxrili

davaleba 1.8

sicocxlis cxrilis nimuSis gamoyenebiT, rogorc es moicemoda (1.11) formuliT, gamoviyenoT rekurentuloba imisaTvis rom vipovoT e_x $x = 0,1,\dots,1-\omega$-sTvis. movaxdinoT fokusireba e_0-ze. ramdenad Semcirdeba Tu 0,000 05 mudmivas SevcvliT 0,000 06-iT? ra moxdeba Tu 0,000 05 igive darCeba da 1,09 Seicvleba 1,092-iT?

Tavi 2. sadazRvevo saqmis Sesavali

2.1. aqtuaruli maTematikis sagani

saxelmZRvaneloSi mocemulia maTematikuri meTodebis da modelebis Sesaxeb sawyisi monacemebi, romlebic gamoiyeneba dazRvevaSi. am samecniero mimarTulebis sayovelTaod miRebuli saxelwodebaa – **aqtuaruli maTematika** (*actuarial mathematics*) da warmoSobilia *actuary* – aqtuaridan, sadazRvevo sazogadoebis statistikosidan. Sesabamis ekonomikur da iuridiul disciplinebTan erTad aqtuaruli maTematika qmnis codnis ufro farTe ares – **aqtuarul mecnierebas** (*actuarial scince*), romelic warmoadgens sadazRvevo biznesis Teoriul safuZvels.

Tumca aqtuaruli maTematika farTod iyenebs albaTobis Teoriis da maTematikuri statistikis meTodebs, Tavisi *sagniT, meTodebiT da gamoyenebis sferoTi* is warmoadgens damoukidebel samecniero mimarTulebas.

aqtuarul ganaTlebas msoflioSi saukunoebrivi tradiciebi gaaCnia, magram Cvens qveyanaSi Tavisufali sabazro urTierTobis

TiTqmis 70 wliani ar arsebobis pirobebSi aqtuaruli mecniereba praqtikulad ar arsebobda XX saukunis 90 wlebamde. amJamad gaCnda didi interesi am sferos mimarT, rac dakavSirebulia sadazRvevo kompaniebis mxridan specialist-aqtuarebis gazrdil moTxovnasTan.

profesiuli momzadebis yvelaze maRal msoflio standarts iZleva aqtuarebis sazogadoebis programa (aSS).

ganasxvaveben aqtuarul maTematikas qonebriv da pirad dazRvevaSi. *qonebriv dazRvevaSi* (*non-life insurance*) igulisxmeba sadazRvevo saqmianobis yvela saxe, romelic araa dakavSirebuli pirad dazRvevasTan (sacxovreblis, avtomobilis, sawarmos, sabanko kapitalis da a.S. dazRveva). *piradi (sicoccxli) dazRvevis* (*life insurance*) qveS moiazreba sicocxlis, janmrTelobis, pensiis da a.S. dazRveva.

gansazRvreba 2.1.1 *fuladi saxsrebs anu* p_1, p_2, ..., p_n *sidideebs, romlebsac mzRvevelebi uxdian sadazRvevo kompaniebs uwodeben sadazRvevo premiebs* (*premiums*).

gansazRvreba 2.1.2 b_1, b_2, ..., b_ν, $\nu \le n$ *sidideebs romlebsac ixdis kompania* ν *sadazRvevo SemTxvevis dadgomisas uwodeben sadazRvevo gadaxdebs* (*benefits*).

cxadia, rom sidideebi $b_j \gg p_j$, winaaRmdeg SemTxvevaSi aravin daizRvevs Tavs. larad sadazRvevo polisis yidvisas dazRveuli Tavidan iSorebs finanasur risks, romelic sadazRvevo SemTxvevis ganusazRvrelobasTanaa dakavSirebuli. am *risks* (*risk*) Tavis Tavze iRebs sadazRvevo kompania, romlisTvisac riski mdgomareobs

SemTxveviT *sarCelSi* (*clain*), romelic SesaZlebelia mas waredginos.

aqtuarul maTematikaSi gansakuTrebuli yuradReba eTmoba terminologiis da aRniSvnebis standartizacias. imisaTvis, rom gaadvildes aqtuarebs Soris urTierToba, gaadvildes samecniero kvlevebis Sedegebis danergva da a.S. jer kidev 1898 wels aqtuarTa saerTaSoriso kongresma gadawyvita moaxdinos im terminologiis da ZiriTadi sidideebis aRniSvnebis standartireba, romlebic gvxvdeba sadazRvevo maTematikaSi. saxelmZRvaneloSi aseTi aRniSvnebis didi raodenoba Segvxvdeba, xSirad sakmaod uCveuloc. aseTi cnebebis da aRniSvnebis Tavisufali floba warmoadgens aqtuarebis profesionaluri codnis nawils. amitom am terminebis da aRniSvnebis damaxsovrebas seriozuli yuradReba unda mieqces.

2.2 sadazRvevo kompaniis umartivesi modeli

sadazRvevo maTematikaSi xdeba Semdegi ZiriTadi problemebis gadawyveta.

1. p premiasa b gadaxdas Soris `swori~ Tanafardobis moZebna, $p \ll b$. amaSi Sedis, magaliTad, neto-premiis gamoTvla, buto-premiis gamoTvla, im gadaxdis gamoTvla ris uflebasac miscems sadazRvevo kompania Tavis Tavs da a.S. SevniSnoT, rom neto-premia Seesabameba kompaniis saSualo nulovan mogebas.

2. gakotrebis albaTobis gamoTvla, romelic mniSvnelovani gadawyvetilebis miRebis safuZvels warmoadgens. Tu U–iT avRniSnavT kompaniis kapitals an mis rezervs, xolo gadaxdebis jams S-iT,

$$S = b_1 + b_2 + ... + b\nu$$

sadac b_j sadazRvevo kompaniis mimarT j-uri sarCelia, maSin gakotrebis albaToba uxeSad $P(S > U)$–is tolia, xolo aragakotrebis albaToba tolia $P(S \le U)$-is. cxadia, rom

$$P(S > U) + P(S \le U) = 1.$$

3. sadazRvevo kompaniis rezervebis gaangariSeba.

ganxiluli problemis gadaWris ilustrireba movaxdinoT sadazRvevo kompaniis muSaobis umartivesi modelis magaliTze, romelic agebulia mxolod albaTobis Teoriis centraluri zRvariTi Teoremebis gamoyenebiT.

ganvixiloT Semdegi idializirebuli sqema. vTqvaT wlis dasawyisSi firmaSi Tavi daizRvia $x = 26$ wlis n mamakacma. vTvliT, rom TiToeuli klienti ixdis p premias, da aqedan gamomdinare, firmam miiRo jamuri $p \cdot n$ Semosavali, romelic SemdgomSi Seadgens mis rezervs $U = pn$. $b_i = 1$ –iT avRniSnoT firmis mimarT wayenebuli sarCeli, Tu wlis ganmavlobaSi i–uri klienti gardaicvleba.

damateba 1-dan mokvdaobis cxrilis gamoyenebiT, nebismieri $i = 1,..., n$-sTvis, vpoulobT

$$P(b_i = 1) = q_{26} = 0{,}0293, \quad P(b_i = 0) = p_{26} = 1 - q_{26} = 0{,}99707.$$

imis albaToba, rom kompania ar gakotrdeba tolia

$$P\left\{\sum_{i=1}^{n} b_i \le U\right\} = P\left\{\frac{\sum_{i=1}^{n} b_i - E\sum_{i=1}^{n} b_i}{\sqrt{D\sum_{i=1}^{n} b_i}} \le \frac{U - E\sum_{i=1}^{n} b_i}{\sqrt{D\sum_{i=1}^{n} b_i}}\right\}. \qquad (2.2.1)$$

aq da Semgom simbo l oebi $E\xi$ da $D\xi$ niSnaven ξ SemTxveviTi sididis maTematikur l odins (saSua l o) da dispersias.

viTva l iswinebT ra b_i SemTxveviTi sididis diskretu l obas, nebismieri $i=1,...,n$-sTvis gvaqvs

$$Eb_i = 1\cdot q_{26} + 0\cdot p_{26} = q_{26} = 0{,}00293,$$

$$Db_i = Eb_i - (Eb_i)^2 = 0{,}00293 - (0{,}00293)^2 \approx 0{,}00292.$$

ramdenadac praqtikaSi b_i sarCe l ebi damoukidebe l ia, amdenad (2.2.1) Semdegi saxiT gardaiqmneba

$$P\left\{\frac{\sum_{i=1}^{n} b_i - n\cdot 0{,}00293}{\sqrt{n\cdot 0{,}00292}} \le \frac{U - n\cdot 0{,}00293}{\sqrt{n\cdot 0{,}00292}}\right\}.$$

dazRveu l Ta $n=$ 3071 raodenobisaTvis miviRebT

$$nEb_1 = 3071\cdot 0{,}00293 \approx 9,\quad nDb_1 = 3071\cdot 0{,}00292 \approx 9.$$

e.i. S SemTxveviTi sididis saSua l o da dispersia erTmaneTs emTxveva, rac zogadad rom vTqvaT miuTiTebs S puasonis ganawi l ebaze. simartivisaTvis visargeb l oT centra l uri zRvru l i TeoremiT, rom l is Tanaxmadac

$$P\left\{\frac{\sum_{i=1}^{n} b_i - 9}{3} \le \frac{U-9}{3}\right\} \approx \Phi\left(\frac{U-9}{3}\right),$$

sadac

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e - u^2/2du$$

warmoadgens a l baTobebis integra l s (standartu l i norma l uri kanonis ganawi l ebis funqcia).

vTqvaT kompaniis xelmZRvanelobas awyobs aragakotrebis 0,95 albaToba. maSin, $\frac{U-9}{3}$-is mimarT

$$\Phi\left(\frac{U-9}{3}\right)=0,95$$

gantolebis amonaxsniT miviRebT $\frac{U-9}{3}=x_{0,95}$, sadac $x_{0,95}=1,645$ standartuli normaluri kanonis 0,95 donis kvantilia.

aqedan gamomdimare, sadazRvevo kompanias am SemTxvevaSi unda hqondes $U=3\cdot 1,645+9\approx 13,935$ rezervi, xolo dazRvevaze gadaxda (premia) unda Seadgendes 0,00453 nawils b =1 sarCelisas, ramdenadac

$$p=\frac{U}{n}=\frac{13,935}{3071}=0,00453.$$

Tu b=100 000 lars, maSin premia Seadgens

p= 100 000 ⋅0,00453 ≈ 453 lars weliwadSi, rac TveSi daaxloebiT 38 lars Seadgens, ris gadaxdac SeuZliaT mcire Semosavlis mqone klientebsac.

sakontrolo davalebebi

davaleba 2.3.1

SeadgineT sxvadasxva riskebi:

1) studentis,
2) meSaxtis,
3) avtomobilis mZRolis
4) ojaxis,
5) organizaciis,

6) mxaris,

7) qveynis,

8) kacobriobis zogadad.

davaleba 2.3.2

ratom ar SeiZleba miiRoT xval Tqveni mezoblis erT-erTi ZaRlis mokvlis riskisagan dacvisTvis dazRveva?

romeli riskebis dazRveva xdeba nawilobriv?

romeli riskebi izRveva aucileblad?

davaleba2.3.3

Tqvens mier davaleba 1.3.1-Si Sedgenili riskebidan romeli aris:

1) dasazRvevi,

2) aradasazRvevi,

3) nawilobriv dasazRvevi,

4) aucileblad dasazRvevi.

davaleba 2.3.4

1.2. ganyofilebis meTodikiT gakotrebis 0,05 da 0,01 albTobisas ipoveT p premia $x=26$ wlis qalebisaTvis. CaatareT qalebisaTvis Sedegebsa da 1.2 ganyofilebaSi mamakacebisTvis miRebul Sedegebs Soris SedarebiTi analizi.

davaleba 2.3.5

aageT sadazRvevo kompaniis muSaobis modeli, im pirobiT, rom S jamuri gadaxdebis ganawileba puasonurs warmodgens.

darwmundiT, rom am SemTxvevaSi 1.2 ganyofileba porobebiT $p \approx 114$ lars gakotrebis 0,05 albaTobisas da $p \approx 130$ lars gakotrebis 0,01 albaTobisas.

Tavi 3. sicocxlis xangrZlivobis albaTuri maxasiaTeblebi

3.1. gadarCenis funqcia (*survival function*)

upirvelesad unda aRiniSnos, rom sicocoxlis dazRvevaSi ganxiluli yvela meTodi da modeli misaRebia sxva saxis dazRvevisaTvisac. vTqvaT X aRniSnavs sicocxlis xangrZlivobas, X_i - i-uri individis sicocxlis xangrZlivobaa. dazRvevis sxva saxeSi X –is qveS SeiZleba vigulsxmoT:

1) daavadebis dadgomamde dro (tkipis encepaliti, laimenis daavadeba);

2) avtomobilis avariis gareSe muSaobis dro;

3) qonebaze zaralis miyenebamde dro da a.S.

sikvdilis momentis, avadmyofobis dawyebis da avariis moxdenis ganusazRvreloba anu winaswargaurkvevloba warmoadgens dazRvevisas SemTxveviTobis wyaros, rac saSualebas iZleva gamoyenebuli iyos SemTxveviTi xdomiloba, sidide, procesi sicocxlis janmrTelobis, avtomobilebis a.S. dazRvevis sxvadasxva aspeqtebis analizis dros.

cxadia, rom konkretuli adamianis sikvdilis momentis Sesaxeb raime gansazRvrulis Tqma, rogorc wesi, SeuZlebelia. magram, Tu ganixileba adamianTa sakmaod did *erTgvarovani* jgufi, maSin misTvis ukve samarTliania kanonzomierebebi, romelic damaxasiaTebelia masobrivi SemTxveviTi movlenebisaTvis, sixSireTa mdgradoba, normaluri an puasonis ganawilebis kanonebisadmi msgavseba da a.S. amitom albaTobis Teoriis terminologiis gamoyenebiT SesaZlebelia vilaparakoT sicocxlis xangrZlivobis rogorc X SemTxveviTi sididis Sesaxeb, amasTan $X \geq 0$.

X SemTxveviTi sididis amomwurav maxasiaTebels warmoadgens Semdegi ganawilebis funqcia

$F(x) = P(X \le x)$.

aqtuarul matematikaSi Cveulebriv ganawilebis funqciis nacvlad iyeneben gadarCenis funqcias

$$\boxed{s(x) = P(X > x) = 1 - F(x)} \qquad (3.1.1)$$

romelic warmoadgens imis albaTobas, rom adamiani miaRwevs x wlamde. rogorc adre vTqviT, x wlamde asakis individums aqtuarul maTematikaSi aRniSnaven (x)-iT.

gadarCenis funqciis formula CarCoSi CavsviT, randenadac $s(x)$ waemoadgens terminebs da aRniSvnebs Soris erT-erT ZiriTads aqtuarul maTematikaSi. SemdgomSi im formulebs romlebsac gansakuTrebuli yuradReba unda mieqces (3.1.1)-is msgavsad ganvalagebT.

gadarCenis funqcias rogorc ganawilebis funqciis damatebiT funqcias gaaCnia Semdegi Tvisebebi:

1) $s(x)$ klebulobs (mkveTrad);

2) $s(0) = 1$, $s(\infty) = 0$;

3) $s(x)$ uwyvetia marjvnidan.

magram mokvdavobis realuri procesisaTvis 1) da 3) Tvisebebi ramdenadme saxes icvlis. marTlac, gadarCenis funqcia *mkacrad* unda *klebulobde*s, ramdenadac iarsebebda ramdenime periodi adamianis cxovrebaSi, magaliTad, $\Delta x = x_2 - x_1$, romlis ganmavlobaSi is ar mokvdeboda, da $s(x)$ unda iyos *uwyveti*, winaaRmdeg

SemTxvevaSi iarsebebda iseTi x_0 momenti adamianis cxovrebaSi, romelSic is mokveboda xolme nulisagan gansxvavebuli albaTobiT $\Delta P = s(x_{0-}) - s(x)$, $s(x_{0-}) = \lim_{x \to x_{0-}} s(x)$, $s(x_0) = \lim_{x \to x_{0+}} s(x)$.

garkveuli xarisxiT gadarCenis funqciis wlovanebaze damokidebulebis realuri damokidebulebis xasiaTis Sesaxeb SeiZleba ganvsajoT qvemoT moyvanili sabWoTa mamakacebis da qalebisaTvis (1984-1985 wlebi), aseve aSS-s mosaxlebisaTvis cxrilebis mixedviT.

მამაკაცები (სსრკ, 1984-1985)

x	0	14	20	30	40	50	60
$s_1(x)$	1,000	0,954	0,947	0,922	0,878	0,795	0,651

70	80	90	100	110
0,434	0,188	0,003	0	0

ქალები (სსრკ, 1984-1985 წ.)

x	0	14	20	30	40	50	60
$s_2(x)$	1,000	0,964	0,961	0,953	0,939	0,908	0,841

70	80	90	100	110
0,700	0,417	0,008	0	0

აშშ-ს მოსახლეობა

x	0	10	20	30	40	50	60
$s_3(x)$	1,000	0,983	0,977	0,965	0,949	0,915	0,837

70	80	90	100	110
0,682	0,432	0,142	0,012	0

ramdenadac realuri X sicocxlis xangrZlivoba SemosazRvrulia zRvruli wlovanebiT (*limiting age*) ω=100 - 120 weli, amdenad

$s(x) = 0, \quad x > \omega.$

amis gamo sicocxlis xangrZlivobis cxrili (sxc) Cveulebriv dgeba mTeli $x \le \omega$ ricxvebisaTvis. magram analitikurad mocemuli $s(x)$ funqciisaTvis sicocxlis dro, rogorc wesi, SeuzRudavia, amasTan $s(x)$ parametrebi ise SeirCeva, rom $P\{X > \omega\}$ albaToba sakmarisad mcire iyos.

gamovarkvioT, rogoraa dakavSirebuli gadarCenis $s(x)$ funqcia sxc-is ZiriTadebidan erT-erT zogad l_x maxasiaTebelTan (danarTi 1). ganvixiloT sakmarisad didi jgufi l_0 axaldabadebulebidan (moxerxebulobisaTvis iReben l_0=100 000) da movaxdinoT maTi sicocxlis xangrZlivobis an sikvdilis momentis fiqsireba: X_1, X_2, ..., X_{l_0}. SemovitanoT xdomilobisaTvis A indikatori:

$I(A)$= {1, Tu xdomiloba moxda; 0, winaaRmdeg SemTxvevaSi}.

maSin, am jgufis warmomadgenelTa raodenoba, romlebmac miaRwies x asaks aris:

$$L(x) = \sum_{i=1}^{l_0} I(X_i > x)$$

SemTxveviTi sidide, romlis maTematikuri lodinic gansazRvravs l_x sidides:

$$l_x = El_x = \sum_{i=1}^{l_0} EI(X_i > x) = \sum_{i=1}^{l_0} P(X_i > x) = \sum_{i=1}^{l_0} s(x) = l_0 s(x),$$

e.i.

$$l_x = l_0 \cdot s(x) \quad (3.1.2)$$

(3.1.2) formulidan gamomdinareobs, rom

1) l_x mrudi icvleba x asakze damokidebulebiT $s(x)$ gadarCenis funqciis analogiurad l_0 mamravl-mudmivi sizustiT;

2) $s(x) = l_x / l_0$ es aris ganxiluli axalSobilebis jgufidan x asakamde miRweulTa saSualo wili.

danarTSi warmodgenili sxc-s Tanaxmad, l_{14}= 95,438 mamakacebisaTvis niSnavs, rom 100 000 axalSobilidan 14 wlamde aRwevs saSualod 95 438 biWi, xolo 14 wlamde miRweulTa saSualo wili xalSobilTa am jgufidan tolia $s(14) = 0{,}95438$-is.

3.2 sikvdilebis mrudi (the curve of deaths)

ganvixiloT wina paragrafidan aSS-s mosaxleobisaTvis sxc-s monacemebi da movaxdinoT maTi sxvagvari interpretacia. Tu aviRebT $l_0 = 1000$, maSin sicocxlis pirveli 10 wlis ganmavlobaSi daiRupeba daaxloebiT 17 kaci, 10-dan 20 wlamde – 6 kaci, 20-dan 30 wlamde – 12 kaci, 30-dan 40 wlamde – 16 kaci, 40-dan 50 wlamde – 34 kaci, 50-dan 60 wlamde – 78 kaci, 60-dan 70 wlamde – 155 kaci, 70 wlidan 80 wlamde – 250 kaci, 80-dan 90 wlamde – 290 kaci, 90-dan 100 wlamde – 130 kaci, 100-dan 110 wlamde – 12 kaci.

cxadia, rom intervalebad dayofili es monacemebi ufro TvalsaCinod axasiaTebs mokvdaobas im monacemebTan SedarebiT, romlebic mocemul cxrilSia warmodgenili. magaliTad, SegviZlia gamovyoT pirveli aTwleuli, romelSic sikvdilianoba samjer metia, vidre meorSi, yvelaze usafrTxo aTwleulSi, an 70 wlidan 90 wlamde periodSi, romlis ganmavlobaSic daiRupa 540 adamiani, rac sawyisi jgufis naxevarze mets $l_0 = 1000$ adamians Seadgens.

axla cxadi xdeba $(x, x+t)$ asakobrivi intervalebisaTvis

$$ {}_tD = L(x) - L(x+t) = \sum_{i=1}^{l_0} I(x < X_i \le x+t), \qquad (3.2.1) $$

SemTxveviTi sididis Semotana, romelic tolia fiqsirebuli l_0 axalSobilTa gjufidan x-dan $x+t$ asakamde daRupulTa ricxvis. am SemTxveviTi sididis maTematikuri lodini gansazRvravs sicocxlis xangrZlivobis saerTo cxrilebis kidev erT ZiriTad maxasiaTebels ${}_td_x$ (danarTi 1):

$$ {}_td_x = E_tD_x $$

cxadia, rom (3.2.1)-is Tanaxmad

$$ {}_td_x = E[L(x) - L(x+t)] = l_x - l_{x+t} = l_0[s(x) - s(x+t)], $$

sadac $s(x) - s(x+t) = P(x < X_i \le x+t)$ $(x, x+t]$ SualedSi sikvdilis albaTobaa.

avRniSnoT, rom indeqsi 1 ${}_1d_x$ aRniSvnaSi Cveulebriv gamoitiveba xolme, amitom:

$$ d_x = l_x - l_{x+t} $$

e.i. gamoisaxeba sicocxlis xangrZlivobis cxrilSi arsebuli l_x da l_{x+t}-iT. miuxedavad amisa, d_x sidide am cxrilebSi moyvenilia rogorc ZiriTadi.

funqcias $f(x) = F'(x) = -s'(x)$ uwodeben X SemTxveviTi sididis ganawilebis simkvrives da aqtuarul maTematikaSi mas aniWeben *sikvdilebis mrudis* (*the curve of deaths*) termins.

vaCvenoT, rom d_x mrudi x cvladis zrdasTan erTad icvleba daaxloebiT iseve rogorc $f(x)$ mokvdavobis mrudi l_0 mamravlamde sizustiT, e.i.

$$\boxed{d_x \approx l_0 f(x)} \qquad (3.2.2)$$

marTlac, teiloris formulis gamoyenebiT (danarTi 4, Teorema 3), gvaqvs

$$s(x) = s(x+1) - s'(\xi),$$

sadac $\xi \in (x, x+1)$. ramdenadac $s(x)$ wlis ganmavlobaSi TiTqmis ar icvleba, amdenad:

$$d_x = -l_0 s'(\xi) \approx -l_0 s'(x) = l_0 f(x)$$

amiT (3.2.2) formula damtkicda.

Tu SevajamebT SeiZleba iTqvas, rom sikvdilebis mrudi garkveuli azriT gadarCenis funqciasTan SedarebiT ufro daxvewili maxasiaTebelia.

3.3 mokvdavobis intensivobis funqcia (*force of mortality*)

Tavis mxriv sikvdilebis mrudTan SedarebiT ufro daxvewil maxasiaTebels warmoadgens mokvdavobis intensivobis funqcia. vipovoT x wlamde miRweuli admianis sikvdilis albaToba uaxloesi t wlis ganmavlobaSi, e.i. $(x,\ x+t]$ SualedSi:

$$P(x < X \le x+t \mid X > x) = \frac{P(x < X \le x+t \cap X > x)}{P(X > x)} =$$

$$= \frac{P(x < X \le x+t)}{P(X > x)} = \frac{s(x) - s(x+t)}{s(x)} = \frac{F(x+t) - F(x)}{1 - F(x)} \qquad (3.3.1)$$

(3.3.1)-Si $F(x+t)$ funqciis mimarT teiloris formulis gamoyenebiT, miviRebT

$$F(x+t) - F(x) = F(x) + F'(\xi)\cdot t - F(x) = f(\xi)\cdot t, \quad \xi \in (x, x+t). \qquad (3.3.2)$$

Tu t sidide mcirea, an $f(\xi)$ mcired icvleba $(x,\ x+t)$ SualedSi, maSin (3.3.1) da (3.3.2)-is Tanaxmad samarTliania Semdegi miaxloebiTi toloba:

$$P(x < X \le x+t \mid X > x) \approx \frac{f(x)}{1-F(x)} \cdot t \,. \qquad (3.3.3)$$

(3.3.3)-is marjvena nawilSi mdebare damokidebulebas

$$\boxed{\mu_x = \frac{f(x)}{1-F(x)} = \frac{f(x)}{s(x)}} \qquad (3.3.4)$$

uwodeben *mokvdavobis intensivobis funqcias* da warmoadgens dazRvevis maTematikis mniSvnelovan maxasiaTebels. (3.3.4) da (3.3.3)-is Tanaxmad $\mu_x \cdot t$ sidide daaxloebiT tolia x wlis asakis adamianis sikvdilis albaTobis $(x, x+t)$ intervalSi.

μ_x funqciis mniSvneloba imiTac mtkicdeba, rom misi miaxloeba q_x moyvenilia sxc-Si (ix. danarTi 1) rogorc ZiriTadi:

$$q_x = \frac{d_x}{l_x} \approx \frac{l_0 f(x)}{l_0 s(x)} = \mu_x \,.$$

avRniSnoT, rom sandoobis TeoriaSi μ_x funqcias uwodeben *uaris funqcias* (*hazard rate function*).

mokvdavobis intensivobis funqcias gaaCnia Semdegi Tvisebebi:

I)

$$\boxed{\mu_x \ge 0} \qquad (3.3.5)$$

OII)

$$\boxed{\int_0^{\infty} \mu_u \, du = +\infty} \qquad (3.3.6)$$

marTlac, im pirobebidan, rom $s(+\infty)=0$, xolo $s(0)=1$, viRebT

$$\int_0^\infty \mu_u du = -\int_0^\infty \frac{ds(u)}{s(u)} = -\ln s(u)|_0^\infty = +\infty$$

III) $\boxed{s(x) = e^{-\int_0^\infty \mu_u du}}$ $\boxed{F(x) = 1 - e^{-\int_0^\infty \mu_u du}}$ (3.3.7)

gamoviyvenoT (3.3.7) damokidebuleba. $\mu_u = -s'(u)/s(u)$ tolobis interpretacia SeiZleba rogorc diferencialuri gantolebis. gavamravloT mocemuli gantolebis orive mxare du–ze

$$\mu_u du = -\frac{s'(u)du}{s(u)} = -\frac{ds(u)}{s(u)}$$

Semdeg movaxdinoT miRebuli tolobis orive mxaris integrireba nulidan x–mde

$$\int_0^x \mu_u du = -\int_0^x \frac{ds(u)}{s(u)} = -\ln s(u)|_0^x =$$
$$= -(\ln s(x) - \ln s(0)) = -(\ln s(x) - \ln 1) = -\ln s(x)$$

saidanac

$$s(x) = 1 - F(x) = e^{-\int_0^x \mu_u du}.$$

(3.3.7) damokidebulebebi damtkicebulia.

I) –III) Tvisebebidan gamomdinareobs, rom mokvdavobis intensivobis funqcia SeiZleba gamoyenebuli iyos sicocxlis xangrZlivobis maxasiaTebeli ganawilebis funqciis, gadarCenis funqciis da ganawilebis simkvrivesTan erTad.

3.4 arauaryofiTi SemTxveviTi sidideebis momentebis Sesaxeb Teoremebi

damtkicebebiT moviyvanoT ori sasargeblo Teorema, romelTa meSveobiT mosaxerxebelia rogorc uwyveti, ise diskretuli tipis arauaryofiTi SemTxveviTi sidideebis momentebis moZebna. aqac da SemdgomSic Teoremis an lemis damtkicebis dasrulebas avRniSnavT ♠ niSnakiT.

Teorema 3.4.1 (uwyveti SemTxveva). *vTqvaT* X *– uwyveti SemTxveviTi sididea ganawilebis* $F(x)$ *funqciiT da Sesabamisi* $f(x)=F'(x)$ *ganawilebis simkvriviT, amasTan*

1) $F(0)=0$,

2) $z(x)$ *funqcia arauaryofiTia, monotonuria da diferencerebadi,*

3) saSualo $Ez(X)=\int_0^\infty z(x)f(x)dx<\infty$. *maSin*

$$Ez(X)=z(0)+\int_0^\infty z'(t)[1-F(t)]dt \tag{3.4.1}$$

damtkiceba. nawilobrivi integrirebiT gvaqvs

$$\int_0^x z(t)f(t)dt=-\int_0^x z'(t)d[1-F(t)]=-z(t)[1-F(t)]|_0^x+\int_0^x [1-F(t)]z'(t)dt.$$

(3.4.1) formulas adgili aqvs, Tu $\lim_{t\to\infty} z(t)[1-F(t)]=0$. ganvixiloT ori SemTxveva:

a. Tu arauaryofiTi $z(x)$ funqcia ar izrdeba, maSin

$\lim_{t\to\infty} z(t)[1-F(t)]\le z(0)\ \ \lim_{t\to\infty}[1-F(t)]=0$,

ramdenadac $\lim_{t\to\infty} F(t)=1$.

b. Tu arauaryofiTi $z(x)$ funqcia ar klebulobs, maSin

$$0 \le z(t)[1-F(t)] = z(t)\int_t^{\infty} f(s)ds \le \int_t^{\infty} z(s)f(s)ds .$$

magram 3) pirobis Tanaxmad dasamtkicebel TeormaSi $\int_0^{\infty} z(x)f(x)dx$ arasakuTrivi integrali krebadia, amitom

$$\lim_{t\to\infty}\int_t^{\infty} z(s)f(s)ds = 0 ,$$

da aqedan gamomdinare, miT umetes

$$\lim_{t\to\infty} z(t)[1-F(t)] = 0 .♠$$

Teorema 3.4.2 (diskretuli SemTxveva). *vTqvaT* $K-$ *uwyveti SemTxveviTi sididea, romelic iRebs arauaryofiT mTel mniSvnelobebs, romelic xasiaTdeba* $F(k)$ *ganawilebis funqciiT da* $p(k) = P(Y=k) = F(k) - F(k-1) = \Delta F(k-1)$ *albaTobebiT, amasTan*

1) $z(k)$ *funqcia arauaryofiTia da monotonuri,*

2)saSualo $Ez(K) = \sum_{k=0}^{\infty} z(k)p(k) < \infty$. *maSin*

$$\boxed{Ez(K) = z(0) + \sum_{k=0}^{\infty}[1-F(k)]\Delta z(k)} \qquad (3.4.2)$$

damtkiceba. nawilobrivi ajamviT gvaqvs

$$\sum_{j=0}^{k-1} z(j)p(j) = -\sum_{j=0}^{k-1} z(j)\Delta[1-F(j-1)] =$$

$$= -z(j)[1-F(j-1)]\Big|_0^k + \sum_{j=0}^{k-1}[1-F(j)]\Delta z(j)$$

(3.4.2) formulas adgili aqvs, Tu $\lim_{k\to\infty} z(k)[1-F(k-1)] = 0$.

ganvixiloT ori SemTxveva:

a. Tu arauaryofiTi $z(k)$ funqcia ar izrdeba, maSin

$\lim\limits_{t\to\infty} z(k)[1-F(k-1)] \le z(0) \quad \lim\limits_{t\to\infty}[1-F(k-1)]=0$,

ramdenadac $\lim\limits_{t\to\infty} F(k-1)=0$.

b. Tu arauaryofiTi $z(k)$ funqcia ar klebulobs, maSin

$$0 \le z(k)[1-F(k-1)] = z(k)\sum_{j=k}^{\infty} p(j) \le \sum_{j=k}^{\infty} z(j)p(j).$$

magram dasamtkicebeli Teoremis 2) pirobis Tanaxmad $\sum\limits_{k=0}^{\infty} z(k)p(k)$ mwkrivi absoluturad krebadia, amitom

$$\lim_{k\to\infty}\sum_{j-k}^{\infty} z(j)p(j)=0,$$

aqedan gamomdinare

$$\lim_{k\to\infty} z(k)[1-F(k-1)]=0 .♠$$

x	0	10	20	30	40	50	60	70	80	85	90
$\overset{\circ}{e}_{x:5]}$	4,853	4,844	4,812	4,792	4,750	4,688	4,583	4,375	3,750	2,500	—
$\mathbf{D}\min(T(x),5)$	0,444	0,496	0,563	0,651	0,771	0,943	1,215	1,632	3,604	2,080	—
$\overset{\circ}{e}_{x:10]}$	9,445	9,375	9,286	9,167	9,000	8,750	8,333	7,500	5,000	—	—
$\mathbf{D}\min(T(x),10)$	2,777	3,836	4,252	4,861	5,666	6,771	8,333	13,542	8,333	—	—

3.5 sicocxlis saSualo dro, misi dispersia, asimetriis koeficienti da eqscesi

cnobilia, rom X SemTxveviTi sididis sicocxlis saSualo xangrZlivoba

$$\overset{0}{e}_0 = EX = \int_0^{\infty} x f(x)dx, \qquad (3.5.1)$$

warmoadgens erT-erT mniSvnelovan maCvenebels, romlis saSualebiTac xdeba sxvadasxva qveynis mosaxleobis cxovrebis xarisxis Sedareba. $\overset{0}{e}_0$-Tan erTad X SemTxveviTi sididis mniSvnelovan praqtikul makromaxasiaTebelebs warmoadgenen misi dispersia

$$DX = E(X-\overset{0}{e}_0)^2 = EX^2 - (\overset{0}{e}_0)^2 = \int_0^{\infty} x^2 f(x)dx - (\overset{0}{e}_0)^2, \qquad (3.5.2)$$

asimetriis koeficinti

$$\gamma = \frac{E(X-\overset{0}{e}_0)^3}{(DX)^{3/2}}, \qquad (3.5.3)$$

da eqscesi

$$oe = \frac{E(X-\overset{0}{e}_0)^4}{(DX)^2} - 3. \qquad (3.5.4)$$

magaliTad, $\overset{0}{e}_0$ sidide iZleva SemTxveviT SerCeuli axalSobilis saSualo sicocxlis xangrZlivobas, DX - $\overset{0}{e}_0$-s mimarT misi sicocxlis xangrZlivobis gabnevis saSualo kvadrats, asimetriis koeficienti $\gamma > 0$ miuTiTebs X SemTxveviTi sididis ganawilebis marjvena mxareSi arsebul grZel kudze, xolo Tu eqscesi $oe \approx 0$, maSin SeiZleba CavTvaloT, rom X ganawilebulia daaxloebiT normaluri kanoniT (normaluri SemTxveviTi sididisaTvis $oe = 0$).

aqve mniSvnelovania aRiniSnos, rom ukidures SemTxvevaSi, pirveli oTxi momentis mniSvneloba an Sefaseba saSualebas iZleva X SemTxveviTi sididis ganawilebis ucnobebi mivuaxlovoT sxvadasxva parametrul ojaxebs.

3.4.1 Teoremis saSualebiT advilad gamovsaxavT (3.5.1) – (3.5.4) formulebs gadarCenis funqciis terminebSi. magaliTad, $\overset{0}{e}_0 = EX$ -sTvis funqcia $z(x) = x$, $z(0) = 0, z'(x) = 1$ da amgvarad

$$EX = \int_0^\infty (1 - F(x))dx = \int_0^\infty s(x)dx \qquad (3.5.5)$$

analogiurad EX^2-sTvis viRebT $z(x) = x^2$, $z(0) = 0$, $z'(x) = 2x$ da

$$EX^2 = 2\int_0^\infty x(1 - F(x))dx = 2\int_0^\infty xs(x)dx \ . \qquad (3.5.6)$$

3.6 mokvdavobis analitikuri kanonebi: de muavris, gompertcis, maikhamis, vaibulis da erlangis modelebi

mokvdavobis procesebis Teoriuli analizisas, realuri situaciebis Tavdapirvel da gamartivebul Seswavlisas, rogorc wesi, gamoiyeneba albaTobebis standartuli modelebi, romlebic mkvlevarisaTvis saintereso ZiriTadi kononzomierebebis gamovlenis saSualebas iZlevian. amasTan mokvdavobis zogierTi realuri procesebi sakmaod kargad aproqsimdeba qvemoT ganxiluli kanonebiT.

de muavris modeli (*de Moivre*)

albaTobis Teoriis erT-erTma fuZemdebelma de muavrma 1729 wels SemogvTavaza CavTvaloT, rom sicocxlis dro Tanabradaa ganawilebuli $(0, \omega)$ intervalze, sadac ω-s, Tanabari ganawilebis kanonis ganmsazRvrel parameters, uwodeben *zRvrul asaks*. cxadia, rom am modelisaTvis $0 < x < \omega$ -sTvis gvaqvs

$$f(x)=\frac{1}{\omega},\quad F(x)=\frac{x}{\omega},\quad s(x)=1-\frac{x}{\omega},\quad \mu_x=\frac{f(x)}{s(x)}=\frac{1}{\omega-x}.$$

de muavris kanoni ar asaxavs mraval damaxasiaTebel Taviseburebas, romelic dakavSirebulia adaminis sicocxlis xangrZlivobasTan. magaliTad, ganxilul modelSi sikvdilebis mrudi warmoadgens horizontalur wrfes, xolo empiriul mrudebs maqsimuli gaaCniaT 80 wlis farglebSi.

gompertcis modeli (*Gompertz*)

am modelSi (1825 weli) mokvdavobis intensivoba mocemulia Semdegi formuliT:

$$\mu_x=\frac{f(x)}{s(x)}=Be^{\alpha x},$$

sadac $\alpha > B > 0$- raRac parametrebia. aq gadarCenis funqcia tolia

$$s(x)=\exp\left[-\int_0^x \mu_u du\right]=\exp\left[-\int_0^x Be^{\alpha u} du\right]=\exp[-B(e^{\alpha x}-1)/\alpha],$$

xolo sikvdilebis mrudi _

$$f(x)=\mu_\alpha s(x)=B\exp[\alpha x-B(e^{\alpha x}-1)/\alpha]$$

da gaCnia maqsimumi $x=(\ln\alpha-\ln B)/\alpha$ wertilSi (ix. davaleba 4.6.2).

am informaciis gamoyeneba SeiZleba α da B parametrebis moiZebnisas. magaliTad, Tu raimeze dayrdnobiT CvenTvis cnobilia, rom maqsimaluri sikvdilianoba daikvirveba 78,3 wlis asakisaTvis, xolo gompertcis ganawilebis kvantili $x_{0,25}$ tolia 33,4-is, maSin Sefasebis mosaZebnad gantolebaTa sistema iRebs aseT saxes:

$$\begin{cases}(\ln\alpha-\ln B)/\alpha=78{,}3\\ 1-\exp[-B(e^{\alpha 33{,}4}-1)/\alpha]=0{,}25\end{cases} \qquad (3.6.1)$$

wrfivi gantolebaTa (3.6.1) sistema SeiZleba amoixsnas kompiuteriT ricxviTi meTodebis gamoyenebiT.

maikehamis modeli (*MAkeham*)

mogvianebiT, 1860 wels, maikehamma SemogvTavaza mokvdavobis intensivobis Semdeg SedarebiT zogad saxis funqciasTan daaxloeba:

$\mu_x = A + Be^{\alpha x}$,

sadac A parametric iTvaliswinebs ubedur SemTxvevebTan dakavSirebul riskebs, xolo $Be^{\alpha x}$ Sesakrebi iTvaliswinebs asakis gavlenas mokvdavobaze. aq

$$s(x) = \exp\left[-\int_0^x (A + Be^{\alpha u})du\right] = \exp[-Ax - B(e^{\alpha x}-1)/\alpha],$$

$$f(x) = -s'(x) = [A + Be^{\alpha x}]\exp[-Ax - B(e^{\alpha x}-1)/\alpha].$$

zemoT moyvanili modelebidan maikehamis kanoni adamianis mokvdavobis procesis Seswavlisatvis yvelaze ufro Sesaferisia, ramdenadac masSi gaTvaliswinebulia is, rom patara asakisaTvis mokvdavobaSi umetes rols ubeduri SemTxvevebi asruleben, xolo asakis gazrdasTan erTad maTi roli sustdeba.

vaibulis modeli (*Weibull*)

1939 wels vaibulma mokvdavobis intensivobis martivi miaxloebis saxiT Semdegi xarisxovani funqcia gamoiyena:

$\mu_x = kx^n$

romlis saxe gansazRvravs gadarCenis funqcias

$$s(x)=\exp\left[-\int_0^x ku^n du\right]=\exp\left[-\frac{k}{n+1}x^{n+1}\right]$$

da sikvdilebis mrudi

$$f(x)=-s'(x)=kx^n\exp\left[-\frac{k}{n+1}x^{n+1}\right]$$

maqsimumiT wertilSi $x=(n/k)^{1/(n+1)}$ (ix. davaleba 4.6.2).

erlangis modeli

ganvixiloT meore rigis erlangis modeli romlisaTvisac sikvdilebis mrudi Semdegi formuliT aRiwereba:

$$f(x)=\frac{x}{a^2}e^{-\frac{x}{a}},\quad x\geq 0.$$

am SemTxvevaSi gadarCenis funqciaa:

$$s(x)=\frac{x+a}{a}e^{-\frac{x}{a}},$$

xolo mokvdavobis intensivoba:

$$\mu_x=\frac{x}{a(x+a)}.$$

dasasruls avRniSnoT, rom analitikuri kanonebis garkveul upiratesobas warmoadgens is, rom maTvis sicocxlis xangrZlivobis albaTuri maxasiTeblebi swrafad SeiZleba iqnas gamoTvlili parametrebis mcire raodenobisaTvis. es SeiZleba mniSvnelovani aRmoCndes aseve im SemTxvevebSic rodesac xelmisawvdomi monacemebi mravalwevrebs ar warmoadgenen.

3.7 sakontrolo davalebebi

davaleba 3.7.1

gadarCenis funqciis asakze damokidebulebis xasiaTi gvaZlevs Semdeg cxrilebs:

მამაკაცები (სსრკ, 1984-1985წ.)

x	0	14	20	30	40	50	60	70	80	90	100	110
$s_1(x)$	1,000	0,954	0,947	0,922	0,878	0,795	0,651	0,434	0,188	0,003	0	0

ქალები (სსრკ, 1984-1985 წ.)

x	0	14	20	30	40	50	60	70	80	90	100	110
$s_2(x)$	1,000	0,964	0,961	0,953	0,939	0,908	0,841	0,700	0,417	0,008	0	0

აშშ-ს მოსახლეობა

x	0	10	20	30	40	50	60	70	80	90	100	110
$s_3(x)$	1,000	0,983	0,977	0,965	0,949	0,915	0,837	0,682	0,432	0,142	0,012	0

1) gadarCenis funqciis azridan gamomdinare gaanalizeT mocemuli cxrilebi.

2) rigiTobiTi statistikis gamoyenebiT aageT gadarCenis funqciis parametruli Sefaseba de muavris modelisaTvis, amisaTvis isargebleT moyvenili cxrilebiT.

3) mocemuli cxrilebis safuZvelze aRwereT vaibulis modelisaTvis gadarCenis funqciis parametruli Sefasebis agebis ramdenime xerxi (vaibulis modelSi mokvdavobis intensivoba μ_x uaxlovdeba kx^n saxis xarisxovan funqcias).

davaleba 3.7.2

gompertcis modelSi mokvdavobis intensivoba μ_x uaxlovdeba $Be^{\alpha x}$ saxis maCveneblian funqcias, sadac $\alpha > B > 0$ raRac parametrebia.

1) gompertcis modelSi mokvdavobis mrudisaTvis ipoveT maqsimumis wertili.

2) sad SeiZleba iqnas gamoyenebuli miRebuli Sedegi?

3) ipoveT sikvdilebis mrudisaTvis maqsimumis wertili vaibulis modelSi.

davaleba 3.7.3

vTqvaT gvaqvs ori gadarCenis funqcia:

$$s_1(x) = e^{-x^3/12}, \quad x \geq 0;$$

$$s_2(x) = \left(1 - \frac{x}{\omega}\right)^{\alpha}, \quad 0 \leq x < \omega, \quad \alpha > 0.$$

$s_2(x)$ gadarCenis funqcia $\alpha = 1$-sTvis aRwers de muavris models.

1) ipoveT $s_i(x)$-is Sesabamisi, μ_{ix} sikvdilianobis intensivoba, $f_i(x)$ mokvdavobis mrudebi da ganawilebis $F_i(x)$ funqcia $i = 1,2$.

2) ipoveT imis albaToba, rom 10-dan 30 wlamde mokvdeba:

a) SemTxveviT SerCeuli adamiani;

b) 10 wlis dazRveuli mozardi.

davaleba 3.7.4

ganvixiloT

$$f(x) = \frac{x}{a^2} e^{-x/a}.$$

1) vaCvenoT, rom $f(x)$ SeiZleba ganixilebodes, rogorc sikvdilebis mrudi.

2) ganvsazRvroT $s(x)$ gadarCenis funqciis Sesabamisi saxe da μ_x mokvdavobis intensivoba, aseve Sesabamisi sicocxlis saSualo xangrZlivoba $\overset{0}{e}_0$.

3) sicocxlis xangrZlivobis cxrilis meSveobiT gaanalizeT mocemuli analizuri Sefasebebis Sesabamisobis xarisxi realur monacemebTan (SeadareT sicocoxlis saSualo dro sikvdilebis mrudis maqsimumis wertils).

4) ipoveT sicocxlis xangrZlivobis dispersia.

davaleba 2.7.5

ganvixiloT 2.18.1 davalebis gadarCenis funqciis mniSvnelobaTa sami cxrili. gamoTvaleT axalSobilebis l_0 sawyis jgufebSi warmomadgenelTa raodenobis saSualo da dispersia, romlebic daixocebian 50-dan 70 wlamde.

davaleba 3.7.6

$n = 2{,}4$ mTeli ricxvebisaTvis vaibulis modelis gamoyenebiT moaxdineT 2.18.1 davalebidan arCeuli romelime $s_i(x)$, $i = 1{,}2{,}3$ gadarCenis funqciis aproqsimacia.

1) umciresi kvadratTa meTodis meSveobiT ipoveT k parametri.

2) aageTY miRebuli aproqsimaciebis grafikebi $n = 2{,}4$-sTvis, gamoarkvieT garkveuli kriteriumis TvalsazrisiT, romelic Tqven TviTon SearCieT, agebuli modelebidan romelia saukeTeso.

3) rogor vipovoT n-is optimaluri mniSvneloba?

4) SesaZlebelia Tu ara k da n parametrebis mixedviT optimizaciis erTdroulad ganxorcieleba?

davaleba 3.7.7

davuSvaT sikvdilebis mrudi aRiwereba Semdegi formuliT

$$f(x) = \frac{x}{a} e^{-x/a}, x \geq 0.$$

1) ipoveT sicocxlis narCeni $T(x) = X - x$ (X, x _ Sesabamisad individis sikvdilis momenti da asakia) drois ganawilebis $F_x(t)$ funqcia, $F_x(t) = P(T(x \leq t))$.

2) aCveneT, rom sicocxlis narCeni drois ganawilebis $f_x(t) = \frac{d}{dt} F_x(t)$ simkvrive warmoadgens $\frac{1}{a} e^{-t/a}$ esqponencialuri da $\frac{t}{a^2} e^{-t/a}$ erlangovis simkvriveebis jams.

3) ipoveT $P(T(x) > t)$–is albaToba, $\lim_{x \to \infty} P(T(x) > t)$ zRvari da gaarkvieT, SesaZlebelia Tu ara sikvdilebis mrudis aseTi aproqsimacia didi x asakebisaTvis.

Tavi 4. narCeni sicocxlis xangrZlivoba

4.1 narCeni sicocxlis xangrZlivoba (*time-untill-death*), misi ganawileba

Tu adamianma miaRwia x wels, maSin sadazRvevo kompanias, misi saqmianobis specifikidan gamomdinare, zogadad rom vTqvaT, ainteresebs ara misi zogadi X sicocxlis xangrZlivoba, aramed narCeni sicocxlis xangrZlivoba $T(x)=X-x$. vipovoT $T(x)$ SemTxveviTi sididis ganawilebis funqcia, romelic warmoadgens $X-x$ sididis ganawilebis pirobiT funqcias, roca $X>x$:

$$
\begin{aligned}
&F_x(t)=P(T(x)\le t)=P(X-x\le t\mid X>x)=\\
&=P(X\le x+t\mid X>x)={}_tq_x=\frac{P(X\le x+t\cap X>x)}{P(X>x)}=\\
&=\frac{P(x<X\le x+t)}{P(X>x)}=\frac{F(x+t)-F(x)}{1-F(x)}.
\end{aligned}
\tag{4.1.1}
$$

Tu cnobilia cxriluri mniSvnelobebi l_x da l_{x+t} maSin maTi meSveobiT $F_x(t)$ ganawilebis funqcia Semdegnairad gamoisaxeba:

$$F_x(t)=\frac{s(x)-s(x+t)}{s(x)}=\frac{l_x/l_0-l_{x+t}/l_0}{l_x/l_0}=\frac{l_x-l_{x+t}}{l_x}.$$

gamoviyenoT (3.1.1) da vipovoT $T(x)$ SemTxveviTi sididis $f_x(t)$ simkvrive:

$$f_x(t)=\frac{d}{dt}F_x(t)=\frac{f(x+t)}{1-F(x)},\quad 0\le t<\infty.$$

magaliTis saxiT ganvixiloT de muavris da maikehamis kanonebi.

de muavris modeli. moxerxebulobisaTvis SemovitanoT aRniSvna

$$I_x(a,b]=\begin{cases}1, x\in(a,b]\\ 0, x\notin(a,b]\end{cases}$$

vTqvaT, sicocoxlis xangrZlivobis ganawileba aRiwereba de muavris kanoniT, romlisTvisac ganawilebis simkvrive da gadarCenis funqcia gamoisaxeba Sesabamisad Semdegi formulebiT:

$$f(x)=\frac{I_x(0,\omega)}{\omega}, \qquad s(x)=I_x(-\infty,\omega)-\frac{xI_x(0,\omega)}{\omega}.$$

ramdenadac $0<X<\omega$, amdenad $0<T(x)<\omega-x$ da

$$F_x(t)=\frac{t}{\omega-x}I_t(0,\omega-x)+I_t[\omega-x,\infty),$$

xolo $t\in(0,\omega-x)$-sTvis

$$f_x(t)=\frac{d}{dt}\left(\frac{t}{\omega-x}\right)=\frac{1}{\omega-x}, \quad t\in(0,\omega-x], \qquad (4.1.2)$$

e.i. sicocxlis narCeni droc aseve Tanabradaa ganawilebuli, magram $(0,\omega-x)$ SualedSi.

maikehamis modeli. am kanonisaTvis

$$\mu_t=A+Be^{\alpha t}, \quad s(x)=\exp\left[-\int_0^x\mu_t dt\right]=\exp[-Ax-B(e^{\alpha t}-1)/\alpha,$$

$$f(x)=-s'(x)=[A+Be^{\alpha x}]\exp[-Ax-B(e^{\alpha x}-1)/\alpha],$$

$$f(x+t)=-s'(x)=[A+Be^{\alpha(x+t)}]\exp[-A(x+t)-B(e^{\alpha(x+t)}-1)/\alpha].$$

amotom sicocxlis narCeni drois ganawilebis simkvrive tolia:

$$f_x(t)=\frac{f(x+t)}{s(x)}=[A+Be^{\alpha x}e^{\alpha t}]\exp[-At-Be^{\alpha x}(e^{\alpha t}-1)/\alpha],$$

saidanac gamomdinareobs, rom $T(x)$ sicocxlis narCeni dros aseve gaaCnia maikehamis ganawileba Semdegi parametrebiT: $A_x=A$, $B_x=Be^{\alpha x}$, $\alpha_x=\alpha$.

4.2 $T(x)$**–sTan dakavSirebuli sidideebi**: $_{t}q_{x}$, $_{tx}$, q_{x}, p_{x}, $_{t|u}q_{x}$, $_{t|}q_{x}$

aqtuarul maTematikaSi $_{t}q_{x}$ da $_{tx}$ simboloebiT Sesabamisad aRniSnuli $P(T(x)\le t)$ albaToba da $P(T(x)>t)$ damatebiTi albaToba ganisazRvreba Semdegi formulebiT:

$$_{t}q_{x}=P(T(x)\le t)=\frac{s(x)-s(x+t)}{s(x)}, \qquad (4.2.1)$$

$$_{tx}=P(T(x)>t)=1-{}_{t}q_{x}=\frac{s(x+t)}{s(x)}. \qquad (4.2.2)$$

$_{t}q_{x}$ sidide gamosaxavs adamianis sikvdilis albaTobas x wlis asakSi drois $(x,x+t]$ SualedSi, xolo $_{tx}$ - imis albaTobas, rom aseTi adamiani aRwevs $x+t$ asaks.

$t=1$-sTvis $_{t}q_{x}$ da $_{tx}$-Si wina indeqsebi gamoitoveba da zogadi (4.2.1) da (4.2.2) formulebidan viRebT

$$q_{x}=P(T(x)\le t)=\frac{s(x)-s(x+1)}{s(x)}=\frac{l_{x}-l_{x+1}}{l_{x}}, \qquad (4.2.3)$$

$$_{x}=P(T(x)>t)=\frac{s(x+t)}{s(x)}=\frac{l_{x+1}}{l_{x}}. \qquad (4.2.4)$$

q_{x} da $_{x}$ cvladebi praqtikaSi ufro xSirad gamoiyeneba; q_{x} sidide tolia individumis x wlis asakSi sikvdilis albaTobis uaxlesi wlis ganmavlobaSi, xolo p_{x}- imis albaTobaa, rom is erT wels mainc icocxlebs.

(4.2.3) formulis gaTvaliswinebiT warmovadginoT

$$\boxed{q_{x}=\frac{d_{x}}{l_{x}}}$$

q_x sidide warmoadgens mesame ZiriTad maxasiaTebels, romelic Sedis sxc-Si (danarTi 1). ramdenadac $l_x = l_0 s(x), d_x \approx l_0 f(x)$, amdenad $q_x \approx \frac{f(x)}{s(x)} = \mu_x$, e.i. q_x mrudis forma daaxloebiT emTxveva mokvdavobis intensivobis funqciis mrudis formas.

amgvarad, gavarkvieT im sami ZiriTadi sididis arsi, romelic Sedis sxc-Si (danarTi 1):

$l_x = l_0 s(x)$ multifikatiuri l_0 mamravliT dakavSirebulia $s(x)$ gadarCenis funqciasTan da imeorebs mis x asakTan dakavSirebiT cvlilebis xasiaTs;

$d_x \approx l_0 f(x)$ multifikatiuri l_0 mamravliT dakavSirebulia miaxloebiTi tolobiT $f(x)$ sikvdilebis mrudTan da miaxloebiT imeorebs mis x asakTan dakavSirebiT cvlilebis xasiaTs;

$q_x \approx \frac{f(x)}{s(x)} = \mu_x$ daaxloebiT emTxveva μ_x mokvdavobis intensivobas.

sadazRvevo saqmeSi SedrebiT udro rTuli SemTxvevebis ganxilvis aucilebloba Cndeba. magaliTad, ganvsazRvroT imis albaToba, rom x asakis adamiani icocxlebs t wels, magram mokvdeba Semdegi u wlis ganmavlobaSi.

am albaTobas Semdegnairad aRniSnaven

$$_{t|u}q_x = P(t < T(x) \le t + u) \qquad (4.2.5)$$

da is SeiZleba gamosaxuli iyos an $_{t|u}q_x$ funqciiT an $_{t}x$-iT, an mxedvelobaSi miviRebT ra (4.2.1) da (4.2.2) damokidebulebebs, sadazRvevo maTematikis ZiriTadi funqciiT – gadarCenis $s(x)$ funqciiT:

$$ {}_{t|u}q_x = P(t < T(x) \le t+u) = \\ = P(T(x) \le t+u) - P(T(x) \le t) = {}_{t+u}q_x - {}_t q_x \qquad (4.2.6) $$

$$ {}_{t|u}q_x = P(t < T(x) \le t+u) = \\ = P(T(x) > t) - P(T(x) > t+u) = {}_t p_x - {}_{t+u}p_x \qquad (4.2.7) $$

$$ {}_{t|u}q_x = \frac{s(x+t) - s(x+t+u)}{s(x)}. \qquad (4.2.8) $$

isev $u=1$ sicocxlis dazRvevis praqtikisaTvis yvelaze saintereso SemTxvevaa, da Cveulebriv es indeqsi gamoitoveba xolme. (4.2.6), (4.2.7) da (4.2.8) formulebis Tanaxmad gvaqvs:

$$ {}_{t|}q_x = \frac{s(x+t) - s(x+t+u)}{s(x)} = {}_{t+1}q_x - {}_t q_x = {}_t p_x - {}_{t+1}p_x = \frac{s(x+t) - s(x+t+1)}{s(x)}. \qquad (4.2.9) $$

4.3. sicocxlis saSualo narCeni dro, misi dispersia. asimetriis koeficienti da eqscesi

aqtuarul maTematikaSi x asakis adamianis sicocxlis saSualo narCeni dro aRiniSneba Semdegnairad:

$$ \overset{0}{e}_x = ET(x); $$

am maxasiaTebels xSirad iyeneben sadazRvevo momsaxurebis bazarze situaciis analizis dros. gasagebia, rom $ET(0) = EX = \overset{0}{e}_0$, da aqedan gamomdinare, $\overset{0}{e}_0$ sicocxlis saSualo xangrZlivoba metia $\overset{0}{e}_x$ sicocxlis saSualo narCeni droze nebismieri $x > 0$-sTvis.

ramdenadac $T(x)$ warmoadgens arauaryofiT SemTxveviT sidides, 3.4.1 Teoremis gamoyenebiT, miviRebT

$$\overset{0}{e}_x = ET(x) = \int_0^\infty t dF_x(t) = \int_0^\infty t s P(T(x) \prec t) = \int_0^\infty P(T(x) > t)dt = \int_0^\infty {}_t p_x dt .$$

gamoviyenebT ra (3.2.2) formulas da gardavqmniT ra bolo integrals

$$\int_0^\infty {}_t p_x dt = \frac{1}{s(x)}\int_0^\infty s(x+t)dt = \frac{1}{s(x)}\int_0^\infty s(u)du ,$$

mivdivarT Semdeg damokidebulebamde

$$\boxed{\overset{0}{e}_x = \frac{1}{s(x)}\int_x^\infty s(u)du} \qquad (4.3.1)$$

analogiuri msjelobiT, $T(x)$ SemTxveviTi sididis meore sawyisi momentisaTvis gvaqvs:

$$E[T(x)]^2 = 2\int_0^\infty tP(T(x) > t)dt = 2\int_0^\infty t\, {}_t p_x dt = \frac{2}{s(x)}\int_0^\infty ts(x+t)dt ,$$

saidanac

$$DT(x) = \frac{2}{s(x)}\int_0^\infty ts(x+t)dt - \left(\overset{0}{e}_x\right)^2 .$$

asimetriis koeficientis da eqscesis mosaZebnad aucilebelia $E[T(x)]^3$–is da $E[T(x)]^4$–is gansazRvra:

$$E[T(x)]^3 = 3\int_0^\infty t^2 P(T(x) > t)dt = 3\int_0^\infty t^2\, {}_t p_x dt = \frac{3}{s(x)}\int_0^\infty t^2 s(x+t)dt ,$$

$$E[T(x)]^4 = \frac{4}{s(x)}\int_0^\infty t^3 s(x+t)dt .$$

magaliTi 4.3.1. ipoveT $T(x)$ sicocxlis narCeni drois $\overset{0}{e}_x$ saSualo, $DT(x)$ dispersia da saSualo kvadratuli gadaxra $\sigma(T(x)) = \sqrt{DT(x)}$ de muavris modelSi.

amoxsna. am modelisaTvis

$$P(T(x) > t) = {}_tp_x = \frac{s(x+t)}{s(x)} = \frac{1-(x+t)/\omega}{1-x/\omega}, \quad 0 \le t \le \omega - x.$$

amgvarad,

$$\overset{0}{e}_x = \int_0^{\omega-x} \frac{\omega-x-t}{\omega-x} dt = \frac{-1}{\omega-x}\int_{\omega-x}^{0} u du = \frac{\omega-x}{2}. \qquad (4.3.2)$$

am formulis miReba ufro advilia (4.3.1)-dan:

$$\overset{0}{e}_x = \frac{\omega}{\omega-x}\int_x^{\omega} \frac{\omega-t}{\omega} dt = \frac{-1}{\omega-x}\int_{\omega-x}^{0} u du = \frac{\omega-x}{2}.$$

aseve Znelia imis Cveneba, rom

$$DT(x) = \frac{(\omega-x)^2}{12}.$$

4.4 Sereuli dazRveva. nawilobrivi narCeni sicocxlis xangrZlivoba

ganvixiloT *asakis miRwevamde* n-*wliani dazRveva* (*n-year endowment insurance*) anu Sereuli dazRveva, romlis arsi imaSi mdgomareobs, rom klienti dazRvevis xelSekrulebas debs n wliT da gadaxda xorcieldeba:

_ dazRveulis sikvdilis nebismier momentSi, Tu is dgeba n-wliani periodis damTavrebamde,

_ an n-wliani periodis bolos, Tu dazRvevuli cocxali rCeba.

sadazRvevo gadaxdis momenti gamoisaxeba $\min(T(x), n)$ formuliT da ewodeba nawilobrivi sicocxlis xangrZlivoba, xolo Sesabamis maTematikur lodins - *nawilobrivi saSualo sicocxlis xangrZlivoba* da $\overset{0}{e}_{x:n]}$-iT aRiniSneba:

$$\overset{0}{e}_{x:n]} = E\min(T(x), n).$$

imisaTvis, rom 3.4.1 Teorema gamoviyenoT, aucilebelia vipovoT $P(\min(T(x),n)>t)$ albaToba. ramdenadac $t<n$ $\min(T(x),n)>t$ xdomiloba tolfasia $T(x)>t$ xdomilobis, amdenad

$$P(\min(T(x),n>t)=\begin{cases} {}_t p_x, 0\le t\le n \\ 0, t\ge n. \end{cases}$$

(3.2.2) formulis gamoyenebiT, miviRebT:

$$\overset{0}{e}_{x:n]}=\int_0^n {}_t p_x dt=\frac{1}{s(x)}\int_0^n s(x+t)dt=\frac{1}{s(x)}\int_x^{x+n} s(u)du\,.$$

nawilobrivi sicocxlis xangrZlivobis dispersiisaTvis gvaqvs

$$D\min(T(x),n)=2\int_0^n t_t p_x dt-(\overset{0}{e}_{x:n]})^2=\frac{2}{s(x)}\int_0^n ts(x+t)dt-(\overset{0}{e}_{x:n})^2.$$

magaliTi 4.4.1. vipovoT nawilobrivi sicocxlis xangrZlivobis saSualo da dispersia de muavris modelSi.

amoxsna. upirvelesad avRniSnoT, rom $x+n<\omega$, amitom

$$\overset{0}{e}_{x:n]}=\frac{1}{s(x)}\int_x^{x+n} s(u)du=\frac{1}{1-x/\omega}\int_x^{x+n}(1-u/\omega)du=\frac{1}{\omega-x}\int_x^{x+n}(\omega-u)du=\frac{2n(\omega-x)-n^2}{2(\omega-x)}=$$
$$=n-\frac{n^2}{2(\omega-x)} \qquad (4.4.1)$$

(3.4.1) formulidan gamomdinareobs, rom:

1) $\overset{0}{e}_{x:n]}<n$;

2) Tu $n=\omega$ da $x=0$, maSin $\overset{0}{e}_{0:\omega]}=\overset{0}{e}=\omega/2$;

3) Tu $x=\omega-n$, maSin $\overset{0}{e}_{(\omega-n):n]}=n/2$.

aseve Znelia araa dispersiisaTvis gamosaxulebis miReba:

$$D\min(T(x),n)=\frac{n^3}{3(\omega-x)}-\frac{n^4}{4(\omega-x)^2}\,. \qquad (4.4.2)$$

(3.4.2) formulidan gamomdinareobs, rom $n=\omega$ da $x=0$-sTvis

$$D\min(T(x),\omega)=\frac{\omega^2}{12},$$

e.i. viRebT $(0,\omega)$ intervalze Tanabari ganawilebis kanonis dispersiisaTvis cnobil Sedegs.

ganvixiloT ω=90 wlisaTvis ori konkretuli SemTxveva roca n =5 da n= 10 wels. nawilobrivi sicocxlis xangrZlivobis saSualo da dispersiis mniSvnelobebi gamoiTvleba (4.4.1) da (4.4.2) formulebiT zogierTi asakisaTvis da n=5, 10-sTvis Sesulia Semdeg cxrilSi:

x	0	10	20	30	40	50	60	70	80	85	90
$\mathring{e}_{x:5]}$	4,853	4,844	4,812	4,792	4,750	4,688	4,583	4,375	3,750	2,500	—
$\mathbf{D}\min(T(x),5)$	0,444	0,496	0,563	0,651	0,771	0,943	1,215	1,632	3,604	2,080	—
$\mathring{e}_{x:10]}$	9,445	9,375	9,286	9,167	9,000	8,750	8,333	7,500	5,000	—	—
$\mathbf{D}\min(T(x),10)$	2,777	3,836	4,252	4,861	5,666	6,771	8,333	13,542	8,333	—	—

4.5 sicocxlis narCeni drois damrgvaleba, misi ganawileba, saSualo da dispersia

aqtuarul maTematikaSi sicocxlis narCeni drosTan $T(x)$ erTad ganixileba misi mTeli nawili $K(x)=[T(x)]$, romelsac *damrgvalebul sicocxlis narCen xangrZlivobas* (*curtate-future-lifetime*) uwodeben. es Semdeg mizezebTanaa dakavSirebuli:

1) adamiani Cveulebriv Tavis asaks mTeli wlebiT iTvlis;

2) sicocxlis dazRvevis xelSekrulebebi, rogorc wesi, wlebis mTel ricxvebiT ideba;

3) sxc-Si monacemebi moyvanilia asakebisaTvis mTel wlebSi.

Tu $T(x)$=10 weli 9Tve=10,75 weli, maSin $K(x)$=10 weli. amgvarad, $K(x)$ SemTxveviTi sidide warmoadgens diskretul SemTxveviT sidides, romelic iRebs mTel mniSvnelobebs. rogorc cnobilia aseTi SemTxveviTi sididis amomwurav maxasiaTebels warmoadgens Semdegi albaTobebis nakrebi

$P(K(x)=k)$, $k=$0, 1, 2, ...

gasagebis, rom

$$P(K(x)=k)=P(k\le T(x)<k+1).$$

ramdenadac $T(x)$ - uwyveti SemTxveviTi sididea, amdenad

$$P(K(x)=k)=P(T(x)=k+1)=0,$$

da amitom

$$P(K(x)=k)=P(k\le T(x)<k+1)=$$
$$=\frac{s(x+k)-s(x+k+1)}{s(x)}={}_kp_x-{}_{k+1}p_x\ .$$

axla, Tu gaviTvaliswinebT, rom $X=T(0)$, SesaZlebelia ganvsazRvroT $K(0)=[X]$ damrgvalebuli sicocxlis xangrZlivobis ganawilebac:

$$P(K(0)=k)=\frac{s(k)-s(k+1)}{s(0)}=s(k)-s(k+1)=\frac{l_k-l_{k+1}}{l_0}=\frac{d_k}{l_0}.$$

magram ramdenadac $d_x\approx l_0 f(x)$, amdenad

$$P(K(0)=k)\approx f(k), \qquad (4.5.1)$$

sadac $f(x)$- X SemTxveviTi sididis simkvrivea, amasTan (4.5.1) miaxloebiTi tolobis marjvena nawilSi, zogadad rom vTqvaT, ufro swori iqneboda dagvewera $f(k)\cdot 1$ weli, ramdenadac mis marcxena nawilSi uzoganzomilo sidide mdebareobs.

amitom 4.5.1 toloba gulisxmobs, rom

$P(K(0)=k)\approx f(k)\cdot 1$ we l i,

saidanac Cans, r om sikvdi l ebis m r udi mWi d r o daa dakavSi r ebu l i dam r gva l ebu l i si c o c x l is xang r Z l ivobis ganawi l ebasTan.

$K(x)$ SemTxvevi T i sididis saSua l os uwo deben *saSua l o dam r gva l ebu l sicocx l is nar Cen xang r Z l ivobas* da aRiniSneba

$e_x = EK(x)$ -iT,

da diskr e t u l i SemTxvevi T i sididisaTvis maTemat ikur i l odinis ganmar tebis Tanaxmad:

$$e_x=\sum_{k=1}^{\infty} kP(K(x)=k .$$

r amdenadac,

$$P(K(x)=k)=\frac{s(x+k)-s(x+k+1)}{s(x)}$$

da

$$\sum_{k=1}^{\infty} k[s(x+k)-s(x+k+1)]=1\cdot s(x+1)+2\cdot s(x+2)+...-1\cdot s(x+2)-2s(x+3)-...=\sum_{k=1}^{\infty} s(x+k)$$

maSin

$$\boxed{e_x=\frac{1}{s(x)}\sum_{k=1}^{\infty} s(x+k)} .$$

an l ogiu r ad vpou l obT meo r e sawyis momen t s:

$$E[K(x)]^2=\sum_{k=0}^{\infty} k^2 P(K(x)=k)=$$

$$=\frac{1}{s(x)}\left\{\sum_{k=0}^{\infty} k^2 s(x+k)-\sum_{k=0}^{\infty} k^2 s(v+k+1)\right\}=$$

$$=\frac{1}{s(x)}\left\{\sum_{k=0}^{\infty} k^2 s(x+k)-\sum_{k=0}^{\infty} (k+1)^2 s(x+k+1)+\sum_{k=0}^{\infty}(2k+1)s(x+k+1)\right\}=$$

$$=\frac{1}{s(x)}\sum_{k=1}^{\infty}(2k-1)s(x+k)=\frac{2}{s(x)}\sum_{k=1}^{\infty} ks(x+k)-e_x$$

r amdenadac,

$$\sum_{k=0}^{\infty} k^2 s(x+k) - \sum_{k=0}^{\infty}(k+1)^2 s(x+k+1) + \sum_{k=0}^{\infty}(2k+1)s(x+k+1) =$$

$$= 1^2 \cdot s(x+1) + 2^2 \cdot s(x+2) + 3^2 \cdot s(x+3) + \cdots - 1^2 \cdot s(x+1) - 2^2 \cdot s(x+2) - 3^2 \cdot s(x+3) - \cdots$$

$$\cdots + 1 \cdot s(x+1) + 3 \cdot s(x+2) + 5 \cdot s(x+3) + \cdots = \sum_{k=0}^{\infty}(2k-1)s(x+k).$$

axla martivad viRebT dispersias

$$\boxed{DK(x) = E[k(x)]^2 - e_x^2 = \frac{2}{s(x)}\sum_{k=1}^{\infty} ks(x+k) - e_x - e_x^2}\ .$$

sicocxlis drois yvela ricxviTi maxasiaTebeli gamoisaxeba gadarCenaze funqciiT, amitom, (3.1.2) tolobis Tanaxmad, is SeiZleba napovni iyos sxc-is monacemebis mixedviT. kerZod,

$$e_x = \frac{1}{l_x}\sum l_{x+k}\ , \quad e_0 = \frac{1}{l_0}\sum_{k=1}^{\infty} l_k\ ,$$

$$E[K(x)]^2 = \frac{2}{l_x}\sum_{k=1}^{\infty} kl_{x+k} - e_x, \quad e[K(0)]^2 = \frac{2}{l_0}\sum_{k=1}^{\infty} kl_k - e_0\ .$$

magaliTi 4.5.1. sxc-is (danarTi 1) monacemebiT calke mamakacebisaTvis da calke qalebisaTvis vipovoT:

1) saSualo damrgvalebuli sicocxlis xangrZlivobebi e_{89}, e_{88}, e_{84};

2) damrgvalebuli sicocxlis xangrZlivobebis dispersia $DK(89)$, $DK(88)$.

amoxsna. mamakacebisaTvis:

$$e_{89} = \frac{290}{1449} = 0{,}2\ , \quad DK(89) = \frac{2 \cdot 290}{1449} - 0{,}2 - (0{,}2)^2 = 0{,}16\ ;$$

$$e_{88} = \frac{1449 + 290}{3623} = 0{,}48\ ,$$

$$DK(88) = \frac{2(1 \cdot 1449 + 2 \cdot 290)}{3623} - 0{,}48 - (0{,}48)^2 = 0{,}41\ ;$$

$$e_{84} = \frac{9063 + 7546 + 6037 + 3623 + 1449 + 290}{10735} = 2{,}6.$$

qalebisaTvis:

$$e_{89} = \frac{806}{4030} = 0{,}2, \quad DK(89) = \frac{2 \cdot 806}{4030} - 0{,}2 - (0{,}2)^2 = 0{,}16;$$

$$e_{88} = \frac{4030 + 806}{10075} = 0{,}48,$$

$$DK(88) = \frac{2(1 \cdot 4030 + 2 \cdot 806)}{10075} - 0{,}48 - (0{,}48)^2 = 0{,}41;$$

$$e_{84} = \frac{24265 + 20988 + 16791 + 10075 + 4030 + 806}{27665} = 2{,}75.$$

4.6 sakontrolo davalebebi

davaleba 4.6.1

davuSvaT, rom mokvdavobis mrudi Semdegi formuliT aRiwereba

$$f(x) = \frac{x}{a^2} e^{-x/a}, \quad x \geq 0.$$

1) vipovoT ganawilebis $F_x(t)$ funqcia sicocxlis narCeni drois $T(x) = X - x$.

2) aCveneT, rom sicocxlis narCeni drois ganawilebis simkvrive $f_x(t) = \frac{d}{dt} F_x(t)$ warmoadgens eqsponencialuri simkvriveebis $\frac{1}{a} e^{-t/a}$ da erlangiuri simkvriveebis $\frac{t}{a^2} e^{-t/a}$ Sewonil jams.

3) ipoveT $P(T(x) > t)$ albaToba, $\lim_{x \to \infty} P(T(x) > t)$ zRvari da gamoarkvieT, SesZlebelia Tu ara sikvdilebis mrudis aseTi aproqsimaciis gamoyeneba didi x asakebisaTvis.

davaleba 4.6.2

danarTi 1-is sxc-is gamoyenebiT SeafaseT albaToba imisa, rom (21) individumi:

1) miaRwevs 70, 80, 90 wels;

2) gardaicvleba 70, 80, 90 wlamde;

3) gardaicvleba 60-dan 70-mde, 70-dan 80-mde, 80-dan 90 wlamde.

davaleba 4.6.3

ganmarteT aqtuaruli maTematikis Semdegi aRniSvnebis Sinaarsi:

$$p_{21},\quad {}_5p_{21},\quad q_{21},\quad {}_5q_{21},\quad {}_{1|}q_{21},\quad {}_{3|}q_{25},\quad {}_{3|4}q_{29},\quad \frac{q_{60}}{q_{20}},\quad \frac{q_{80}}{q_{20}}.$$

danarTi 1-is sxc-is gamoyenebiT SeafaseT zemoT moyvanili sidideebi.

davaleba 4.6.4

daamTkiceT, rom

$${}_{t|u}q_x = {}_tp_x \cdot {}_uq_{x+t}.$$

davaleba 4.6.5

3.18.1 davalebidan gadarCenaze funqciisaTvis romelime cxrilis gamoyenebiT gansazrvreT albaToba imisa, rom narCeni sicocxlis xangrZlivoba (20) mdebareobs 30-dan 40 wlamde, 40-dan 50 wlamde, 70-dan 80 wlamde.

davaleba 4.6.6

ipoveT sicocxlis nawilobrivi xangrZlivoba muavris modelSi, aageT misi grafiki dazRveulis asakze damokidebulebiT $n=5$ da $n=10$ wlebisaTvis.

davaleba 4.6.7

vTqvaT gadarCenaze funqcia Semdegi formulebiT moicema:

$$s_1(x)=\frac{x+a}{a}e^{-x/a},\ x\geq 0;\quad s_2(x)=\sqrt{1-\frac{x}{110}},\ 0\leq x\leq 110.$$

ipoveT:

1) sicocxlis saSualo xangrZlivoba $\overset{0}{e}_0$;

2) sruli albaTuri sicocxlis xangrZlivoba $\overset{0}{e}_x$;

3) saSualo sicocxlis narCeni drois dispersia $DT(x)$;

4) nawilobrivi sicocxlis saSualo xangrZlivoba $\overset{0}{e}_{x:n]}$;

5) nawilobrivi sicocxlis xangrZlivobis dispersia $D\{\min(T(x),n)\}$.

grafikulad gamosaxeT miRebuli damokidebulebebi, CaatareT SedarebiTi analizi.

davaleba 4.6.8

danarTi 1-is sxc-is monacemebis mixedviT calk-calke qalebisaTvis da mamakacebisaTvis ipoveT:

1) damrgvalebuli sicocxlis saSualo xangrZlivoba e_0, e_{10}, e_{21}, e_{40}, e_{70};

2) damrgvalebuli sicocxlis xangrZlivobis dispersiebi $DK(0)$, $DK(21)$, $DK(70)$, $DK(84)$.

imsjeleT.

Tavi 5.sicocxlis wiladuri xangrZlivoba

5.1 splainuri aproqsimaciebi

wiladuri asakebisaTvis (*fractional ages*)

realuri statistika Cveulebriv xelmisawvdomia mxolod x-is (wlebSi) mTeli mniSvnelobebisaTvis rac ganpirobebulia rogorc statistikuri monacemebis Segrovebis tradiciebiT da mosaxerxeblobiT, ise maTi sxc-Si warmodgenis formiT, sadac argumenti x, rogorc wesi iRebs 0, 1, 2, … mniSvnelobebs. radganac umetesoba klientebisa sadazRvevo kompaniaSi Tavis dabadebis dRes ar modian, amdenad am konkretul individumebTan muSaobisas unda SevZloT movZebnoT zogierTi albaTuri maxasiTeblebis miaxloebebi mTeli x-sTvis cnobili mniSvnelobebis mixedviT maTi wiladuri asakebisaTvis.

ganxiluli amocana warmoadgens interpolaciis tipiur amocanas, amasTan SesaZlebelia SemovifargloT misi mxolod gadarCenis $s(x)$ funqciisaTvis amoxsniT, ramdenadac sxva sidideebi SeiZleba gamoisaxos $s(x)$-iT.

aqtuarul maTematikaSi am amocanas xsninan e.w. splainebis meSveobiT. ganvixiloT sami postulati, romlebic gvaZleven sxvadasxva miaxlovebebs:

1. sikvdilebis Tanabari ganawileba,
2. mokvdavobis mudmivi intensivoba,
3. balduCis (*Balducci*) daSveba

sikvdilebis Tanabari ganawileba

am SemTxvevaSi gadarCenis funqcia interpolirdeba $s(x)=a_n+b_nx$ wrfivi funqciis saxiT $n \le x \le n+1$-sTvis. ramdenadac $s(n)$ da $s(n+1)$ cnobilia (magaliTad, sxc-dan), vadgenT gantolebas:

$$a_n+b_nn=s(n)$$

$$a_n+b_n(n+1)=s(n+1)$$

da vpoulobT ucnob a_n da b_n–s (meore gantolebidan pirvelis gamoklebiT):

$$b_n=s(n+1)-s(n),$$

$$a_n=s(n)(n+1)-s(n+1)n.$$

SevniSnoT, rom $b_n<0$.

aqedan gamomdinare, $n \le x \le n+1$ monakveTze $s(x)$ funqcia aproqsimirdeba Semdegi saxis wrfivi splainiT

$$\boxed{s(x)=(n+1-x)s(n)+(x-n)s(n+1)}\,, \qquad (5.1.1)$$

$n \le x \le n+1$. aqedan $f(x)$ sikvdilebis mrudisaTvis da μ_x mokvdavobis intesivobisaTvis Sesabamisad viRebT:

$$f(x)=-s'(x)=s(n)-s(n+1),\quad n<x<n+1,$$

$$\mu_x=\frac{f(x)}{s(x)}=\frac{s(n)-s(n+1)}{(n+1-x)s(n)+(x-n)s(n+1)}= \qquad (5.1.2)$$

$$=\frac{s(n)-s(n+1)}{(n+1)s(n)-ns(n+1)-x[s(n)-s(n+1)]}$$

amgvarad,

$$\boxed{f(x)=-b_n=s(n)-s(n+1)}$$

$n<x<n+1$.

adre Semotanili sididis saSualebiT $q_n=\dfrac{s(n)-s(n+1)}{s(n)}$, romelic imis albaTobis tolia, rom adamiani n wlis asakSi SeiZleba gardaicvalos uaxloesi wlis ganmavlobaSi, gardavqmnaT (5.1.2) formula ufro mosaxerxebeli saxiT:

$$\mu_x = \frac{s(n)-s(n+1)}{s(n)+(x-n)(s(n+1)-s(n))} = \frac{q_n}{1-(x-n)q_n}, \quad n < x < n+1.$$ vxedavT, rom

aseTi miaxloeba iwvevs mokvdavobis intensivobis zrdas

$$\mu_x = \frac{q_n}{1-(x-n)q_n}, \qquad (5.1.3)$$

interpolaciis kvanZebs Soris ($n < x < n+1$), xolo ganawilebis simkvrive $f(x) = const$ ar icvleba, amasTan mTelricxvian wertilebSi $f(x)$ da μ_x araa gansazRvruli.

SevniSnoT, rom $q_n > s(n) - s(n+1)$, ramdenadac $s(n) < 1$.

mokvdavobis mudmivi intensivoba

$n \le x \le n+1$ monakveTze $s(x)$ funqcia miuaxlovoT klebad maCvenebli an $a_n e^{-b_n x}$ funqcias. am dros gantolebebi iReben Semdeg saxes:

$$a_n e^{-b_n n} = s(n)$$

$$a_n e^{-b_n (n+1)} = s(n+1),$$

da meore gantolebis pirvelze gayofiT miviRebT

$$-b_n = \ln\frac{s(n+1)}{s(n)}, \quad a_n = s(n)e^{b_n n} = s(n)\left(\frac{s(n+1)}{s(n)}\right)^{-n},$$

e.i.

$$b_n = -\ln p_n, \; a_n = s(n) p_n^{-n},$$

sadac p_n aris imis albaToba, rom adamiani n wlis asakSi icocxlebs sul cota kidev erTi weli.

am SemTxvevaSi

$$s(x)=a_n e^{-b_n x}=s(n)p_n^{-n}e^{(\ln p_n)x}=s(n)p_n^{x-n},\quad n\le x\le n+1,$$

da ZiriTadi albaTuri maxasiaTeblebi miaxloebiT Semdegnairad gamoisaxeba:

$$\boxed{s(x)=s(n)p_n^{x-n}},\quad n\le x\le n+1$$

$$\boxed{f(x)=-s'(x)=-s(n)p_n^{x-n}\ln p_n},\qquad \boxed{\mu_x=\frac{f(x)}{s(x)}=-\ln p_n},\qquad n<x<n+1,$$

e.i. interpolaciis kvanZebs Soris $\mu_x=const$.

SevniSnoT, rom $-\ln p_n>-s(n)\ln p_n$, ramdenadac $s(n)<1$.

balduCis daSveba

am SemTxvevaSi wrfiv funqciad $s(x)$ _is nacvlad interpilirdeba $s^{-1}(x)$. Tu (5.1.1)-Si $s(x)$-s SevcvliT $\frac{1}{s(x)}$–iT uceb gadavalT Semdeg damokidebulebaze

$$\frac{1}{s(x)}=\frac{n+1-x}{s(n)}+\frac{x-n}{s(n+1)},\quad n\le x\le n+1.$$

saidanac,

$$s(x)=\frac{s(n)s(n+1)}{(n+1-x)s(n+1)+(x-n)s(n)}=\frac{s(n+1)}{(n+1-x)p_n+x-n}=\frac{s(n+1)}{p_n+(x-n)q_n},\quad n\le x\le n+1,$$

$$f(x)=-s'(x)=\frac{s(n)s(n+1)(s(n)-s(n+1))}{(s(n+1)(n+1-x)+(x-n)s(n))^2}=\frac{s(n+1)q_n}{(p_n+(x-n)q_n)^2},\quad n<x<n+1,$$

$$\mu_x=\frac{f(x)}{s(x)}=\frac{q_n}{p_n+(x-n)q_n},\quad n<x<n+1.$$

magaliTi 5.1.1. daTvaleT albaToba imis, rom ssrk-s mamakaci (80) 80-iani wlebis SuaSi sikvdilebis Tanabari ganawilebis Sesaxeb daSvebisas gardaicvleba $80\frac{1}{2}$ wlidan $81\frac{1}{2}$ wlamde asakSi.

amoxsna: visargebloT Semdegi formuliT

$$s(x)=(n+1-x)s(n)+(x-n)s(n+1),\quad n\le x\le n+1,$$

romlis Tanaxmadac

$$s\left(80\frac{1}{2}\right)=\left(81-80\frac{1}{2}\right)s(80)+\left(80\frac{1}{2}-80\right)s(81)=0,5(s(80)+s(81)),$$

$$s\left(81\frac{1}{2}\right)=0,5(s(81)+s(82)).$$

amgvarad,

$$P\left(\frac{1}{2}<T(80)<1\frac{1}{2}\right)=\frac{s(80\frac{1}{2})-s(81\frac{1}{2})}{s(80)}=0,5\frac{s(80)+s(81)-s(81)-s(82)}{s(80)}=$$

$$=0,5\left(1-\frac{s(82)}{s(80)}\right)=0,5\left(1-\frac{s(82)}{s(81)}\frac{s(81)}{s(80)}\right)=0,5(1-p_{81}p_{80})=0,5(1-(1-q_{81})(1-q_{80})=$$

$$=0,5(1-(1-0,12548)(1-0,11672))=0,5(1-0,874520\cdot 0,88328)=0,11378$$

5.2 wiladuri asakis ganawileba

SemoviyvanoT SemTxveviTi sidide $\tau=\{X\}$, sadac $\{X\}$ aRniSnavs X sididis wiladur nawils. axla davuSvaT sicocxlis xangrZlivoba X SesaZlebelia warmovadginoT mTeli da wiladuri nawilebis jamis saxiT: $X=K(0)+\tau$, sadac $K(0)=[X]$ _ sicocxlis damrgvalebuli droa. gasagebia, rom τ sidide aRwers sikdilis moments wlis SigniT. vipovoT τ-s pirobiTi ganawileba im pirobiT, rom sikvdili n wlis asakSi dadga:

$$P\{\tau \le t \mid K(0)=n\} = P\{X - K(0) \le t \mid K(0)=n\} =$$
$$= P\{X \le t+n \mid n \le X < n+1\} = \frac{P\{X \le t+n, n \le X < n+1\}}{P\{n \le X < n+1\}} = \qquad (5.2.1)$$
$$= \frac{P\{n \le X \le n+t\}}{P\{n \le X < n+1\}} = \frac{s(n)-s(n+t)}{s(n)-s(n+1)}, 0<t<1$$

$s(n+t)$ sidide, ufro zustad l_{n+t}, rodesac n mTelia, xolo $0<t<1$, scx cxrilSi araa, amitom mis mosaZebnad vsargeblobT wiladuri asakebisaTvis miaxloebebiT.

sikvdilebis Tanabari ganawilebis postulatis dros (5.2.1) formula, Tu $s(x)$-Si argumentad aviRebT $x=n+t$, Semdegnairad gardaiqmneba:

$$P\{\tau \le t \mid K(0)=n\} = \frac{s(n)-[(n+1-n-t)s(n)+(n+t-n)s(n+1)]}{s(n)-s(n+1)} =$$
$$= \frac{t(s(n)-s(n+1))}{s(n)-s(n+1)} = t, 0<t<1.$$

$$\mu_{t|.} = \frac{f(t\mid\cdot)}{1-F(t\mid\cdot)} = \frac{1}{1-t}.$$

amgvarad, am interpolaciis dros

1) adamianis sikvdili or dabadebis dRes Soris nebismier dRes Tanabaralbaturia;

2) ganawilebis piroba $P\{\tau \le t \mid K(0)=n\}$ araa damokidebuli n-ze da amotom emTxveva upirobo ganawilebas $P(\tau \le t)$;

3) SemTxveviTi sidideebi $K(0)$ da τ damoukidebelni arian.

mokvdavobis mudmivi intensivobis postulatisaTvis analogiurad gvaqvs:

$$P\{\tau \le t \mid K(0)=n\} = \frac{s(n)-s(n+t)}{s(n)-s(n+1)} = \frac{s(n)-s(n)p_n^{n+t-n}}{s(n)-s(n+1)} = \frac{s(n)(1-p_n^t)}{s(n)-s(n+1)} = \frac{1-p_n^t}{1-p_n}, \quad 0<t<1.$$

$$f(t\,|\,\cdot)=F'(t\,|\,\cdot)=\frac{-p_n^t\ln p_n}{1-p_m},$$

$$s(t\,|\,\cdot)=1-F(t\,|\,\cdot)=1-\frac{1-p_n^t}{1-p_n}=\frac{1-p_n-1+p_n^t}{1-p_n}=\frac{p_n(p_n^{t-1}-1)}{1-p_n},$$

$$\mu_{t|\cdot}=\frac{f(t\,|\,\cdot)}{s(t\,|\,\cdot)}=\frac{-p_n^t\ln p_n}{p_n(p_n^{t-1}-1)}=\frac{-p_n^{t-1}\ln p_n}{p_n^{t-1}-1}.$$

ana l ogiuri msjelobiT aseve SesaZlebelia miviRoT formulebi $f(t|\cdot)$, $s(t|\cdot)$, $\mu(t|\cdot)$ balduCis postultis drosac.

SeniSvna. zogadad rom vTqvaT, mokvdavobis mudmivi intensivobis dros formulebi mosaxerxebelia gamoisaxos $q_n=1-p_n$-iT, ramdenadac q_n sidide gvaqvs sxc-Si.

5.3 wiladuri asakis saSualo da dispersia

vipovoT wiladuri τ asakis saSualo im pirobiT, rom sikvdili dgeba n wlis asakSi:

$$a(n)=E\{\tau\,|\,K(0)=n\}=\int_0^1 P\{\tau>t\,|\,K(0)=n\}dt.$$

cxadia, rom

$$P\{\tau>t\,|\,K(0)=n\}=1-P\{\tau\le t\,|\,K(0)=n\}=1-\frac{s(n)-s(n+t)}{s(n)-s(n+1)}=$$

$$=\frac{s(n)-s(n+1)-s(n)+s(n+t)}{s(n)-s(n+1)}=\frac{s(n+t)-s(n+1)}{s(n)-s(n+1)}$$

aqedan,

$$a(n)=\frac{1}{s(n)-s(n+1)}\int_0^1[s(n+t)-s(n+1)]dt.$$

davTvaloT axla $a(n)$ sidide wiladuri asakebisaTvis mokvdavobis xasiaTis Sesaxeb samive daSvebisaTvis.

sikvdilebis Tanabari ganawileba. cxadia, rom

$$a(n)=\frac{\int_0^1[(n+1-n-t)s(n)+(n+t-n)s(n+1)]dt}{s(n)-s(n+1)}=$$

$$=\frac{\int_0^1[(1-t)s(n)+tS(n+1)-s(n+1)]dt}{s(n)-s(n+1)}=\frac{1}{2}$$

e.i $a(n)$ emTxveva drois erTwliani Sualedis Suas, rasac Cven intuiciuradac moveloditT.

mokvdavobis mudmivi intensivoba. am SemTxvevaSi

$$a(n)=\frac{1}{s(n)-s(n+1)}\int_0^1[s(n)p_n^{n+t=n}-s(n+1)]dt=$$

$$=\frac{s(n)}{s(n)-s(n+1)}\int_0^1[p_n'-p_n]dt=\frac{1}{q_n}\left[\frac{p_n^t}{\ln p}\Big|_0^1-p_n\right]=$$

$$=\frac{1}{q_n}\left(\frac{p_n-1}{\ln p_n}-p_n\right)=\frac{1}{q_n}\left(-p_n-\frac{q_n}{\ln p_n}\right)=-\frac{1}{\ln p_n}-\frac{p_n}{q_n}$$

ramdenadac $p_n=1-q_n$, xolo q_n sidide sakmaod mcirea, amdenad Semdegi warmodgenis gamoyenebiT

$$\ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-...-\frac{x^n}{n}+o(x^n),$$

gavSaloT $\ln p_n$ mwkrivad q_n-is xarisxebis mixdviT

$$\ln p_n=-q_n-\frac{q_n^2}{2}-\frac{q_n^3}{3}-... ,$$

ris Semdegac gadavdivarT $a(n)$-sTvis Semdeg Sefasebebze:

$$a(n)=\frac{1}{2}-\frac{q_n}{12}+O(q_n^2)=\frac{1}{2}-\frac{q_n}{12}+o(q_n) \qquad (5.3.1)$$

davamtkicoT (5.3.1) toloba. ramdenadac

$$-\frac{p_n}{q_n}-\frac{1}{\ln p_n}=-\frac{1-q_n}{q_n}+\frac{1}{q_n+\frac{q_n^2}{2}+\frac{q_n^3}{3}+...}=$$

$$=1+\frac{1}{q_n+\frac{q_n^2}{2}+\frac{q_n^3}{3}+...}-\frac{1}{q_n}=1-\frac{\frac{q_n^2}{2}+\frac{q_n^3}{3}+...}{q_n^2+\frac{q_n^3}{2}+\frac{q_n^4}{3}+...}$$

amdenad, davSliT ra orcvladian funqcias (damokidebulebas) teiloris mwkrivad $\left(\frac{q_n^2}{2},q_n^2\right)$ wertilis midamoSi, miviRebT

$$-\frac{p_n}{q_n}-\frac{1}{\ln p_n}=1-\left[\frac{1}{2}+\frac{1}{q_n^2}\left(\frac{q_n^3}{3}+...\right)-\frac{\frac{q_n^2}{2}}{q_n^4}\left(\frac{q_n^3}{2}+...\right)\right]=$$

$$=\frac{1}{2}-\frac{q_n}{3}+\frac{q_n}{4}+o(q_n)=\frac{1}{2}-\frac{q_n}{12}+o(q_n)$$

rac unda dagvemtkicebina.

Tu q_n arc ise mcirea, maSin azri aqvs gaviTvaliswinoT $O(q_n^2)$ rigis Sesakrebebic:

$$a(n)=1-\frac{1}{q_n^2}\left(\frac{q_n^3}{3}+\frac{q_n^4}{4}+...\right)+\frac{1}{2q_n^2}\left(\frac{q_n^3}{2}+\frac{q_n^4}{3}+...\right)=$$

$$=\frac{1}{2}-\frac{q_n}{3}+\frac{q_n}{4}-\frac{q_n^2}{4}+\frac{q_n^2}{6}+...=\frac{1}{2}-\frac{q_n}{12}-\frac{q_n^2}{12}+o(q_n^2).$$

baluCis postulati. aq,

$$a(n)=\frac{1}{s(n)-s(n+1)}\int_0^1\left(\frac{s(n+1)}{p_n+tq_n}-s(n+1)\right)dt=$$

$$=\frac{s(n+1)}{s(n)-s(n+1)}\int_0^1\left(\frac{1}{p_n+tq_n}-1\right)dt=\frac{p_n}{q_n}\int_0^1\left(\frac{1}{p_n+tq_n}-1\right)dt=$$

$$=\frac{p_n}{q_n}\left[\left.\frac{\ln(p_n+q_n)}{q_n}\right|_0^1-1\right]=\frac{p_n}{q_n}\left[-\frac{\ln p_n}{q_n}-1\right]=$$

$$=-\frac{p-n}{q_n^2}(q_n+\ln p-n)=\frac{q_n-1}{q_n^2}\left(q_n-q_n-\frac{q_n^2}{2}-\frac{q_n^3}{3}-...\right)=$$

$$=(q_n-1)\left(-\frac{1}{2}-\frac{q_n}{3}-\frac{q_n^2}{4}-...\right)=\frac{1}{2}-\frac{q_n}{2}+\frac{q-n}{3}-\frac{q_n^2}{3}+\frac{q_n^2}{4}+o(q_n)=$$

$$=\frac{1}{2}-\frac{q_n}{6}-\frac{q_n^2}{12}+o(q_n)$$

dispersias Semdegi formuliT vipoviT:

$$b(n)=D\{\tau\mid k(0)=n\}=\frac{2}{s(n)-s(n+1)}\int_0^1[s(n+t)-s(n+1)]dt-a^2(n).$$

davTvaloT axla $b(n)$ sidide wiladuri asakebisaTvis mokvdavobis samive postulatisaTvis.

sikvdilebis Tanabari ganawileba. am postulatisaTvis

$$b(n)=\frac{2}{s(n)-s(n+1)}\int_0^1 t(1-t)[s(n)-s(n+1)]dt-\frac{1}{4}=$$

$$=2\int_0^1(1-t^2)dt-\frac{1}{4}=2\left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{4}=\frac{1}{12}$$

mokvdavobis mudmivi intensivoba. adre Cven miviReT, rom $a(n)=\frac{1}{q_n}\int_0^1[p_n^t-p_n]dt$, amitom

$$b(n)=\frac{2}{q_n}\int_0^1 t(p_n^t-p_n)dt-a^2(n)=\frac{2}{q_n}\left[\int_0^1 tp_n^t dt-\int_0^1 tp_n dt\right]-\left[\frac{1}{2}-\frac{q_n}{12}+o(q_n)\right]^2=$$

$$=\frac{2}{q_n}\left[\frac{p_n}{\ln p_n}-\frac{p_n-1}{\ln^2 p_n}-\frac{p_n}{2}\right]-\left[\frac{1}{4}-\frac{q_n}{12}+o(n)\right]=2\left[\frac{1-q_n}{q_n\ln p_n}+\frac{q_n}{q_n\ln^2 p_n}-\frac{1-q_n}{2q_n}\right]-[...]=$$

$$=2\left[\frac{1}{q_n\ln p_n}-\frac{1}{\ln p_n}+\frac{1}{\ln^2 p_n}-\frac{1}{2q_n}+\frac{1}{2}\right]-[...]=2\left[\frac{2\ln p_n-2q_n\ln p_n+2q_n-\ln^2 p_n}{2q_n\ln^2 p_n}+\frac{1}{2}\right]-[...]=$$

$$=2\left[\frac{-2q_n-q_n^2-\frac{2}{3}q_n^3-\frac{1}{2}q_n^4-...+2q_n^2+q_n^3+\frac{2}{3}q_n^4+...}{2q_n^3+2q_n^4+...}\right]+2\left[\frac{2q_n-q_n^2-q_n^3-\frac{11}{12}q_n^4-...}{2q_n^3+2q_n^4+...}+\frac{1}{2}\right]-[...]=$$

$$=2\left[\frac{-\frac{2}{3}q_n^3-\frac{3}{4}q_n^4-...}{2q_n^3+2q_n^4+...}+\frac{1}{2}\right]-[...].$$

damokidebulebis $\left(-\frac{2}{3}q_n^3, 2q_n^3\right)$ wertilis midamoSi teiloris mwkrivad daSlis Semdeg gvaqvs

$$b(n)=2\left[\frac{1}{2}-\frac{1}{3}-\frac{1}{2q_n^3}\cdot\frac{3}{4}q_n^4+\frac{\frac{2}{3}q_n^3}{4q_n^6}2q_n^4-...\right]-[...]=$$

$$=2\left[\frac{1}{6}-\frac{3}{8}q_n+\frac{1}{3}q_n-...\right]-\left[\frac{1}{4}-\frac{q_n}{12}+...\right]=\frac{1}{12}+o(q_n)$$

balduCis postulati. aq

$$b(n)=\frac{2p_n}{q_n}\int_0^1 t\left(\frac{1}{p_n+tq_n}-1\right)dt-a^2(n)=\frac{2p_n}{q_n}\left[\int_0^1\frac{t}{p_n+tq_n}dt-\int_0^1 tdt\right]-a^2(n)=$$

$$=\frac{2p_n}{q_n^2}\left[\int_0^1\frac{p_n+tq_n-p_n}{p_n+tq_n}dt\right]-\frac{1}{2}\cdot\frac{2p_n}{q_n}-a^2(n)=\frac{2p_n}{q_n^2}\left[1-p_n\int_0^1\frac{dt}{p_n+tq_n}\right]-\frac{p_n}{q_n}-a^2(n)=$$

$$=\frac{2p_n}{q_n^2}\left[1-\frac{p_n}{q_n}\ln(p_n+tq_n)\Big|_0^1\right]-\frac{p_n}{q_n}-a^2(n)=\frac{2p_n}{q_n^2}\left[1+p_n\frac{p_n\ln p_n}{q_n}\right]-\frac{p_n}{q_n}-a^2(n)=$$

$$=\frac{2(1-q_n)}{q_n^2}+\frac{2(1-q_n)^2\ln p_n}{q_n^3}-\frac{1-q_n}{q_n}-a^2(n)=\frac{2}{q_n^2}-\frac{2}{q_n}+\frac{2\ln p_n}{q_n^3}-\frac{4\ln p_n}{q_n^2}+\frac{2\ln p_n}{q_n}-\frac{1}{q_n}+1-a^2(n)=$$

$$=\frac{2}{q_n^3}q_n^2-\frac{2}{q_n}+\frac{2}{q_n^3}\left(-q_n-\frac{q_n^2}{2}-\frac{q_n^3}{3}-\frac{q_n^4}{4}-\dots\right)+\frac{4}{q_n^2}\left(q_n+\frac{q_n^2}{2}+\frac{q_n^3}{3}+\dots\right)+$$

$$+\frac{2}{q_n}\left(-q_n-\frac{q_n^2}{2}-\dots\right)-\frac{1}{q_n}+1-a^2(n)=\frac{2}{q_n^2}-\frac{2}{q_n}-\frac{2}{q_n^2}-\frac{1}{q_n}-\frac{2}{3}-\frac{1}{2}q_n-\dots+\frac{4}{q_n}+2+\frac{4}{3}q_n+\dots-$$

$$-2-q_n-\dots-\frac{1}{q_n}+1-\frac{1}{4}+\frac{q_n}{6}+\dots=1-\frac{1}{4}-\frac{2}{3}-\frac{q_n}{2}+\frac{4}{3}q_n-q_n+\frac{q_n}{6}+\dots=\frac{1}{12}+o(q_n).$$

ganvixiloT miRebuli Sedegebi. 30 wlis asakis ssrk-s qalebisaTvis $q_{30}=0,0010$. amitom mokvdavobis mudmivi intensivobis da balduCis postulatis saSualoebisaTvis da dispersiebisaTvis Secdomis rigi Seadgens $O(q_{30}^2)\approx 10^{-6}$. didi asakebisaTvis $a(n)$ da $b(n)$–sTvis formulebi gamoyenebuli iqneba q_n^2 Sesakrebis gaTvaliswinebiT, ramdenadac, magaliTad, 82 wlis asakis ssrk-s qalebisaTvis $q_{82}=0,1015$, da am SemTxvevaSi $O(q_{82}^2)\approx 10^{-2}$.

aseve praqtikaSi zogierT SemTxvevSi zemoT miRebuli Sedegis Tanaxmad, $K(0)$-s da τ-s damoukideblobis daSvebiT, SesaZlebelia sicocxlis narCeni drois saSualosaTvis da dispersiisaTvis gamoviyenoT Semdegi umartivesi aproqsimaciebi:

$$\overset{0}{e}_x\approx e_x+\frac{1}{2},$$

$$DT(x) \approx DK(0) + \frac{1}{12}.$$

5.4 L_x da T_x cxriluri sidideebi da maTi kavSiri erTmaneTTan da $a(x)$-sTan

sidideebi L_x da T_x gamoiyeneba ufro dawvrilebiT sxc-ebSi. magaliTis saxiT danarT 3-Si moyvanilia aSS-s mosaxleobisaTvis (1979-81) sxc-is fragmenti.

L_x simboloTi avRniSnavT wlebis saSualo jamur ricxvs, romelic gavlilia x da $x+1$ momentebs Soris, x - mTelia sawyisi l_0 jgufis yvela warmomadgenlisaTvis. gasagebia, rom is Sedgeba ori mdgenisagan:

1) im sawyisi jgufis warmomadgenlebis mier x da $x+1$ momentebs Soris ganvlili wlebis saSualo jamuri ricxvisgan, romlebic gardaicvalnen x-dan $x+1$-mde asakSi (es sidide tolia $d_x a(x)$-is);

2) wlebis saSualo jamuri ricxvisagan, romlebic eZleva sawyisi jgufis cocxal warmomadgenlebs $x+1$ momentisaTvis (es sidide tolia $l_{x+1} \cdot 1$weli $= l_{x+1}$).

amgvarad,

$$L_x = d_x a(x) + l_{x+1} = \frac{d_x}{s(x) - s(x+1)} \int_0^1 [s(x+t) - s(x+1)]dt + l_{x+1} =$$

$$= \frac{d_x}{l_x - l_{x+1}} \int_0^1 [l_{x+t} - l_{x+1}]dt + l_{x+1} = \int_0^1 [l_{x+t} - l_{x+1}]dt + l_{x+1} = \int_0^1 l_{x+t} dt. \qquad (5.4.1)$$

ramdenadac scx-Si xdeba l_x da L_x sidideebis Sejameba, amdenad maTi saSualebiT SesaZlebelia $a(x)$–is gaangariSeba. marTlac, (5.4.1)-dan gamomdinareobs, rom

$$L_x - l_{x+1} = d_x a(x),$$

saidanac

$$a(x) = \frac{L_x - l_{x+1}}{l_x - l_{x+1}} = \frac{L_x - l_{x+1}}{d_x}. \qquad (5.4.2)$$

magaliTi 5.4.1. saerTo scx-is monacemebis mixedviT (danarTi 3) vipovoT $a(20)$, $a(80)$, $a(106)$, $a(107)$, $a(108)$.

amoxsna. (4.4.2) formulis Tanaxmad gvaqvs

$$a(20) = \frac{L_{20} - l_{21}}{d_{20}} = \frac{97682 - 97623}{118} = \frac{59}{118} = 0{,}5;$$

$$a(80) = \frac{L_{80} - l_{81}}{d_{80}} = \frac{41694 - 40208}{3972} = \frac{1486}{2972} = 0{,}5;$$

$$a(100) = \frac{983 - 815}{335} = 0{,}501; \quad a(106) = \frac{99 - 78}{41} = 0{,}512;$$

$$a(107) = \frac{99 - 78}{41} = 0{,}512; \quad a(108) = \frac{42 - 33}{18} = 0{,}5.$$

ramdenadac $a(i)$ ~0,5, amdenad pirveli magaliTidan gamomdinareobs, rom albaT nebismieri asakis individebisaTvis or dabadebis dRes Soris SualedSi adgili aqvs Tanabar mokvdavobas.

T_x simboloTi avRniSnavT wlebis saSualo jamur ricxvs, romelic ganvles l_0 axalSobilTagan Semdgari jgufebidan yvela warmomadgenelma (x,∞) intervalSi. gasagebia, rom

$$T_x = l_0 E[(X-x)I(X-x>0)] = l_0\int_0^\infty (X-x)dP\{X-x\le t\} =$$

$$= l_0\int_0^\infty P\{X-x>t\}dt = l_0\int_0^\infty P\{X>x+t\}dt = l_0\int_0^\infty s(x+t)dt = l_0\int_x^\infty s(u)du$$

axla $\overset{0}{e}_x = ET(x)$ formulas SeiZleba mivceT Semdegi saxe:

$$\overset{0}{e}_x = ET(x) = \frac{1}{s(x)}\int_0^\infty s(u)du = \frac{1}{l_0 s(x)} l_0\int_0^\infty s(u)du = \frac{T_x}{l_x}. \qquad (5.4.3)$$

magaliTi 5.4.2. saerTo scx-is monacemebis mixedviT (danarTi) vipovoT $\overset{0}{e}_{20}$, $\overset{0}{e}_{80}$.

amoxsna. (4.4.3) formulis Tanaxmad gvaqvs

$$\overset{0}{e}_{20} = \frac{T_{20}}{l_{20}} = \frac{5420937}{97741} = 55{,}462; \quad \overset{0}{e}_{80} = \frac{T_{80}}{l_{80}} = \frac{344612}{43180} = 7{,}98.$$

aseve L_n, $n \ge x$ sididis meSveobiT SesaZlebelia T_x sididis gamoangariSeba:

$$T_x = l_0\int_x^\infty s(u)du = l_0\sum_{k=0}^\infty \int_{x+k}^{x+k+1} s(u)du = l_0\sum_{k=0}^\infty\int_0^1 s(x+k+t)dt = \sum_{k=0}^\infty\int_0^1 l_{x+k+t}dt = \sum_{k=0}^\infty L_{x+k} = \sum_{n=x}^\infty L_n .$$

amgvarad,

$$\boxed{T_x = \sum_{n=x}^\infty L_n} \qquad (5.4.4)$$

magaliTi 5.4.3. saerTo scx-is monacemebis mixedviT (danarTi 3) (5.4.4) formulis mixedviT gamoiTvaleT T_{100} da T_{105}.

amoxsna. mosaxerxebelia jer T_{105}-is povna:

$$T_{105} = L_{105}+L_{106}+L_{107}+L_{108}+L_{109}+\sum_{n=10}^\infty L_n = L_{105}+L_{106}+L_{107}+L_{108}+L_{109}+(T_{109}-L_{109}) =$$

$$=150+99+64+42+27+(73-27)=428.$$

Semdeg,

$$T_{100}=T_{105}+L_{104}+L_{103}+L_{102}+L_{101}+L_{100}=428+223+330+481+692+983=3137.$$

zogadad SeiZleba interpolaciisaTvis gamoyenebuli iyos aseve paraboluri da kuburi splainebic, magram albaT, Sedegebis sizustis gazrda ar iqneba gamarTlebuli gamosaTvleli formulebis garTulebis gamo.

5.5. sakontrolo davalebebi

davaleba 5.5.1

gamoiyeneT sxc danarTi 1-dan da gamoiTvaleT imis albaToba, rom mamakaci (77) gardaicvleba $77\frac{5}{12}$-dan $78\frac{11}{12}$-mde asakSi (intervali 1,5 weli):

1) sikvdilebis Tanabari ganawilebis Sesaxeb daSvebisas;

2) wiladuri asakebisaTvis balduCis daSvebisas;

3) mokvdavobis musmivi intensivobis daSvebisas.

awarmoeT miRebuli Sedegebis SedarebiTi analizi.

davaleba 5.5.2

gamoiyvaneT formulebi $f(t\,|\,\cdot)$, $s(t\,|\,\cdot)$, $\mu_{t|\cdot}$–sTvis wiladuri asakebisaTvis baluCis daSvebisas.

davaleba 5.5.3

aageT sikvdilebis mrudebis grafikebi (77,79) SualedSi 5.5.1 davalebis daSvebisas.

davaleba 5.5.4

aageT mokvdavobis intensivobis funqciis mrudebis grafikebi (77,79) SualedSi 5.5.1 davalebis daSvebisas.

Tavi 6. koleqtiuri dazRveva

6.1 ramdenime piris sicocxlis dazRveva. gaerTianebuli sicocxlis statusi (*joint-life status*)

2-5 TavebSi warmodgenili Sedegebi SesaZlebelia ganzogaddes mravalganzomilebian SemTxvevisTvis. aseTi ganzogadeba aucilebelia sapensio dazRvevasTan, sicocxlis, janmrTelobis koleqtiuri dazRvevasTan da sxva dakavSirebul gamoTvlebis dros.

mokled CamovayaliboT ramdenime piris sicocxlis dazRvevis Taviseburebani. ganvixiloT sicocxlis koleqtiuri dazRvevis SemTxveva, romlisTvisac sasargeblo abstraqcias warmoadgens statusis cneba. davuSvaT $(x_1, x_2, ..., x_m)$ asakebis mqone m individebs survili aqvT dadon sadazRvevo xelSekruleba. k–uri individis momavali cxovrebis dro avRniSnoT $T(x_k) = X - x_k$-iT.

$T(x_1)$, $T(x_2)$, ... , $T(x_m)$ ricxvebis m erToblioba CavayenoT U statusis (*status*) SesabamisobaSi, romelsac Seesabameba sakuTari $T(U)$ sicocxlis xangrZlioba. or yvelaze gavrcelebul statuss warmoadgens gaerTianebuli sicocxlis statusi da ukanasknelis cocxlad darCenilis statusi.

gaerTianebuli sicocxlis statusi aRiniSneba $U := x_x : x_2 : ... : x_m$-iT an $(x_1 : x_2 : ... : x_m)$-iT da iTvleba daSlilad, Tu erT-erTi individi mainc gardaicvleba, e.i.

$T(U) = \min\{T(x_1), T(x_2), ..., T(x_m)\}$.

gasagebia, rom

$$P\{T(U) > t\} = P\{\min(T(x_1), T(x_2), ..., T(m)) > t\} = P\{T(x_1) > t, T(x_2) > t, ..., T(x_m) > t\},$$

sikvdilebis damoukideblobis daSvebisas

$$P\{T(U)>t\}=\prod_{i=1}^{m}{}_t p_{x_i}\ . \qquad (6.1.1)$$

${}_t p_i = P\{T(x_i)>t\}$ albaTobebis arsi ganmartebulia 4.2 paragrafSi.

axla martivad gamoiyvaneba sicocxlis xangrZlivobis sxva albaTuri maxasiaTebebi $T(U)$-sTvis, magaliTad,

$${}_t q_{x_1:x_2:...:x_m} = 1 - {}_t p_{x_1:x_2:...:x_m} = 1-\prod_{i=1}^{m}(1-{}_t q_{x_i})\,.$$

mocemuli statusis daSlis drois ganawilebis simkvrivisaTvis samarTliania Semdegi damokidebuleba:

$$f_{x_1:x_2:...:x_m}(t) = -\frac{d}{dt}P\{T(U)>t\} = -\frac{d}{dt}\prod_{i=1}^{m}{}_t p_{x_i} = -\frac{d}{dt}\prod_{i=1}^{m}\frac{s(x_i+t)}{s(x_i)}\,. \qquad (6.1.2)$$

magaliTi 6.1.1. sikvdilebis damoukideblobis daSvebisas ipoveT ori individis gaerTianebuli sicocxlis statusis $U:=x_1:x_2$ ganawilebis simkvrive

1) zogad SemTxvevaSi;

2) muavris modelisaTvis.

amoxsna. ramdenadac $m=2$, xolo $T(U)=\min(T(x_1),T(x_2))$, amdenad statusis ganawilebis simkvrivisaTvis (6.1.2)-is Sesabamisad gvaqvs:

$$f_{x_1:x_2}(t) = \frac{d}{dt}\{-P\{\min(T(x_1),T(x_2))>t\}\} = \left\{-\frac{s(x_1+t)}{s(x_1)}\frac{s(x_2+t)}{s(x_2)}\right\}'_t =$$

$$= \frac{f(x_1+t)}{s(x_1)}\frac{s(x_2+t)}{s(x_2)} + \frac{f(x_2+t)}{s(x_2)}\frac{s(x_1+t)}{s(x_1)} = f_{x_1}(t)s_{x_2}(t) + f_{x_2}(t)s_{x_1}(t) < f_{x_1}(t)+f_{x_2}(t)\ ,$$

sadac $s_x(t)=\dfrac{s(x+t)}{s(x)}$ - $T(x)$ SemTxveviTi sididis gadarCenis funqciaa. rogorc mogvianebiT vaCvenebT, utolobis damtkiceba saSulebas aZlevs sadazRvevo kompaniebs dawion koleqtiuri dazRvevis monawileTa premiis zoma individualuri dazRvevis SemTxvevasTan SedarebiT.

de muavris modelisaTvis $f_x(t)$ moicema (5.1.2) formuliT, xolo gadarCenis funqcia - $s_x(t) = I_t(-\infty, \omega - x) - \frac{tI_t(0, \omega - x)}{\omega - x}$, amitom

$$\begin{aligned} f_{x_1:x_2}(t) &= \frac{I_t(0,\omega - x_1)}{\omega - x_1}\left[I_t(-\infty,\omega - x_2) - \frac{tI_t(0,\omega - x_2)}{\omega - x_2}\right] + \\ &+ \frac{I_t(0,\omega - x_2)}{\omega - x_2}\left[I_t(-\infty,\omega - x_1) - \frac{tI_t(0,\omega - x_1)}{\omega - x_1}\right] = \\ &= \left[\frac{\omega - x_2 - t}{(\omega - x_1)(\omega - x_2)}\right] I_t(0, \min(\omega - x_1, \omega - x_2)) \end{aligned} \tag{6.1.3}$$

gadavideT axla gaerTianebuli sicocxlis statusis daSlis intensivobis funqciaze. sicocxlis narCeni drois intensivobis $T(x) = X - x$ funqcia akmayofilebs tolobebs

$$\begin{aligned} \mu_x(t) &= \frac{f_x(t)}{s_x(i)} = \frac{f(x+t)}{s(x+t)} = \mu_{x+t} = -\frac{d}{dt}\ln s(x+t) = \\ &= -\frac{d}{dt}[\ln s(x+t) - \ln s(x)] = -\frac{d}{dt}\ln\frac{s(x+t)}{s(x)} = -\frac{d}{dt}\ln {}_t p_x , \end{aligned}$$

e.i.

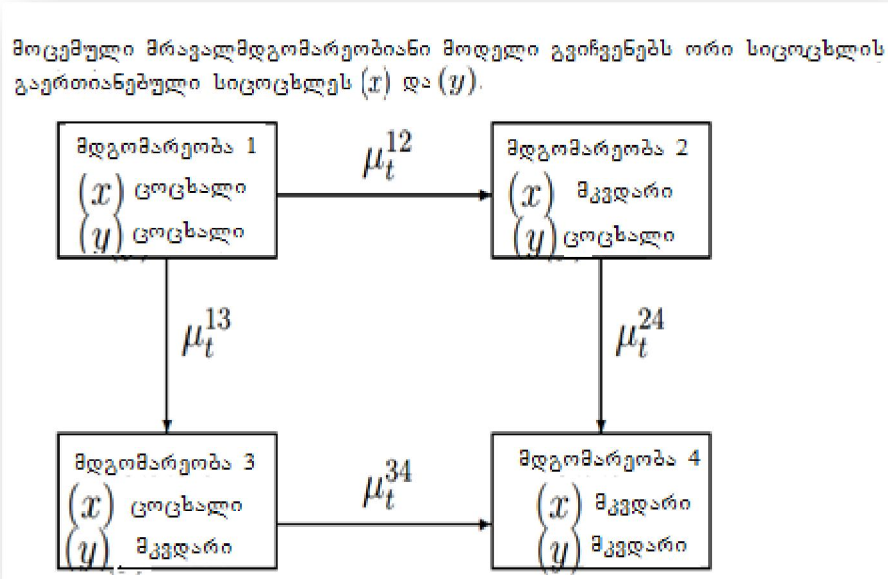
$$\boxed{\mu_x(t) = \mu_{x+t} = -\frac{d}{dt}\ln {}_t p_x} \tag{6.1.4}$$

gaviTvaliswinebT ra (6.1.4)-s, miviRebT

$$\mu_{x_1:x_2:\ldots:x_m}(t) = -\frac{d}{dt}\ln {}_t p_{x_1:x_2:\ldots:x_m} = -\frac{d}{dt}\sum_{i=1}^{m}\ln {}_t p_{x_i} = \sum_{i=1}^{m}\mu_{x_i}(t)$$

da, aqedan gamomdinare,

$$\boxed{\mu_{x_1:x_2:\ldots:x_m}(t) = \sum_{i=1}^{m}\mu_{x_i}(t)} \tag{6.1.5}$$



6.2 gamartiveba hompertcis da maikhamis modelebisaTvis

Tu ganxiluli jgufis yvela adamianis mokvdaoba ganawilebulia hompertcis erTidaimave kanoniT (ix. p. 3.6), maSin

$$\mu_{x_i}(t) = \mu_{x_i+t} = Be^{\alpha(x_i+t)} = Br^{x_i+t}, \qquad (6.2.1)$$

sadac $r = e^{\alpha}$, $t \geq 0$, $i = 1,\ldots,m$.

(6.1.5) toloba (6.2.1)-is gaTvaliswinebiT saSualeba gveZleva SevadginoT Semdegi gantoleba:

$$r^{x_1} + r^{x_2} + \ldots + r^{x_m} = r^{\omega},$$

romlis ω-s mimarT amoxsniT miviRebT

$$\omega = \frac{1}{\alpha}\ln\{r^{x_1} + r^{x_2} + \ldots + r^{x_m}\} = \frac{1}{\alpha}\ln\{e^{\alpha x_1} + e^{\alpha x_2} + \ldots + e^{\alpha x_m}\}. \qquad (6.2.2)$$

amgvarad, gaerTianebuli sicocxlis statusis daSlis intensivobis funqcia SeiZleba Semdegi sxiT warmovadginoT

$$\boxed{\mu_{x_1:x_2:\ldots:x_m}(t) = \mu_{\omega}(t), t \geq 0},$$

saidanac Cans, rom gaerTianebuli sicocxlis statusis daSlis intensivoba aseve emorCileba sawyisi ω asakis mqone romeliRac pirobiTi individis hompertcis kanons, romelic gamoTvlilia (6.2.2) formulis mixedviT da, ra Tqma unda, gamosaxulia koleqtiuri dazRvevis yvela monawilis sawyisi $x_1,\ x_2,\ \ldots,\ x_m$ asakebiT. es saSualebas iZleva yvela gamoTvla, romelic gaerTianebuli sicocxlis mdgomareobas exeba, warmoebuli iyos erTi (ω) individis terminebSi.

garkveuli gamartivebebi warmoiSoba aseve rodesac yvela individis mokvdaoba ganawilebulia maikhamis erTidaimave kanoniT (ix. p. 3.6):

$$\mu_{x_i}(t) = \mu_{x_i+t} = A + Be^{\alpha(x_i+t)} = A + Br^{x_i+t}, \ , \ t \geq 0, \ i = 1,\ldots,m, \ r = e^{\alpha}.$$

SevadginoT toloba

$$A + Br^{x_1+t} + \ldots + A + Br^{x_m+t} = m(A + Br^{\omega+t})$$

da igve msjelobiT, rogorc es hompertcis kanonis dros iyo, mivdivrT Semdeg gantolebamde

$$r^{x_1} + r^{x_2} + \ldots + r^{x_m} = mr^{\omega},$$

romlis amonaxssac Semdegi saxe aqvs:

$$\omega = \frac{1}{\alpha}[\ln\{r^{x_1} + r^{x_2} + \ldots + r_{x_m}\} - \ln m] =$$

$$= \frac{1}{\alpha}[\ln\{e^{\alpha x_1} + e^{\alpha x_2} + \ldots + e^{\alpha x_m}\} - \ln m]. \qquad (6.2.3)$$

da amgvarad, hompertcis kanonisTvis

$$\boxed{\mu_{x_1:x_2:\ldots:x_m}(t) = m\mu_{\omega}(t) = \mu_{\omega:\omega:\ldots:\omega}(t)}$$

da, aqedan gamimdinare, x_1, x_2, . . . , x_m asakebis mqone m piri SeiZleba Secvlili iyos EerTnairi ω `sawyisi~ asakis mqone m piriT, romelic gamoiTvleba (6.2.3) formuliT.

6.3 ukanasknelis cocxlad darCenilis statusi (*last-survivor status*)

ukanasknelis cocxlad darCenilis statusi aRiniSneba

$U := \overline{x_1 : x_2 : \ldots : x_m}$ an $(\overline{x_1 : x_2 : \ldots : x_m})$-iT

da iTvleba daSlilad, Tu koleqtivis yvela warmomadgeneli gardaicvala, e.i.

$T(U) = \max(T(x_1), T(x_2), \ldots, T(m))$.

gaerTianebuli sicocxlis mdgomareoba Seesabameba eleqtroqselSi naTurebis mimdevrobiT SeerTebas, ukanasknelis cocxlad darCenis mdgomareoba ki – paralelur SeerTebas. gasagebia, rom

$$
{}_t p_{\overline{x_1:x_2:\ldots:x_m}} = P\{T(U) \le t\} = P\{\max(T(x_1), T(x_2), \ldots, T(x_m)) \le t\} =
$$

$$
= P\{T(x_1) \le t, T(x_2) \le t, \ldots, T(x_m) \le t\},
$$

xolo damoukidebeli sikvdilebis daSvebisas

$$
{}_t q_{\overline{x_1:x_2:\ldots:x_m}} = \prod_{i=1}^{m} {}_t q_{x_i}\,, \qquad {}_t q_{\overline{x_1::x_2:\ldots:x_m}} = 1 - \prod_{i=1}^{m} (1 - {}_t p_{x_i})\,.
$$

ukanasknelis cocxlad darCenilis statusis daSlis drois ganawilebis simkvrive tolia

$$
f_{\overline{x_1:x_2:\ldots:x_m}}(t) = \frac{d}{dt} P\{T(U) \le t\} = \frac{d}{dt} \prod_{i=1}^{m} (1 - {}_t p_{x_i})\,.
$$

magaliTi 6.3.1. sikvdilebiis damoukideblobis daSvebisas ipoveT bolo ori individis cocxlad darCenilis statusis ganawilebis simkvrive $U = \overline{x_1 : x_2}$

1) zogad SemTxvevaSi;

2) de muavris modelisaTvis.

amoxsna. ramdenadac $m=2$, xolo $T(U)=\max(T(x_1),T(x_2))$, amdenad statusis ganawilebis simkvrivisaTvis

$$f_{\overline{x_1:x_2:\ldots:x_m}}(t)=\frac{d}{dt}\{P(T(U)\le t)P(T(x_2)\le t)\}=\frac{d}{dt}\{F_{x_1}(t)F_{x_2}(t)\}=$$
$$=\frac{d}{dt}\left\{\frac{F(x_x+t)}{s(x_1)}\frac{F(x_2+t)}{s(x_2)}\right\}=\frac{f(x_1+t)}{s(x_1)}\frac{F(x_2+t)}{s(x_2)}+$$
$$+\frac{f(x_2+t)}{s(x_2)}\frac{F(x_1+t)}{s(x_1)}=f_{x_1}(t)F_{x_2}(t)+f_{x_2}(t)F_{x_1}(t)<f_{x_1}(t)+f_{x_2}(t).$$

de muavris modelisaTvis

$$f_{\overline{x_1:x_2}}(t)=\frac{I_t(0,\omega-x_1)}{\omega-x_1}\left[\frac{tI_t(0,\omega-x_2)}{\omega-x_2}+I_t(-\infty,\omega-x_2)\right]+$$
$$+\frac{I_t(0,\omega-x_2)}{\omega-x_2}\left[\frac{tI_t(0,\omega-x_1)}{\omega-x_1}+I_t(-\infty,\omega-x_1)\right]= \qquad (6.3.1)$$
$$=\frac{2tI_t(0,\min(\omega-x_1,\omega-x_2))}{(\omega-x_1)(\omega-x_2)}+\frac{I_t(\min(\omega-x_1,\omega-x_2),\max(\omega-x_1,\omega-x_2))}{\min(\omega-x_1,\omega-x_2)}$$

gasagebia, rom

$$\mu_{\overline{x_1:x_2:\ldots:x_m}}(t)=\frac{f_{\overline{x_1:x_2:\ldots:x_m}}(t)}{s_{\overline{x_1:x_2:\ldots:x_m}}(t)}=\frac{d}{dt}\prod_{i=1}^{m}(1-{}_tp_{x_i})\Big/\left(1-\prod_{i=1}^{m}(1-{}_tp_{x_i})\right).$$

6.4 orive statusze magaliTebi

upirvelesad movaxdinoT ilustrireba sxc-is maxasiaTeblebis SesaZleblobis im albaTobebis gamoTvlisas, romlebic dakavSirebulia ganxilul statusebTan.

magaliTi 6.4.1. Tu vivaraudebT, rom $T(70)$ da $T(75)$ damoukidebelni arian miiReT imis albaTobebis gamosaxuleba, rom

(i) pirveli sikvdili moxdeba 5-dan 10 wlamde SualedSi;

(*ii*) bolo sikvdili moxdeba imave SualedSi;

(*iii*) gamoiTvaleT es albaTobebi ssrk-s qalebisaTvis da mamakacebisaTvis, SeadareT Sesabamis individualur albaTobebs.

amoxsna. (*i*) ori piris (70:75) gaerTianebuli sicocxlis statusis saZiebeli albaTobisaTvis, (6.1.1) formulis gaTvaliswinebiT, samarTliania tolobaTa Semdegi jaWvi:

$$P\{5<T(70:75)\le 10\}=P\{T(70:75)>5\}-P\{T(70:75)>10\}=$$
$$={}_5p_{70:75}-{}_{10}p_{70:75}={}_5p_{70}\,{}_5p_{75}-{}_{10}p_{70}\,{}_5p_{75}=\frac{l_{75}}{l_{70}}\frac{l_{80}}{l_{75}}-\frac{l_{80}}{l_{70}}\frac{l_{85}}{l_{75}}=\frac{l_{80}}{l_{70}}\left(1-\frac{l_{85}}{l_{75}}\right).$$

(*ii*) aq ipoveT bolo ori piris ($\overline{70:75}$) cocxlad darCenilis statusisaTvis miviRebT:

$$P\{5<T(\overline{70:75})\le 10\}=P\{T(\overline{70:75})\le 10\}-P\{T(\overline{70:75})\le 5\}=$$
$$={}_{10}q_{70:75}-{}_5q_{70:75}={}_{10}q_{70}\,{}_{10}q_{75}-{}_5q_{70}\,{}_5q_{75}=(1-{}_{10}p_{70})(1-{}_{10}p_{75})-(1-{}_5p_{70})(1-{}_5p_{75})=$$
$$=\left(1-\frac{l_{80}}{l_{70}}\right)\left(1-\frac{l_{85}}{l_{75}}\right)-\left(1-\frac{l_{75}}{l_{70}}\right)\left(1-\frac{l_{80}}{l_{75}}\right).$$

(*iii*) sxc-is (danarTi 1) Tanaxmad ssrk-s mamakacebisaTvis gvaqvs

$$P_m\{5<T(70:75)\le 10\}=\frac{18{,}787}{43{,}405}\left(1-\frac{9{,}063}{30{,}857}\right)=0{,}3057,$$

$$P_m\{5,T(\overline{70:75})\le 10\}=\left(1-\frac{18{,}787}{43{,}405}\right)\left(1-\frac{9{,}063}{30{,}857}\right)-\left(1-\frac{30{,}857}{43{,}405}\right)\left(1-\frac{18{,}7870}{30{,}857}\right)=0{,}2875,$$

$$P_{m_1}=P\{5<T(70)\le 10\}=\frac{l_{75}}{l_{70}}-\frac{l_{80}}{l_{70}}=\frac{30{,}857}{43{,}405}-\frac{18{,}787}{43{,}405}=0{,}2781,$$

$$P_{m_2}=P\{5<T(75)\le 10\}=\frac{l_{80}}{l_{75}}-\frac{l_{85}}{l_{75}}=\frac{18{,}787}{30{,}857}-\frac{9{,}063}{30{,}857}=0{,}3151,$$

xolo ssrk-s qalebisTvis

$$P_w\{5<T(70:75)\le 10\}=\frac{41{,}674}{70{,}043}\left(1-\frac{42{,}265}{57{,}679}\right)=0{,}3447,$$

$$P_w\{5,T(\overline{70:75})\le 10\}=\left(1-\frac{41,674}{70,043}\right)\left(1-\frac{24,265}{57,679}\right)-\left(1-\frac{57,679}{70,043}\right)\left(1-\frac{41,674}{57,679}\right)=0,1856,$$

$$P_{w_1}=P\{5<T(70)\le 10\}=\frac{57,679}{70,043}-\frac{41,674}{70,043}=0,2285,$$

$$P_{w_2}=P\{5<T(75)\le 10\}=\frac{41,674}{57,679}-\frac{24,265}{57,679}=0,30182.$$

SesZlebelia agreTve $U:=\overline{x_1:x_2:...:x_m}$ da $U:=x_1:x_2:...:x_m$ statusebisaTvis sxvadasxva ricxviTi maxasiaTeblebis moZebnac, kerZod, p. 3.3-is Tanaxmad

$$\overset{0}{e}_U=ET(U)=\int_0^\infty tf_U(t)dt=\int_0^\infty {}_t\,p_U\,dt,$$

$$DT(U)=\int_0^\infty t^2 f_U(t)dt-\left(\overset{0}{e}_U\right)^2=2\int_0^\infty t\,{}_tp_U\,dt-\left(\overset{0}{e}_U\right)^2.$$

magaliTi 6.4.2. sikvdilebis damoukideblobis daSvebisas de muavris modelisaTvis ipoveT $\overset{0}{e}_{\overline{x_1:x_2}}$.

amoxsna. visargebloT (6.3.1) formuliT, gveqneba

$$\overset{0}{e}_{\overline{x_1:x_2}}=\int_0^\infty tf_{\overline{x_1:x_2}}(t)dt=\int_0^{\min(\omega-x_1,\omega-x_2)} t\frac{2t}{(\omega-x_1)(\omega-x_2)}dt+\int_{\min(\omega-x_1,\omega-x_2)}^{\max(\omega-x_1,\omega-x_2)} t\frac{dt}{(\omega-x_1)(\omega-x_2)}=$$

$$=\frac{2\min^3(\omega-x_1,\omega-x_2)}{3(\omega-x_1)(\omega-x_2)}+\frac{\max^2(\omega-x_1,\omega-x_2)-\min^2(\omega-x_1,\omega-x_2)}{2\max(\omega-x_1)(\omega-x_2)}=$$

$$=\frac{2\min^2(\omega-x_1,\omega-x_2)}{3\max(\omega-x_1)(\omega-x_2)}+\frac{\max^2(\omega-x_1,\omega-x_2)-\min^2(\omega-x_1,\omega-x_2)}{2\max(\omega-x_1)(\omega-x_2)}=$$

$$=\frac{\min^2(\omega-x_1,\omega-x_2)+3\max^2(\omega-x_1,\omega-x_2)}{6\max(\omega-x_1)(\omega-x_2)}$$

magaliTi 6.4.3. sikvdilebis damoukideblobis daSvebisas de muavris modelisaTvis ipoveT $\overset{0}{e}_{x_1:x_2}$.

amoxsna. Tu visargeblenT (6.1.3) formuliT, miviRebT

$$\overset{0}{e}_{x_1:x_2} = 2\int_0^{\omega-x} t\frac{\omega-x-t}{(\omega-x)^2}dt = \frac{\omega-x}{3}.$$

6.5. k cocxlad darCenilebis statusebi, Sereuli statusebi (*compound statuses*)

k ukanasknelі cocxlad darCenilis statusi (*k-survivor status*) aRiniSneba ase

$$U := \frac{\quad k}{x_1 : x_2 : ... : x_m} \qquad (6.5.1)$$

da arsebobs iqamde sanam (x_1), (x_2), . . . , (x_m) individebidan k mainc cocxalia, e.i. iTvleba daSlilad $(m-k+1)$-e sikvdilis dadgomisas. gasagebia, rom

$$\left(\frac{\quad m}{x_1 : x_2 : ... : x_m}\right) = (x_1 : x_2 : ... : x_m),$$

$$\left(\frac{\quad 1}{x_1 : x_2 : ... : x_m}\right) = (\overline{x_1 : x_2 : ... : x_m}),$$

da, aqedan gamomdinare, erToblivi cxovrebis mdgomareoba $(k=m)$ da ukanasknelis cocxlad darCenis mdgomareoba $(k=1)$ warmoadgenen (6.5.2) statusis kerZo SemTxvevebs. k cocxlad darCenilebis zusti statusi ($|k|$*-deferred status*) aRiniSneba Semdegnairad

$$U := \frac{\quad |k|}{x_1 : x_2 : ... : x_m} \qquad (6.5.2)$$

da arsebobs Tu cocxalia m individebidan (x_1), (x_2), . . . , (x_m) zustad k, e.i. is iwyeba $(m-k)$-e sikvdilis momentSi da wydeba $(m-k+1)$-e sikvdilis dadgomis momentSi. es statusi farTo gamoyenebas povebs anuitetebis (xangrZlivobis SezRuduli vadiT gadaxdebis mimdevrobebi) gamoTvlisas.

amgvarad, Cven ganvsazRvreT statusebi individebis jgufisTvis k cocxlad darCenilis saerTo statusis mixedviT. avRniSnoT, rom SeiZleba moxdes axali statusebis kombinirebac am TavSi ganxiluli sabaziso statusebis meSveobiT.

Sereuli statusi vuwodoT mdgomareobas, romelsac safuZvlad udevs statusebis kombinacia, amasTan maTgan erTi maincaa mocemuli erT individze metisaTvis.

magaliTi 6.5.1. aRwereT Semdegi Sereuli statusebi:

(i) $(\overline{x_1 : x_2} : \overline{x_3 : x_4})$; (ii) $(\overline{\overline{x_1 : x_2} : x_3 : x_4})$; $(iii)(x_1 : x_2 : \overline{x_3 : x_4})$.

amoxsna. (i) es mdgomareoba narCundeba Tu (x_1) da (x_2) -dan erTi mainc cocxalia da ukidures SemTxvevaSi (x_3) da (x_4) -dan erTi mainc. $(\overline{x_1 : x_2} : \overline{x_3 : x_4})$ statusis daSlis moments warmoadgens

$$T(U) = \min\{maxT(x_1), T(x_2)\}, \max\{T(x_3), T(x_4)\}\}.$$

(ii) aseTi mdgomareoba narCundeba, Tu oTxidan ori maincaa cocxali, kerZod (x_3) da (x_4), an roca mxolod erTia cocxali da es an (x_1) –ia an (x_2). $(\overline{\overline{x_1 : x_2} : x_3 : x_4})$ statusis daSlis moments warmoadgens

$$T(U) = \max\{maxT(x_1), T(x_2)\}, \min\{T(x_3), T(x_4)\}\}.$$

(iii) mdgomareoba narCundeba, Tu cocxlebi arian (x_1) da (x_2) da, roca kidev erTia cocxali an (x_3), an (x_4). $(x_1{:}x_2{:}\overline{x_3{:}x_4})$ statusis daSlis moments warmoadgens

$T(U) = \min\{T(x_1), T(x_2), \max\{T(x_3), T(x_4)\}\}$.

6.6 sakontrolo davalebebi

davaleba 6.6.1

darwmundiT, rom ori individis $U \coloneqq x_1{:}x_2$ gaerTianebuli sicocxlis (6.1.2) statusis ganawilebis simkvrive de muavris modelisaTvis akmayofilebs normirebis pirobas.

davaleba 6.6.2

darwmundiT, rom bolo ori individis $U \coloneqq \overline{x_1{:}x_2}$ cocxlad darCenilis (6.2.1) statusis ganawilebis simkvrive de muavris modelisaTvis akmayofilebs normirebis pirobas.

davaleba 6.6.3

aCveneT, rom $f(t) = e^{-t}, t \geq 0$ ganawilebis simkvrivis mqone eqsponencialuri modelisaTvis samarTliania Semdegi formulebi:

$f_{x_1:x_2}(t) = 2e^{-2t}$, $f_{\overline{x_1:x_2}}(t) = 2e^{-t}(1 - e^{-t})$, $t \geq 0$.

daxateT am simkvriveebiis grafikebi da SeadareT isini de muavris modelis SemTxvevisaTan.

davaleba 6.6.4

Tu vivaraudebT, rom $T(65)$, $T(70)$ da $T(75)$ damoukidebelni arian miiReT imis albaTobebis gamosaxuleba, rom

(ii) pirveli sikvdili moxdeba 5-dan 10 wlamde SualedSi;

(*ii*) bolo sikvdili moxdeba imave SualedSi;

(*iii*) gamoiTvaleT es albaTobebi ssrk-s qalebisaTvis da mamakacebisaTvis, SeadareT Sesabamis individualur albaTobebs.

davaleba 6.6.5

de muavris modelis da 6.6.3 davalebidan eqsponencialuri modelisaTvis sikvdilebis damoukideblobis daSvebisas ipoveT $\overset{0}{e}_{x_1:x_2}$.

davaleba 6.6.6

darwmundiT, rom sikvdilebis damoukideblobis daSvebisas de muavris modelisaTvis

$$\overset{0}{e}_{\overline{x_1:x_2}} = \frac{2(\omega - x)}{3}, \quad DT(x:x) = DT(\overline{x:x}) = \frac{(\omega-x)^2}{18}.$$

davaleba 6.6.7

sikvdilebis damoukideblobis daSvebisas $f(t) = \lambda e^{-\lambda t}, \lambda > 0, t \geq 0$ ganawilebis simkvrivis mqone eqsponencialuri modelisaTvis, miiReT formulebi SemdegebisaTvis

$$f_{x_1:x_2:x_3}(t), \; f_{\overline{x_1:x_2}:x_3}(t), \quad f_{\overline{x_1:x_2:x_3}}(t),$$

$$\overset{0}{e}_{x_1:x_x:x_3}, \overset{0}{e}_{\overline{x_1:x_2}:x_3}, \overset{0}{e}_{\overline{x_1:x_2:x_3}}.$$

davaleba 6.6.8

aRwereT Semdegi Sereuli statusebi

$$\left(\overline{x_1:x_2}:\overline{(x_3:x_4):x_5}\right); \quad \left(x_1:\overline{(x_2:x_3):(x_4:x_5)}:x_6\right).$$

Tavi 7.aqtuaruli maTematikis ZiriTadi albaTuri maxasiaTeblebis Sefasebis statistikuri meTodebi

am TavSi naCvenebi iqneba sadazRvevo saqmeSi maTematikuri statistikis meTodebis gamoyenebis SesaZleblobebi. amasTan aqve upriania kidev erTxel moxdes imis Sexseneba, rom wina Tavebis ideologia p. 3.1-is Sesabamisad cxadi saxiT vrceldeba dazRvevis iseT sferoebze, romlebic araa dakavSirebuli pirad dazRvevasTan, kerZod:

— sawarmoebis da firmebis aRWurviloba (*equipment of organizations and firms*).

— samewarmeo riskebi (*business ventures*), samewarmeo sesxebi (*business lians*), kreditebi (*credits*) da sxva.

aseve SegaxsenebT, iseve rogorc adre, Teoremebis da lemebis damtkicebas avRniSnavT ♠ maCvenebliT.

7.1 albaTobebis Sefasebebi

sadazRvevo maTematikaSi albaTobebis magaliTebs, romelTa Sefasebebic xdeba warmoadgenen ${}_tq_x$, ${}_tp_x$, q_x, p_x, ${}_{t|u}q_x$, ${}_{t|}q_x$.

vTqvaT, A _ SemTxveviTi xdomilobaa, $A_1, A_2, ..., A_N$ _ damoukidebeli cdebis erTgvarovani seriaa, romelTagan TiToeulis Sedegad A dgeba an $P(A)$ albaTobiT, an ar dgeba $1 - P(A)$ albaTobiT. am SemTxvevaSi $P(A)$-s Sesafaseblad bunebrivia aviRoT

$$P_N(A) = \frac{1}{N}\sum_{i=1}^{N} I(A_i) = \frac{k}{N},$$

sadac $I(A)$- SemTxveviTi sidideა, romelic xdomilobis Semdegi indikatoris tolia

$$I(A) = \begin{cases} 1, & \text{თუ } \omega \in A, \\ 0, & \text{თუ } \omega \notin A, \end{cases}$$

ω - elementaruli xdomilobaa, xolo $\frac{k}{N}$ fardoba warmoadgens A xdomilobis sixSires N erTgvarovan damoukidebel cdebSi.

$P_N(A)$ Sefasebas gaaCnia Semdegi Tvisebebi:

1) $P_N(A)$ – $P(A)$-sTvis Caunacvlebeli Sefasebaa, e.i.

$EP_N(A) = P(A)$;

2) misi dispersia cdebis ricxvis zrdasTan erTad mcirdeba:

$DP_N(A) = \frac{1}{N} P(A)\big(1 - P(A)\big) < \frac{1}{N} \to 0$, როცა $N \to$;

3) $NP_N(A)$ SemTxveviTi sidide ganawilebulia binomialuri kanonis mixedviT:

$P\{NP_N(A) = k\} = C_N^k [P(A)]^k [1 - P(A)]^{N-k}$;

4) $P_N(A)$ – $P(A)$-sTvis Zaldebuli Sefasebaa, e.i.

$\lim_{N\to} P\{|P_N(A) - P(A)| < \varepsilon\} = 1$ ნებისმიერი > 0 – სთვის;

5) $P_N(A)$ – minimaluri dispersiiT Sefasebaa $\sum_{i=1}^{N} c_i I(A_i)$ $c_i > 0$ saxis wrfiv miukerZoebel SefasebaTa klasSi.

6) $P_N(A)$ 1-is toli albaTobiT miiswrafis $P(A)$-sken, e.i.

$P\{\lim_{N\to} |P_N(A) - P(A)| = 0\} = 1$.

1-4 Tvisebis damtkiceba SesaZlebelia moiZebnos maTematikuri statistikis saxelmZRvaneloebis umetesobaSi; 5 Tviseba mtkicdeba lagranJis ganusazRvreli mamravlebis meTodiT (pirobiTi eqstremumis amocana), xolo 6 Tviseba gamomdinareobs didi ricxvebis gaZlierebuli kanonidan.

SeniSvna 7.1.1. *Canacvlebul Sefasebas*

$$\hat{P}(A) = P_N(A)\left[1 + \frac{\hat{D}P_N(A)}{P_N^2(A)}\right]^{-1} = P_N(A)\left[1 + \frac{1 - P_N(A)}{P_N(A)}\right]^{-1}$$

gaaCnia ufro naklebi saSualokvadratuli gadaxra (skg), vidre Caunacvlebeli Sefasebis $DP_N(A)$ *dispersias, e.i.*

$u^2\big(\hat{P}(A)\big) = S\hat{P}(A) + b^2(\hat{P}(A)) < DP_N(A)$, *sadac*, $b(\hat{P}(A)) = E\hat{P}(A) - P(A)$ - $\hat{P}(A)$*-s gawonaswo rebaa (sapi rwonea).*

garkveu l i pi robebis da dakvi rvebaTa mci re mocu l obebis d r os $\hat{P}(A)$ Se f asebis skg z og j e r SeiZ l eba P_N Se f asebis d i spe r s i aze o r-sam j e r nak l ebi aRmoCndes.

7.2 pa r ame t r u l i da a r apa r ame t r u l i Se f asebebi. ganawi l ebis da gada r Cenis empi r iu l i f unqc iebi

ucnobi pa r ame t r ebis sabo l oo nak r ebze damokidebu l ebi ganawi l ebis da gada r Cenis f unqciebis Se f asebisas, f unqc i i s ucnobebis moZebnis amocanis daiyaneba xdeba ucnobi pa r ame t r ebis Se f asebamde. ma r T l ac, z ogie r Ti ganawi l eba sakmao d zus t ad aRwe r s individumebis mokvdavobis p r o cess (amaTuim e l emen t ebis mwyob r idan gamosv l is gamoCena). aseTi ganawi l ebis maga l i Tebad aq t ua r u l maTema t ikaSi SeiZ l eba gamo dges de muav r is, homp e r t c i s, maikhamis, vaibu l is da e r l angis mo de l ebis ganawi l ebebi, r om l ebic p. 3.6-Sia ganxi l u l i.

r ogo r c ad r e iyo aRniSnu l i, saimedoobis Teo r iaSi, r ome l sac m r ava l i Sexebis we r t i l i gaaCnia aq t ua r u l maTema t ikasTan, f a r Ted gamo iyeneba eqsponenc ia l u r i $F(t,\lambda) = 1 - e^{-\lambda t}, t \geq 0$ ganawi l eba. r ome l i c aRwe r s im e l emen t ebis mwyob r idan gamosv l as, r ome l Tac eqsp l uatac i i s na r Ceni d r o a r aa damokidebu l i wina muSaobis xang r Z l ivobaze. SevniSnoT, r om Tu mwyob r idan gamosv l is ganawi l ebis f unqc ia eqsponenc ia l u r i ganawi l ebaa, maSin in t ensivobis f unqc ia λ mudmivis t o l ia. sxva ganawi l ebebidan SeiZ l eba gamovyoT vaibu l is ganawi l eba $F(t,\lambda,\alpha) =$

$1 - e^{-(\lambda t)^{\alpha}}$, $t \geq 0, \alpha > 0$, romelic gamoiyeneba daRlilobis, eleqtrovakuumuri xelsawyoebis mwyobridan gamosvlis, sakisrebis damtvrevis movlenebis aRwerisas.

Tu ganawilebis funqcia cnobilia ucnobi parametrebis ukanasknel nakrebamde, maSin adgili aqvs parametrul apriorul *ganusazRvrelobis SemTxvevas*. klasikuri meTodebi (magaliTad: maqsimaluri albaTobis meTodis, momentebis meTodis, umciresi kvadratebis meTodis) saSualebas iZlevian sakmarisad efeqturad moxdes dakvirvebis mixedviT ucnobi parametrebi.

eqsperimentaluri sistemebis sandoobis Sefasebis amocanebSi statistikur monacemebs warmoadgenen gamosakvlevi elementebis mwyobridan gamosvlis momentebi, romlebsac, rogorc wesi, iReben didi raodenobis ZviradRirebuli eqsperimentebis Catarebis Sedegad. amasTan mkvlevarebi xSirad ar floben TviTon elementebis Sesaxeb da maTi mwyobridan gamosvlis warmoSobis bunebis Sesaxeb sakmaris informacias. aseve SesaZlebelia SemTxveva rodesac es informacia realur obieqtis Sesatyvisi arc aRmoCndes, rac arTulebs, zogjer ki SeuZlebels xdis adekvaturi paramertuli modelis agebas.

Tu aprioruli informacia mwyobridan gamosvlis Sesaxeb atarebs zogad xasiaTs (magaliTad, cnobilia, rom mwyobridan gamosvlis ganawilebis warmoebuli funqciebi garkveul rigamde arseboben, uwyvetni arian da a.S.), ganawilebis funqciis ucnobebis Sefasebis, saimedoobis, intensivobis da sxv. amocana bunebrivia ganvixiloT statistikis erT-erTi ganyofilebis - araparametruli statistikis TvalsazrisiT. zemoT ganxilul problemas, ra Tqma unda adgili aqvs aqtuarul maTematikaSi Sefasebebis sxvadasxva amocanebis amoxsnis drosac, kerZod, dazRvevis axal arastandartul saxeobebSi neto-premiis gamoTvlisas.

ganvsazRvroT termini `araparametruli~.

f. tarasenkos mixedviT `araparametruli amocana – es aris statistikuri amocana, romelic gansazRvrulia iseT klasikur ganawilebebze, romelTa Soris erTi mainc ar daiyvaneba funqciebis parametrul ojaxebze~.

araparametruli procedurebis parametrulisagan mTavar gansxvaveba imaSi mdgomareobs, rom isini Sromisunarianebi arian maSin, rodesac ganawilebis Sesaxeb aprioruli informacia obieqtis maTematikuri modelis gansazRvrisas ganawilebis raime parametruli ojaxis gamoyenebis saSualebas ar iZleva.

Semogvyavs simboloebi: ⇒ _ ganawilebis mixedviT krebadobis; $\mathcal{N}_s\{\mu,\sigma\}$ _ s- ganzomilebiani SemTxveviTi sididis, romelic ganawilebulia normalurad saSualoebis veqtoriT $\mu = (\mu_{1,} \dots, \mu_s)$ da kovariaciuli matricebis

$$\sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1s} \\ \cdots & \cdots & \cdots \\ \sigma_{s1} & \cdots & \sigma_{ss} \end{bmatrix},\ 0 \le \sigma_{ij} = \sigma_{ij}(x) < \quad, ij = \overline{1,s}.$$

Nramdenadac $F(x) = P(X \le x)$, $s(x) = P(X < x)$, amdenad erTnairad ganawilebuli SemTxveviTi sidideebis $\{X_i, i = \overline{1,N}\}$, romlebic warmoadgenen N individumis sicocxlis xangrZlivobebs N moculobis arCevisas, umartivesi Sefasebis saxiT bunebrivia aviRoT:

$$F_N(x) = \frac{1}{N}\sum_{i=1}^{N} I(X_i \le x),\ s_N(X) = \frac{1}{N}\sum_{i=1}^{N} I(X_i > t). \qquad (7.2.1)$$

$F_{N(x)}$ Sefasebas ewodeba *ganawilebis empiriuli funqcia*, zogadad warmoadgens araparametrul Sefasebas $F(x)$–sTvis da gaaCnia ganawilebis funqciis yvela Tviseba. $F_{N(x)}$ ganawilebis empiriuli funqciis statistikuri Tvisebebi kargadaa cnobili. gasagebia, rom isini SinaarsiT $P_N(A)$ Sefasebis identurini arian. aq maTgan mxolod imaT moviyvanT, romlebic Cven SemdgomSi gamogvadgebian gadarCenis empiriuli funqciis TvisebebTan SedarebiTi analizis Casatareblad da ganawilebis da gadarCenis funqciebis gluvi empiriuli SefasebisaTvis:

1) $EF_N(x) = F(x)$;

2) $DF_N(x) = \frac{1}{N}F(x)(1 - F(x))$;

3) centraluri zRvruli Teoremis ZaliT

$$F_N(x) = F(x) + \frac{\xi_N(x)}{\sqrt{N}},$$

sadac $\xi_N(x)$ SemTxveviTi sididis ganawileba ikribeba normaluri ganawilebisaken kanoniT, romlis parametrebia $\{0, F(x)(1 - F(x))\}$, anu

$$\xi_N(x) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}[I(X_i \le x) - F(x)] \Rightarrow \mathcal{N}_1\ \{0, \mathrm{F(x)(1 - F(x))}\} \qquad (7.2.2)$$

4) $F_{N(x)}$ - aris minimaluri dispersiis mqone Sefaseba $\sum_{i=1}^{N} c_i I(X_i \le x)$, $c_i > 0$ saxis mqone wrfivi Caunacvlebeli Sefasebebis klasSi.

ramdenadac

$$s_N(x) = \frac{1}{N}\sum_{i=1}^{N} I(X_i > x) = \frac{1}{N}\sum_{i=1}^{N}[1 - I(X_i \le x)] = 1 - F_N(x),$$

amdenad gadarCenis (saimedoobis) funqciis Sefasebas $s_N(x)$ gaaCnia $F_N(x)$ empiriuli ganawilebis funqciis analogiuri Tvsebebi. kerZod,

1) $Es_N(x) = s(x)$;

2) $Ds_N(x) = \frac{1}{N}F(x)\big(1-F(x)\big) = \frac{1}{N}s(x)(1-s(x))$.

zemoT moyvenil $s_N(x)$ da $F_N(x)$ Sefasebebs gaaCniaT ori nakli:

1) isini ganicdian wyvetas $X_1,\dots,X_n$ wertilebSi;

2) Sefaseba $F_N(x) = 0$ $_0 = [0, \min(X_1,\dots,X_N)]$ areSi, xolo $s_N(x) = 0$ $_0 = [\max(X_1,\dots,X_N), \ ]$ areSi.

amgvarad, Casmis meTodiT miRebuli funqciis intensivobis Sefaseba

$$\widehat{\mu_x} = \frac{f_N(x)}{s_N(x)}, \qquad (7.2.3)$$

2) naklis gamo areSi Sromisuunaroa $f_N(x)$ simkvrivis nebismieri SefasebisaTvis

7.3 gluvi empiriuli gadarCenis funqcia, misi asimptoturi waunacvloba da wanacvlebis krebadobis xarisxi

gansazRvreba 7.3.1. *borelevis Suv (aRniSvnebSi: $S(u) \in Suv$) funqcia miekuTvneba gadarCenis funqciaTa klass, Tu $S(u)$ uwyveti, mkacrad monotorulad klebadi funqciaa, iseTi, rom* $S(u): R^1 \to R^1, S(-\infty) = 1, S(\infty) = 0$.

wina punqtSi miTiTebuli $s_N(x)$ Sefasebis naklebs ar gaaCniaT gluvi empiriuli gadarCenis funqcia:

$$\widehat{s_N}(x) = \frac{1}{N}\sum_{i=1}^{N} S\left(\frac{x-X_i}{a_N}\right). \qquad (7.3.1)$$

sadac $S(u) \in Suv$, warmoadgens $a_N \downarrow 0$, ricxvTa mimdevrobas. $S(u)$ funqcias vuwodebT (7.3.1) Sefasebis *birTvs*.

SeniSvna 7.3.1. *gluvi empiriuli gadarCenis funqcia* $\widehat{s_N}(x)$ *(7.3.1)* a_N *= 0-s dros emTxveva* $s_N(x)$ *(7.2.1) empiriuli gadarCenis funqcias, e.i.* $\widehat{s_N}(x)|_{a_N=0} = s_N(x)$.

gansazRvreba 7.3.2. $H(z): R^s \to R^1$ *funqcia miekuTvneba* $\mathcal{N}_{v,s}(x)$ *klass, Tu* $H(z)$ *funqcia da misi* v *rigamde CaTvliT kerZo warmoebulebi uwyvetni arian* x *wertilSi;* $H(z) \in \mathcal{N}_{v,s}(R)$, *Tu* $H(z)$ *funqcis aRniSnuli Tviseba sruldeba yvela* $x \in R^s$*-sTvis.*

amoviweroT piroba, romlis drosac (7.3.1) Sefaseba warmoadgens asimptoturad waunacvlebels $s(x)$ –sTvis.

lema 7.3.1 ($\widehat{s_N}$-is asimptoturi waunacvlebloba). Tu gadarCenis funqcia $s(z) \in \mathcal{N}_{0,1}(x)$, e.i. $s(z)$ x wertilSi uwyvetia, $S(u) \in Suv$, xolo realur ricxvTa mimdevroba $a_N \downarrow 0$, maSin

$$\lim_{N\to} E\widetilde{s_N}(x) = s(x). \qquad (7.3.2)$$

damtkiceba. viTvaliswinebT ra $s(\cdot)$ gadarCenis funqciis uwyvetobas x wertilSi, maTematikuri lodinis ganmartebis Tanaxmad, gvaqvs

$$E\widetilde{s_N}(x) = E\left[\frac{1}{N}\sum_{i=1}^{N} S\left(\frac{x-X_i}{a_N}\right)\right] = \int_{R^{1+}} S\left(\frac{x-y}{a_N}\right) dF(y) =$$

$$= \int_0^x S\left(\frac{x-y}{a_N}\right) dF(y) + \int_x S\left(\frac{x-y}{a_N}\right) dF(y), \qquad (7.3.3)$$

sadac R^{1+} = [0,).

ramdenadac

$$\lim_{N\to} S\left(\frac{x-y}{a_N}\right) = \begin{cases} 0, \text{თუ } y < x, \\ 1, \text{თუ } y > x, \end{cases}$$

amdenad maJorirebuli krebadobis Sesaxeb lebegis Teoremis Tanaxmad (ix. danarTi 4 Teorema III)

$\lim_{N\to\infty} \int_{R^{1+}} S\left(\frac{x-y}{a_N}\right) dF(y)$ = $\int_x$ dF(y) = 1 – F(x) = s(x) ♠.

iseTi $\widetilde{s_N}(x)$–is Sefasebebis asagebad, romelTa skg-is mTavari nawili $u^2(\widetilde{s_N}(x)) = E(\widetilde{s_N}(x) - s(x))^2$ emTxveva (7.2.1)-Si ganmartebuli gadarCenis empiriuli funqciis $\frac{s(x)(1-s(x))}{N}$ dispersias, aucilebelia dadgindes nulisaken krebadi Semdegi wanacvlebis krebadobis xarisxi

$b(\widetilde{s_N}(x)) = E\widetilde{s_N}(x) - s(x)$.

$\widetilde{s_N}(x)|_{a_{N=0}} = s_N(x)$.

ramdenadac mkvlevars SeuZlia TviTon SearCios $\widetilde{s_N}(x)$–is Sefasebis Sesaferisi $S(u)$ birTvebi, jer SeviswavloT $\widetilde{s_N}(x)$ Sefasebis Tvisebebi laplasis birTviT $S_{LAP}(u) = 1 - \mathcal{F}_{LAP}(u)$, sadac $\mathcal{F}()$ – laplasis birTv-ganawilebaa:

$$\mathcal{F}_{LAP}(u) = \begin{cases} 0{,}5e^{u}, & -\infty < u < 0, \\ 1 - 0{,}5e^{-u}, & 0 \le u < \infty. \end{cases}$$

am SemTxvevaSi

$$S_{LAP}(u) = \begin{cases} 1 - 0{,}5e^{u}, & -\infty < u < 0, \\ 0{,}5e^{-u}, & 0 \le u < \infty. \end{cases} \qquad (7.3.4)$$

vipovoT Sefasebis wanacvlebis krebadobis xarisxi

$\tilde{s}_{N\,LAP}(x) = \frac{1}{N}\sum_{i=1}^{N} S_{LAP}\left(\frac{x-X_i}{a_N}\right)$.

lema 7.3.2 ($\tilde{s}_{N\,LAP}(x)$-is wanacvlebis krebadobis xarisxi). *vTqvaT* $s(z) \in \mathcal{N}_{0,1}(x)$, $\sup_{x\in R^{1+}} f(x) \le C < \infty$, $a_N \downarrow 0$, *maSin* $N \to \infty$*-sTvis*

$|b(\tilde{s}_{N\,LAP}(x)| = O(a_N)$. (7.3.5)

damtkiceba. warmovadginoT

$E\tilde{s}_{N\,LAP}(x) = \int_{R1+} S_{LAP}\left(\frac{x-y}{a_N}\right) dF(y) =$

$$= s(x) + \int_0^x S_{LAP}\left(\frac{x-y}{a_N}\right) dF(y) + \int_x^{\infty} \left[S_{LAP}\left(\frac{x-y}{a_N}\right) - 1 \right] dF(y) =$$

$$= s(x) + \int_0^x 0{,}5 e^{-\frac{x-y}{a_N}} f(y) dy + \int_x^{\infty} \left[1 - 0{,}5 e^{\frac{x-y}{a_N}} - 1 \right] f(y) dy$$

Sevcv l iT ra integra l ebSi cv l adebs $u = \frac{x-y}{a_N}$ da gaviTva l iswinebT sikvdi l ebis mrudebis SemosazRvru l obas, miviRebT

$|b(\tilde{s}_{N\,LAP}(x)| \le C\frac{a_N}{2}\left(\int_x^{x/a_N} e^{-u}du + \int_{-\infty}^{0} e^{u}du\right) = C\frac{a_N}{2}\left(-e^{x/a_N} + 1 + 1 -\right.$

e−∞=0aN. ♠

7.3.2 l emidan gamomdinareobs, r om $\widetilde{s_N}(x)$–is wanacv l ebis nu l isaken krebadobis xar isxis moZebnis d r os warmoiSoba N → ∞-sTvis Semdegi integra l ebis nu l isken krebadobis xar isxis gansaz Rv r is auc i l eb l oba

$$\int_x^{\infty} \left|S\left(\frac{x-y}{a_N}\right) - 1\right| dy, \qquad \int_0^x S\left(\frac{x-y}{a_N}\right) dy \qquad (7.3.6)$$

z ogad SemTxvevaSi, r oca bi r Tvebi $S(u) \in S_{uw}$, aseTi amocanis amoxsna ga r kveu l winaaRmdegobebs awydeba. maga l iTad, $S(u) = 1 - \mathcal{F}(u)$ bi r Tvis a r CeviT, sadac $\mathcal{F}(u)$ - koSis ganawi l ebis f unqc iaa:

$$S(u) = \frac{1}{2} - \frac{\arctan(u)}{\pi},$$

miviRebT, r om (7.3.6)-Si pi r ve l i integra l i ganS l adia, xo l o meo r e ikr ibeba r aRac mudmivisken.

amasTan dakavSi r ebiT ganvsaz Rv r oT bi r Tvebis iseTi k l asi, r one l TaTvisac ukve SesaZ l ebe l ia moiZebnos wanacv l ebis nu l isken k r ebadobis xa r isxi.

gansaz Rv r eba 7.3.3. $S(u)$ *f unqc ia miekuTvneba gada r Cenis f ini t u r f unqc iaTa k l ass* $Fin_S(S(u) \in Fin_S)$, *T u*

$$S(u)=\begin{cases}1, & -\infty < u < C_1,\\ Z(u), & C_1 \le u \le C_2,\\ 0, & C_2 \le u \le \infty,\end{cases}$$

sadac $Z(u)$ *- uwyveti, mkacrad monotonurad klebadi funqciaa, iseTi, rom* $Z(C_1)=1$, $Z(C_2)=0$, $C_1 < C_2$.

gadarCenis empiriuli gluvi funqciebis finitur birTvad SeiZleba aRebuli iyos, magaliTad, $[C_1, C_2]$-Si erTgvarovani birTvi, romlisTvisac $Z(u)=1-\frac{u-C_1}{C_2-C_1}$.

vipovoT $S(u)\in Fin_S$ finituri birTvis mqone $\tilde{s}_{N\,Fin}(x)$ Sefasebis wanacvlebis nulisken krebadobis siCqare.

lema 7.3.3. ($\tilde{s}_{NFin}(x)$-is wanacvlebis krebadobis xarisxi). Tu $s(z)\in\mathcal{N}_{0,1}(x)$, $\sup_{x\in R^{1+}} f(x)\le C<\infty$, $a_N\downarrow 0$, *maSin* $N\to\infty$*-saTvis*

$$|b(\tilde{s}_{N\,Fin}(x)| = O(a_N). \qquad (7.3.7)$$

damtkiceba. warmovadginoT

$$E\tilde{s}_{NFin}(x)=\int_0^\infty S\left(\frac{x-y}{a_N}\right)dF(y) =$$

$$= s(x)+\int_0^x S\left(\frac{x-y}{a_N}\right)dF(y)+\int_x^\infty\left[S\left(\frac{x-y}{a_N}\right)-1\right]dF(y).$$

$dF(y)=f(y)dy$-is da sikvdilianobis mrudis Semosazrvrulobis ZaliT

$$E\tilde{s}_{NFin}(x)\le s(x)+C\left[\int_0^x S\left(\frac{x-y}{a_N}\right)dy+\int_x^\infty\left|S\left(\frac{x-y}{a_N}\right)-1\right|dy\right].$$

Tu integralSi SevcvliT cvladebs $u=\frac{x-y}{a_N}$, miviRebT

$$|b(\tilde{s}_{N\,Fin}(x)|\le C a_N\left[\int_0^{x/a_N} S(u)du+\int_{-\infty}^0|S(u)-1|du\right].$$

radganac nebismieri N-saTvis integralebi

$$\int_0^{x/a_N} S(u)du\le\int_{C_1}^{C_2} S(u)du<\infty,$$

$\int_{-\infty}^{0}|S(u)-1|\,du \le \int_{C_1}^{C_2}|S(u)-1|du < \infty$, amdenad $|b(\tilde{s}_{N\,Fin}(x)| = O(a_N)$. ♠

7.4 zRvruli dispersia, skg-is krebadobis siCqare da empiriuli gluvi gadarCenis funqciis asimptoturi normaluroba

vipovoT $S(u) \in S_{uw}$ birTvis mqone $\widetilde{s_N}(x)$ Sefasebebis zRvruli dispersia (ix. 7.3.1 ganmarteba) da $\tilde{s}_{NFin}(x)$.

lema 7.4.1. ($\widetilde{s_N}$ dispersia). *Tu sruldeba 7.3.1 lemis pirobebi, maSin $N \to \infty$-saTvis.*

$$D\widetilde{s_N}(x) = \frac{s(x)(1-s(x))}{N} + o\left(\frac{1}{N}\right). \qquad (6.4.1)$$

damtkiceba. dispersiis ganmartebis Tanaxmad, (7.3.3)-is gaTvaliswinebiT, gvaqvs:

$$D\widetilde{s_N}(x) = \frac{1}{N} DS\left(\frac{x-X_1}{a_N}\right) =$$

$$= \frac{1}{N}\left\{\int_{R^{1+}} S^2\left(\frac{x-y}{a_N}\right)dF(y) - \left[\int_{R^{1+}} S\left(\frac{x-y}{a_N}\right)dF(y)\right]^2\right\}. \qquad (7.4.2)$$

integralebis mimarT lema 7.3.1-dan xerxebis gamoyenebiT, miviRebT:

$$D\widetilde{s_N}(x) = \frac{1}{N}[s(x) - s^2(x) + o(1)] = \frac{s(x)(1-s(x))}{N} + o\left(\frac{1}{N}\right). ♠$$

Sedegi 7.4.1. ramdenadac $S_{LAP}(u) \in S_{uw}$, amdenad 7.4.1 lemis ZaliT

$$D\tilde{s}_{N\,LAP}(x) = \frac{s(x)(1-s(x))}{N} + o\left(\frac{1}{N}\right).$$

lema 7.4.2. ($\tilde{s}_{N\,Fin}$ dispersia). *Tu sruldeba 7.3.3 lemis pirobebi, maSin $N \to \infty$-saTvis.*

$$D\tilde{s}_{N\,Fin}(x) = \frac{s(x)(1-s(x))}{N} + O\left(\frac{a_N}{N}\right) = \frac{s(x)(1-s(x))}{N} + o\left(\frac{1}{N}\right). \qquad (7.4.3)$$

damtkiceba. Tu integralebisaTvis gamoviyenebT (7.4.2) warmodgenas 7.3.3 lemidan mivalT (7.4.3) damokidebulebamde.♠

vipovoT $\tilde{s}_{N\,Fin}$ da $\tilde{s}_{N\,LAP}$ Sefasebebis mTavari nawilebi asimptoturi skg.

Teorema 7.4.1 ($\tilde{s}_{N\,Fin}$ skg). *vTqvaT sruldeba 7.3.3 lemis piroba. maSin $N \to \infty$-saTvis.*

$$u^2(\tilde{s}_{N\,Fin}) = \begin{cases} \dfrac{s(x)(1-s(x))}{N} + o\left(\dfrac{1}{N}\right), & a_N = o\left(1/\sqrt{N}\right); \\ O\left(\dfrac{1}{N}\right), & a_N = O\left(1/\sqrt{N}\right). \end{cases}$$

damtkiceba. Teoremis damtkiceba gamomdinareobs Semdegi warmodgenis $u^2(\tilde{s}_{N\,Fin}(x)) = D\tilde{s}_{N\,Fin}(x) + b^2((\tilde{s}_{N\,Fin}(x))$ da (7.4.3) da (7.3.7)-dan.♠

Teorema 7.4.2 ($\tilde{s}_{N\,Fin}$ skg). *vTqvaT sruldeba 7.3.3 lemis piroba. maSin $N \to \infty$-saTvis.*

$$u^2(\tilde{s}_{N\,LAP}) = \begin{cases} \dfrac{s(x)(1-s(x))}{N} + o\left(\dfrac{1}{N}\right), & a_N = o\left(1/\sqrt{N}\right); \\ O\left(\dfrac{1}{N}\right), & a_N = O\left(1/\sqrt{N}\right). \end{cases}$$

damtkiceba. 7.4.1 Sedegs da 7.3.2 lemas dauyonebliv mivyavarT Teoremis damtkicebamde.♠

ganvsazRvroT $\tilde{s}_{N\,Fin}(x)$ Sefasebis zRvruli ganawileba.

SemoviRoT aRniSvna $\{\xi_{J,N}\}_{j=1}^{N}, N = 1,2,\ldots$ _ seriis sqemaSi damoukdebeli erTnairad ganawilebuli SemTxveviTi sidideebis mimdevrobaa ($\xi_{J,N}$-is ganawileba damokidebulia N-ze).

Teorema 7.4.3 Tu sruldeba 7.3.3 lemis piroba da $a_N = o\left(N^{-1/2}\right)$, roca $N \to \infty$, maSin

$$\sqrt{N}\left[\widetilde{s_{N\,Fin}}(x) - s(x)\right] \Rightarrow \mathcal{N}_1\{0, s(x)\left(1-s(x)\right)\}. \qquad (7.4.4)$$

damtkiceba. warmovadginoT

$$\sqrt{N}\left[\widetilde{s_N}_{Fin}(x) - s(x)\right] = \sqrt{N}\left[\widetilde{s_N}_{Fin}(x) - E\widetilde{s_N}_{Fin}(x)\right] + \sqrt{N}b(\widetilde{s_N}_{Fin}(x)) \qquad (7.4.5)$$

cxadia, rom (7.4.5)-is marjvena nawilSi meore Sesakrebi (7.3.7)-is Tanaxmad nulsaken ikribeba roca $N \to \infty$:

$$\sqrt{N}b(\widetilde{s_N}_{Fin}(x) = \sqrt{N}\left[o\left(N^{-1/2}\right)\right] \to 0. \qquad (7.4.6)$$

vaCvenoT rom (7.4.5)-is marjvena nawilis pirveli SesakrebisaTvis sruldeba seriebis sqemaSi centraluri zRvruli Teoremis yvela piroba. vTqvaT,

$$\xi_{j,N} = \frac{1}{\sqrt{N}}\left[S\left(\frac{x-X_j}{a_N}\right) - ES\left(\frac{x-X_j}{a_N}\right)\right].$$

amgvarad, $\widetilde{s_N}_{Fin}(x) - E\widetilde{s_N}_{Fin}(x) = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\xi_{j,N}$. cxadia, rom $E\xi_{j,N} = 0$ da, 7.4.1 lemis Sedegis gaTvaliswinebiT

$$E\xi_{j,N}^2 = \frac{1}{N}DS\left(\frac{x-X_j}{a_N}\right) < \infty.$$

aseve, 7.4.1 lemis Tanaxmad $\lim_{N\to\infty} nE\xi_{1,N}^2 = s(x)(1 - s(x))$.

SevamowmoT lindebergis pirobis Sesruleba, romelic warmoadgens centraluri zRvruli Teoremis gamoyenebadobis sakmaris pirobas. imis gaTvaliswinebiT, rom $\sup_{u\in R^1} S(u) \leq 1$, nebismieri τ–sTvis gvaqvs

$$\beta_N = nE\left(\left|\xi_{1,N}\right|^2, \left|\xi_{1,N}\right| > \tau\right) < \frac{N}{\tau}E\left|\xi_{1,N}\right|^3 \leq$$

$$\leq \frac{C}{\sqrt{N}}\left[E\left|S\left(\frac{x - X_j}{a_N}\right)\right|^3 + \left|ES\left(\frac{x - X_j}{a_N}\right)\right|^3\right] < \frac{2C}{\sqrt{N}}$$

aq $C < \infty$ _ raRac dadebiTi mudmivaa. aqedan gamomdinare,

$\beta_N = O\left(N^{-1/2}\right) \to 0$, roca $N \to \infty$, e.i. lindebergis piroba sruldeba. seriebis sqemaSi centraluri zRvruli Teoremis gamoyenebiT, miviRebT, rom

$\sqrt{N}\xi_N = \sqrt{N}\sum_{j=1}^{N}\xi_{j,N} \Rightarrow \mathcal{N}_1\{0, s(x)(1 - s(x))\}.$

(7.4.6)-is gaTvaliswinebiT, viRebT saWiro (7.4.4) debulebas.♠

7.5. gluvi empiriuli ganawilebis funqcia

gansazRvreba 7.5.1. *borelevskis* $\mathcal{F}(u)$ *funqcia miekuTvneba* $Dis(\mathcal{F}(u) \in Dis)$*-is ganawilebis funqciaTa klass, Tu* $\mathcal{F}(u)$ *– uwyveti, mkacrad zrdadi funqciaa, iseTi, rom* $\mathcal{F}(.): R^1 \to R^1$, $\mathcal{F}(-\infty) = 0$, $\mathcal{F}(\infty) = 1$.

gluvi empiriuli gadarCenis $\tilde{s}_N(x)$ funqciis analogiis mixedviT aigeba gluvi empiriuli ganawilebis funqcia

$$\widetilde{F_N}(x) = \frac{1}{N}\sum_{i=1}^{N}\mathcal{F}\left(\frac{x - X_i}{a_N}\right), \qquad (7.5.1)$$

sadac $\mathcal{F}(u) \in Dis$, xolo ricxvTa mimdevroba $a_N \downarrow 0$.

(7.5.1) tipis Sefaseba pirvelad SemoTavazebuli iyo e. nadaraias mier 1964 wels. $\mathcal{F}$(u) funqcias uwodeben (7.5.1) tipis Sefasebis *birTv-ganawilebas*. Tu birTv-ganawilebad aviRebT $\mathcal{F}$(u) = 1 – S(u)–s, sadac $S(u) \in Suv$ an $S(u) \in Fin_S$, maSin

$$\widetilde{F_N}(x) = 1 - \frac{1}{N}\sum_{i=1}^{N} S\left(\frac{x - X_i}{a_N}\right) = 1 - \widetilde{s_N}(x). \qquad (7.5.2)$$

$\widetilde{F_N}(x)$ Sefasebis Tvisebebi asaxulia im lemebSi da TeoremebSi, romelTa damtkicebac mkiTxvelisadmia warmodgenili

lema 7.5.1. ($\widetilde{F_N}$-is asimptoturi Caunacvlebloba da dispersia). Tu ganawilebis funqcia $F(z) \in \mathcal{N}_{0,1}$, $\mathcal{F}(u) \in Dis$, xolo namdvili ricxvebis mimdevroba $a_N \downarrow 0$, maSin

$$E\widetilde{F_N}(x) = F(x) + o(1), \quad D\widetilde{F_N}(x) = \frac{1}{N}F(x)\big(1 - F(x)\big) + o\left(\frac{1}{N}\right).$$

finituri birTvebis klasis Fin_S, analogiis mixedviT SemoviyvanoT finituri birTv-ganawilebis klasi Fin_F.

gansazRvreba 7.5.2. $\mathcal{F}$(u) *funqcia miekuTvneba finitur birTv-ganawilebis klass* Fin_F *(*$\mathcal{F}(u) \in Fin_F$*, Tu*

$$\mathcal{F}(\mathrm{u}) = \begin{cases} 0, & -\infty < u < \mathrm{C}_1, \\ \mathrm{Y}(\mathrm{u}), & \mathrm{C}_1 \le \mathrm{u} \le \mathrm{C}_2, \\ 1, & \mathrm{C}_2 < u < \infty, \end{cases}$$

sadac $Y(u)$*- uwyveti mkacrad monotonurad zrdadi funqciaa, iseTi, rom* $Y(C_1) = 0$, $Y(C_2) = 1$, $C_1 < C_2$.

$\mathcal{F}(u) \in Fin_F$ birTvs vuwodebT finitur birTv-ganawilebas. cxadia, rom Tu $\mathcal{F}(u) \in Fin_F$, maSin $\mathcal{F} \in Dis$.

ganvsazRvroT $\mathcal{F}(u) \in Fin_F$-is mqone $\tilde{F}_{NFin}(x)$ Sefasebis wanacvlebis nulisaken krebadobis xarisxi, asimptoturi skg-is mTavari nawili da zRvruli ganawileba.

lema 75.2. ($\tilde{F}_{NFin}(x)$-is wanacvlebis krebadobis xarisxi). Tu $F(z) \in \mathcal{N}_{0,1}$, $\sup_{t \in R^{1+}} f(x) < \infty$, $a_N \downarrow 0$, maSin $N \to \infty$-sTvis

$$\left|b\left(\tilde{F}_{NFin}(x)\right)\right| = \mathrm{O}(a_N).$$

Teorema 7.5.1. ($\widetilde{F_N}$-is skg). vTqvaT sruldeba 7.5.2 lemis pirobebi. maSin $N \to \infty$-sTvis

$$u^2\left(\tilde{F}_{NFin}\right) = \begin{cases} \dfrac{F(x)\left(1 - F(x)\right)}{N} + o\left(\dfrac{1}{N}\right), & a_N = o\left({}^{1}/_{\sqrt{N}}\right) \\ \mathrm{O}\left(\dfrac{1}{N}\right), & a_N = \mathrm{O}\left({}^{1}/_{\sqrt{N}}\right). \end{cases}$$

Teorema 7.5.2 Tu sruldeba 7.5.2 lemis pirobebi da $a_N = o\left(N^{-1/2}\right)$, maSin

$$\sqrt{N}\left[\widetilde{F_N}(x) - F(x)\right] \Rightarrow \mathcal{N}_1\{0, F(x)(1 - F(x))\}.$$

7.6 sikvdilebis mrudis araparametruli Sefasebebi

$\mathcal{F}(u)$ birTv-ganawilebis absolutur uwyvetobis varaudiT, $f(x) = F'(x)$ sikvdilebis mrudis araparametruli Sefasebebis saxiT bunebrivia aviRoT Semdegi saxis Sefaseba:

$$f_{N(x)} = \frac{d}{dx}\widetilde{F_N}(x) = \frac{1}{Nh_N}\sum_{i=1}^{N} K\left(\frac{x-X_i}{h_N}\right), \qquad (7.6.2)$$

sadac ricxvTa mimdevroba $h_N \downarrow 0$, $K(u)$ – birTvs, zogagad rom vTqvaT, araa aucilebeli hqondes ganawilebis simkvrivis Tvisebebi.

(7.6.2) Sefasebas Cveulebriv uwodeben birTvuls an parzenovaskis, an rozenblat-parzenis tipis Sefasebas. h_N parametri, rogorc es (7.6.2) statistikis struqturidan Cans, asrulebs parametris $K(.)$ birTvis masStabis rols da amitom uwodeben sikvdilianobis mrudis birTvuli Sefasebis *bundovanobis parameters*.

(7.6.2) Sefasebis klasi pirvelad SemoTavazebuli iyo m. rozenblatonis mier 1956 wels. am naSromSi damtkicebuli iyo birTvuli Sefasebis asimptoturi Caunacvlebloba da Zaldebuloba. mogvianebiT, 1962 wels e. parzenma daamtkica am Sefasebebis asimptoturi normaluroba.

simkvrivis araparametruli SefasebisaTvis cnobilia Semdegi saintereso Sedegi.

lema 7.6.1. *araparametruli aprioruli ganusazRvrelobis pirobebSi ar arsebobs $f(x)$ ganawilebis ucnobi simkvrivis Caunacvlebadi Sefasebebi.*

gansazRvreba 7.6.1. borelevis funqcia $K(u)$ miekuTvneba $\mathcal{A}$ klass, Tu

$\sup_{u\in R^1} |K(u)| \le \infty$, $\int_{-\infty}^{\infty}|K(u)|du < \infty$, $\int_{-\infty}^{\infty} K(u)du = 1$, $K(u)\epsilon\mathcal{A}_{\nu}$,

Tu $K(u) \in \mathcal{A}$ da $K(u)$ akmayofileben Semdeg damatebiT pirobebs

$\int_{-\infty}^{\infty}|u^{\nu}K(u)|du < \infty$, $\mathcal{T}_j = \int_{-\infty}^{\infty} u^j K(u)du = 0$, $j = 1, \dots, \nu - 1$.

gamovarkvioT, ra pirobebis Sesrulebis dros warmoadgens (7.6.2) Sefaseba sikvdilebis mrudebisaTvis $f(x)$, e.i X SemTxveviTi sididis arauaryofiTi ganwilebis simkvrivisaTvis, asimptoturad Caunacvlebels. avRniSnoT, rom am SemTxvevaSi rTul deba cnobili klasikuri Sedegebis damtkiceba, raSic SesaZlebelia davrwmundeT mocemuli ganyofilebis yvela lemis da Teoremis magaliTze.

lema 76.2. (asimptoturi Caunacvlebloba f_N).

Tu sikvdilebis mrudi $f(x) \in \mathcal{N}_{0,1}(R)$, e.i. $f(x)$ uwyvetia $R^1 = (-\infty, \infty)$ -ze, $\sup_{x\in R^{1+}} f(x) \le C_1 < \infty$, $\int_{-\infty}^{\infty}|K(u)|du < \infty$, $\int_{-\infty}^{\infty} K(u)du = 1$, namdvil ricxvTa mimdevroba $h_N \downarrow 0$, maSin

$\lim_{N\to\infty} Ef_N(x) = f(x)$ (7.6.3)

damtkiceba. maTematikuri lodinis ganmartebis Tanaxmad gvaqvs

$Ef_N(x) = \frac{1}{h_N}\int_0^{\infty} K\left(\frac{x-y}{h_N}\right) f(y)dy$ (7.6.4)

integral (7.6.4) cvladebis SecvliT $u = \frac{x-y}{h_N}$, miviRebT

$$Ef_N(x) = \frac{1}{h_N}\int_{x/h_N}^{-\infty} (-h_N)\, K(u)f(x - uh_N)du =$$

$$= \int_{-\infty}^{\infty} K(u)\, f(x - uh_N)du - \int_{x/h_N}^{\infty} K(u)\, f(x - uh_N)du.$$

(7.6.5)

lebegis maJorirebadi krebadobis Teoremis ZaliT (ix. Teorema III danarT 4-Si) (7.6.5)-Si miviRebT: pirveli integralisaTvis

$\lim_{N\to\infty}\int_{-\infty}^{\infty}K(u)f(x-uh_N)\,du = f(x)\int_{-\infty}^{\infty}K(u)\,du = f(x)$, (7.6.6)

xolo meore integralisaTvis

$\lim_{N\to\infty} -\int_{x/h_N}^{\infty}K(u)\,f(x-uh_N)du \le C_1 \lim_{N\to\infty}\int_{x/h_N}^{\infty}K(u)du = 0.$ (7.6.7)

axla (7.6.6) da (7.6.7) dan pirdapir gamomdinareobs damtkiceba (7.6.3). ♠

vipovoT (6.6.2) Sefasebis dispersia.

gansazRvreba 7.6.2. namdvil ricxvTa $h_N > 0$, mimdevrobisaTvis pirobebi $\lim_{N\to\infty} h_N = 0$, $\lim_{N\to\infty}\left(h_N + \frac{1}{Nh_N}\right) = 0$ avRniSnoT Sesabamisad $\mathcal{H}_1$–iT, $\mathcal{H}_2$–iT.

lema 7.6.3 (f_N dispersia). *Tu sruldeba 7.6.2 lemis piroba* $\int_{-\infty}^{\infty}K^2(u)du < \infty$ *da* $h_N \epsilon \mathcal{H}_2$, *maSin* $N\to\infty$*-sTvis gvaqvs.*

$Df_N(x) = \frac{f(x)}{Nh_N}\int_{-\infty}^{\infty}K^2(u)du + o\left(\frac{1}{Nh_N}\right).$ (7.6.8)

damtkiceba. dispersiis ganmartebis Tanaxmad

$$Df_n(x) = \frac{1}{Nh_N^2}DK\left(\frac{x-X_1}{h_N}\right) = \frac{1}{Nh_N^2}\left[\int_0^{\infty}K^2\left(\frac{x-y}{h_N}\right)f(y)dy - \left(\int_0^{\infty}K\left(\frac{x-y}{h_N}\right)f(y)dy\right)^2\right].$$

(7.6.9)

Tu (7.6.9) integralSi SevcvliT cvlads $u = \frac{x-y}{h_N}$, miviRebT

$$Df_N(x) =$$
$$= \frac{1}{Nh_N}\left[\int_{-\infty}^{\infty}K^2(u)f(x-uh_N)du - h_N\left(\int_{-\infty}^{\infty}K(u)f(x-uh_N)du - \int_{x/h_N}^{\infty}K(u)f(x-uh_N)du\right)^2 - \int_{x/h_N}^{\infty}K^2(u)f(x-uh_N)du\right]$$

Semdeg, iseTive msjelobiT rogorc lema 7.6.2-is SemTxvevaSi iyo, mivdivarT dasamtkicebel debulebamde:

$$Df_N(x) = \frac{1}{Nh_N}\left[f(x)\int_{-\infty}^{\infty}K^2(u)du + h_N\left(f(x)+o(1)\right)^2 + o(1)\right] =$$

$$= \frac{1}{Nh_N}\int_{-\infty}^{\infty}K^2(u)du + o\left(\frac{1}{Nh_N}\right). \spadesuit$$

siCqarebis mixedviT optimaluri skg-is krebadobis $f_N(x)$ Sefasebebis asagebad, aucilebelia ganisazRvros $b(f_N(x))$ wanacvlebis nulisaken krebadobis xarisxi.

SemoviRoT aRniSvna $w_\nu(x) = \frac{f^{(\nu)}}{\nu!}\mathcal{T}_\nu$, sadac

$f^\nu(x) = \frac{\partial^\nu f(x)}{\partial x^\nu}$, $f^{(0)}(x) = f(x)$.

lema 76.4 ($b(f_N)$ wanacvlebis krebadobis siCqare).

vTqvaT

1) *sikvdilebis mrudi* $f(z) \in \mathcal{N}_{\nu,1}(R)$;

2) $\sup_{x\in R^1}\left|f^{(m)}(x)\right| < \infty$, $m = 0, \nu$;

3) *birTvi* $K(u) \in \mathcal{A}_\nu$ $\sup_{u\in R^1}|K(u)| < \infty$ *pirobis gareSe;*

4) $1 - \mathcal{K}(x) = o(x^{-\nu})$ $x \to \infty$*-sTvis, sadac* $\mathcal{K}(x) = \int_{-\infty}^{\infty}K(u)du$;

5) $h_N \downarrow 0$.

maSin $N \to \infty$*-sTvis* ($b(f_N)$ *wanacvleba akmayofilebs Tanafardobas*

$$\left|b\left(f_N(x)\right) - w_\nu(x)h_N^\nu\right| = o(h_N^\nu). \qquad (7.6.10)$$

damtkiceba. adre, (6.6.5)-Si, naCvenebi iyo, rom

$$Ef_N(x) = \int_{-\infty}^{\infty}f(x-uh_N)K(u)du - \int_{x/h_N}^{\infty}f(x-uh_N)K(u)du. \qquad (7.6.11)$$

$f(x)$-is SezRudulobis da dasamtkicebeli lemis 4) pirobis gaTvaliswinebiT, (7.6.11) warmodgenaSi meore integralisaTvis N → ∞-sTvis gvaqvs

$$\int_{x/h_N}^{\infty} f(x-uh_N)K(u)du \le C\int_{x/h_N}^{\infty} K(u)du = C\left[1-\mathcal{K}\left(\frac{x}{h_N}\right)\right] = o(h_N^{\nu}).$$

(7.6.12)

(7.6.11)-is pirvel integralSi $f(x-uh_N$ funqciis lagranJis formaSi naSTiTi wevris teiloris formulis mixedviT daSliT, miviRebT

$$Ef_N(x) = f(x) + \sum_{i=1}^{\nu-1}(-1)^i f^{(i)}(x)\frac{h_N^i}{i!}\mathcal{T}_i +$$

$$+(-1)^{\nu} f^{(\nu)}(x)\frac{h_N^{\nu}}{\nu!}\mathcal{T}_{\nu} + \gamma_N - \int_{x/h_N}^{\infty} f(x-uh_N)K(u)du, \qquad (7.6.13)$$

sadac

$$\gamma_N = \frac{(-1)^{\nu}h_N^{\nu}}{\nu!}\int_{-}[f^{(\nu)}(x+(-1)^{\nu}uh_N\theta) - f^{(\nu)}(x)]u^{\nu}K(u)du,$$

$0 < \theta < 1$.

ramdenadac $\int_{-\infty}^{\infty}|u^{\nu}K(u)|du < \infty$, xolo N → ∞-sTvis TiToeuli $x \in R^1$-sTvis

$f^{(\nu)}(x+(-1)^{\nu}uh_N\theta) \to f^{(\nu)}(x)$ amdenad lebegis Teoremis maJorirebadi krebadobis piroebebi Sesrulebulia, da, Sesabamisad $|\gamma_N| = o(h_N^{\nu})$. imis gaTvaliswinebiT, rom (7.6.13) jami nulis tolia (piroba 3)-dan: $\mathcal{T}_i = 0, i = 1, \dots, \nu-1$), axla gamomdinareobs (7.6.10) mtkicebulebis samarTlianoba.♠

lema 7.6.4 migviTiTebs ganawilebis simkvrivis araparametruli Sefasebis povnis gzas maTi $b(f_N(x))$ wanacvlebis nebismieri didi krebadobis siCqariT. amisaTvis aucilebelia iseTi $K(u)$ birTvebis gamoyeneba, rom $K(u) \in \mathcal{A}_\nu$, $\nu \geq 4$.

magaliTi 7.6.1. birTv-simkvrive $K(u) \in \mathcal{A}_2$, Tu $\sup_u K(u) < \infty$, $K(u) = K(-u)$ da $\int u^2 K(u)du < \infty$.

magaliTi 7.6.2. birTvi $K(u) \in \mathcal{A}_4$, Tu

$$K(u) = \begin{cases} \frac{15(3-10u^2+7u^4)/2^5}{2^5}, & |u| \leq 1-\text{სთვის} \\ 0, |u| > 1-\text{სთვის} \end{cases}.$$

birTvi $K(u) \in \mathcal{A}_6$, Tu

$$K(u) = \begin{cases} \frac{105(5-35u^2+63u^4-33u^6)/2^8}{2^5}, & |u| \leq 1-\text{სთვის} \\ 0, |u| > 1-\text{სთვის} \end{cases}.$$

miTiTebuli birTvebi miiReba rekurentuli procedurebiT, $\rho(u) = \{1-u^2, |u| \leq 1; 0, |u| > 1\}$: $K(u) \in \mathcal{A}_\nu$ funqciis wonasTanA orTonormirebuli iakobis polinomis gamoyenebiT, Tu

$$K(u) = \rho(u)\sum_{j=0}^{\nu-2} p_j(0)p_j(u)(2j+3)(j+2)/8(j+1),$$

$$p_{j+2}(u) = \frac{j+3}{j+4}\left[\frac{2j+5}{j+2}up_{j+1}(u) - p_j(u)\right],$$

$$p_0(u) = p_0(0) = 1,\ p_1(u) = 2u,\quad p_1(0) = 0.$$

SeniSvna 7.6.1. birTvebs $K(u) \in \mathcal{A}_\nu$, $\nu \geq 4$ ar gaaCniaT $K(u) \geq 0$ simkvrivis Tviseba da SeuZliaT miiRon uaryofiTi mniSvnelobebi.

SeniSvna 7.6.2. magaliTi 2-is birTvebi $K(u) \in \mathcal{A}_\nu$, $\nu \geq 4$ $K(u)$-s zogierTi SezRudvebis dros warmoadgenen polinomis klasSi optimalurebs.

gansazRvreba 7.6.3. *vityviT, rom t-sTvis t_N Sefasebas gaaCnia $O(N^{-\alpha}), \alpha > 0$ krebadobis siCqare (aRniSvnebSi $t_N \in \mathcal{V}(N^{-\alpha})$), TuU $u^2(t_N)$ skg-sTvis samarTliania Semdegi zRvruli damokidebuleba*

$\lim_{N\to\infty} N^{\alpha} u^2(t_N) = C$, $0 < C < \infty$.

gansazRvreba 7.6.4. mimdevrobebs α_N da β_N-s uwodeben eqvivalenturs ($\alpha_N \sim \beta_N$ aRniSvnebSi), Tu $\lim_{N\to\infty} |\alpha_N / \beta_N| = 1$.

axla vaCvenoT $b(f_N(x))$ wanacvlebis krebadobis siCqaris gaumjobesebis SesaZlebloba rogor iZleva sikvdilebis mrudis Sefasebis krebadobis siCqaris gazrdas gansazRvrebis 7.6.3 Sesabamisad.

Teorema 7.6.1 (optimaluri skg $u^2(f_N)$). *vTqvaT sruldeba 7.6.2 lemis pirobebi da* $h_N \in \mathcal{H}_2$. *maSin* $N \to \infty$*-sTvis moiZebneba iseTi optimaluri mimdevroba*

$$h_{N,0} = argmin_{h_N} > u^2(F_N(x)) \sim \left(\frac{A}{2\nu B^2 N}\right)^{\frac{1}{2\nu+1}} , \qquad (7.6.14)$$

sadac

$$A = f(x) \int_{-\infty}^{\infty} K^2(u) du, \quad B = \frac{f^{(\nu)} \mathcal{T}^{\nu}}{\nu!},$$

romlisTvisac optimaluri skg

$$u^2\left(f_N(x)|_{h_N = h_{N,0}}\right) \sim O\left(N^{-\frac{2\nu}{2\nu+1}}\right). \qquad (7.6.15)$$

damtkiceba. 6.6.4 da 6.6.3 lemebis ZaliT

$$u^2(f_N(x)) = \frac{A}{N h_N} + B^2 h_N^{2\nu} + o\left(\frac{1}{N h_N} + h_N^{2\nu}\right). \qquad (7.6.16)$$

Tu (7.6.16) skg-is mTavar nawils gavadiferencialebT h_N–iT da gavutolebT nuls, miviRebT

$$h_{N,0} \sim \left(\frac{A}{2\nu B^2 N}\right)^{\frac{1}{2\nu+1}} = O\left(N^{-\frac{1}{2\nu+1}}\right). \qquad (7.6.17)$$

(7.6.17)-is (7.6.16)-Si CasmiT gvaqvs

$$u^2\left(f_{N,0}(x)\right) \sim \frac{A}{N}\left(\frac{2\nu B^2 N}{A}\right)^{\frac{1}{2\nu+1}} + B^2\left(\frac{A}{2\nu B^2 N}\right)^{\frac{2\nu}{2\nu+1}} =$$

$$= \left(\frac{A}{N}\right)^{\frac{2\nu}{2\nu+1}} B^{\frac{2}{2\nu+1}} \left[(2\nu)^{\frac{1}{2\nu+1}} + \left(\frac{1}{2\nu}\right)^{\frac{2\nu}{2\nu+1}}\right] = O\left(N^{-\frac{2\nu}{2\nu+1}}\right). \spadesuit$$

amgvarad, 7.6.1. Teoremidan gamomdinareobs, rom $\nu \to \infty$-sTvis

$$f_{N,0}(x) \in \mathcal{V}\left(N^{-\frac{2\nu}{2\nu+1}}\right) \to \mathcal{V}(N^{-1}). \qquad (7.6.18)$$

(7.6.18)-dan Cans, rom 7.6.3 ganmartebis mixedviT sikvdilebis mrudis optimaluri Sefasebis krebadobis siCqarem $f_{N,0}(x)$ SeiZleba miaRwios paramertuli Sefasebebis, aseve ganawilebis funqciis Sefasebebis $F_N(x)$, $\widetilde{F_N}$ da saimedoobis funqciebis s_N, $\widetilde{s_N}$ krebadobis siCqareebs.

7.7 sikvdilebis mrudis birTvul SefasebebSi bundovnobis optimaluri parametrebis moZebna

simkvrivis araparametruli Sefasebisas erT-erTi ZiriTadi problema _ es aris gansazRvruli $X_1, \ldots, X_N$ nakrebisaTvis optimaluri $h_{N,0}(X_1, \ldots, X_N)$ mniSvnelobebis povna. 7.6.1 Teorema gviCvenebs im klasis simkvriveebis adaptirebuli birTvuli Sefasebis $f_{N,o(x)} = f_N(x)|_{h_N = h_{N,o}}$ $h_{N,o}$ parametris optimaluri mniSvnelobis moZebnis gzas, romlebic gansazRvrulia lema 7.6.4-is 1), 2) pirobebiT. aseve SevniSnoT, rom birTvebis klasidan lema 7.6.4-is 3) pirobis damakmayofilebeli $K(u)$ birTvis SerCeviT, SesaZlebelia $f_{N,o(x)}$ Sefasebis skg-is krebadobis siCqaris gaumjobeseba (ix. 7.6.1). (7.6.4)-dan Cans, rom im $h_{N,o}$ mimdevrobis amowera, romelic axdens asimptoturi skg-is mTavari nawilis minimizirebas, cxadi saxiT Znelia, ramdenadac is gamoisaxeba ucnobi $f(x)$ simkvrivis da misi ν–uri $f^{(\nu)}(x)$ warmoebulis meSveobiT.

am ganyofilebaSi ganvixilavT simkvriveebis adaptirebuli birTvuli Sefasebebis meTodebs, romlebic or ZiriTad tipad iyofa.

pirvel tips miekuTvnebian meTodebi, romlebic dakavSirebulni arian xarisxis romeliRac kriteriumis asimptoturi gaSlis mTavari nawilis ucnobi parametrebis arCevis mixedviT SefasebasTan (magaliTad, $h_{N,o}$-is moZebna (7.6.14)-Si A da B ucnobi mudmivebis Sefasebis gziT).

bundovnobis optimaluri parametris moZebnis meTodebi zogierTi kriteriumis (magaliTad, dasajerobis maqsimumis saxeSecvlili kriteriumis, romelsac qvemoT ganvixilavT) uSualo minimizaciis gziT, miekuTvneba meore tips.

pirvel tips miekuTvneba parametruli meTodi erTganzomilebiani sibrtyeebisaTvis da kriteriumebisaTvis $J_N = E\left(\int_{R^1} (f_N(x) - f(x))^2 dt\right)$. zogierTi naSromis mixedviT, Tu $K(u)$-SezRuduli simetriuli birTv-simkvrivea, $K(u) \in \mathcal{A}_2$, $f(x)$-SezRuduli da orjer uwyvetad diferencirebadi simkvrivea da $\int_{R^1} (f''(x))^2 dx < \infty$, maSin $h_N \in \mathcal{H}_2$-sTvis

$$J_N \sim (Nh_N)^{-1} \int_{R^1} K^2(u)du + \frac{1}{4} h_N^4 \left(\int_{R^1} u^2 K(u)du\right)^2 \int_{R^1} (f''(x))^2 dt. \qquad (7.7.1)$$

(6.7.1)-dan gamomdinareobs, rom

$$h_{N,0} = \left[C/N \int_{R^1} (f''(x))^2 dx \right]^{1/5}, \qquad (7.7.2)$$

sadac koeficienti $C=\int_{R^1}K^2(u)du\Big/\left(\int_{R^1}u^2K(u)du\right)^2$ damokidebulia mxolod $K(u)$ birTvis SerCevaze. amgvarad, (7.7.2)-Si ucnob sidides warmoadgens $\int_{R^1}(f''(x))^2dx$ integrali, romelic magaliTad, normaluri simkvrivisaTvis tolia $3/(8\sqrt{\pi}\sigma^5)$-is.

zogierT naSromSi $\int_{R^1}(f''(x))^2dx$ integrali fasdeba araparametruli meTodebis gamoyenebiT mocemul SemTxvevaSi adaptiuri birTvuli Sefasebis agebis procedura ori etapisagan Sedgeba: Tavdapirvelad igeba $\int_{R^1}(f''(x))^2dx$ integralis Sefasebebi, Semdeg xdeba am Sefasebebis gamoyeneba (7.7.2) formulaSi da fasdeba $f(x)$ ucnobi.

adaptiuri birTvuli Sefasebis araparametruli meTodis dros aucileblad warmoiSoba $f(x)$-is romelime parametrul ojaxTan mikuTvnebis daSvebis Semotanis aucilebloba. am meTodis nakls warmoadgens is, rom Tu daSveba araswori a, maSin Cven viRebT krebadobis siCqaris araoptimalur Sefasebas (SerCeuli kriteriumis mixedviT).

araparametruli meTodis gamoyenebiT adaptiuri birTvuli Sefasebebis agebisas kvlav warmoiSoba araparametruli Sefasebebis parametrebis moZebnis amocana, romelic sirTulis mixedviT imis analogiuria rasac vwyvetT.

zemoT aRniSnuli naklis Tavidan acileba SesaZlebelia, Tu h_N parametrs ise SevarCevT, rom movaxdinoT raRac empiriuli kriteriumis maqsimizireba (minimizireba). mocemuli procesura miekuTvneba adaptiuri birTvuli Sefasebis meore tips. avRniSnoT kriteriumi, romelic efuZneba dasajerobis maqsimumis principს. $h_{N,o}$ parametri airCeva dasajerobis empiriuli funqciis maqsimumis pirobidan:

$$h_{N,o} = h_{N,o}(X_1, \dots, X_N) = argmax_{h>0}[\prod_{i=1}^{N} f_{N-1}(X_i)], \qquad (7.7.3)$$

$$f_{N-1}(X_i) = \frac{1}{(N-1)h} \sum_{j=1, j\neq 1}^{N-1} K\left(\frac{X_i - X_j}{h}\right), \qquad (7.7.4)$$

da miRebuli mniSvneloba $h_{N,o}$ gamoiyeneba adaptur birTvuli Sefasebis agebisas $f_{N,o}(x) = f_N(x)|_{h_N = h_{N,o}}$. amgvarad miRebul Sefasebebs uwodeben simkvrivis kros-ganmeorebad birTvul Sefasebas.

7.8. saSualoebis $\overset{0}{e}_o, \overset{o}{e}_x, \overset{o}{e}_{x:n]}, \overset{0}{e}_{x_1:x_2}$ da dispersebis $DX, DT(x)$ Sefaseba

Tavdapirvelad umartivesi aqtuaruli albaTuri maxasiaTeblis – sicocxlis drois saSualos $\overset{0}{e}_0$ magaliTze ganvixiloT statistikuri Sefasebis ideologia.

ramdenadac $\overset{0}{e}_0 = EX = \int_0^\infty x df(x) = \int_0^\infty s(x)dx$, amdenad Casmebis Sefasebad bunebrivia aviRoT

$$\hat{\overset{o}{e}}_o = \int_0^\infty x dF_N(x) \quad \text{an} \quad \hat{\hat{\overset{o}{e}}}_o = \int_0^\infty s_N(x)dx.$$

iolad davrwmundebiT, rom es Sefasebebi emrTmaneTs emTxveva. namdvilad,

$$\hat{\overset{o}{e}} = \int_0^\infty x \frac{1}{N}\sum_{i=1}^{N} \delta(x - X_i)dx = \frac{1}{N}\sum_{i=1}^{N} X_i = \bar{x},$$

$$\hat{\hat{\overset{o}{e}}} = \int_0^\infty \frac{1}{N}\sum_{i=1}^{N} I(X_i > x) = \frac{1}{N}\sum \int^{X_i} dx = \bar{x}.$$

axla SeiZleba Sefasebebis sinqronizeba ufro rTuli aqtuaruli maxasiaTeblebisaTvis. magaliTad, mocemuli dispersiidan

$$DX = E(X - \overset{o}{e}_x)^2 = \int_0^{\infty} (x - \overset{o}{e}_0)^2 dF(x),$$

$$Dx = 2\int_0^{\infty} xs(x)dx - (\overset{o}{e}_0)^2,$$

gamomdinareobs, rom Sesabamisad

$$\widehat{DX} = \frac{1}{N}\sum_{i=1}^{\infty}(X_i - \bar{x})^2$$

$$\widehat{\widehat{D}X} = 2\int_0^{\infty} x\frac{1}{N}\sum_{i=1}^{N} I(X_i > x)dx - (\bar{x})^2 =$$

$$= \frac{1}{N}\sum_{i=1}^{N} X_i^2 - (\bar{x})^2 = \frac{1}{N}\sum_{i=1}^{N} (X_i - \bar{x})(X_i - \bar{x})^{\ 2}$$

formula $\overset{o}{e}_x = \frac{1}{s(x)}\int_0^{\infty} s(x+t)dt$-dan miviRebT, rom

$$\hat{\overset{o}{e}}_x = \frac{1}{s_N(x)}\int_0^{\infty}\frac{1}{N}\sum_{i=1}^{N} I(X_i > x+t)dt =$$

$$= \frac{1}{s_N(x)N}\sum_{i=1}^{N}\int_0^{(X_i - x)I(X_i - x > o)} dt = \qquad (7.8.1)$$

$$= \sum_{i=1}^{N}(X_i - x)I(X_i - x > 0)\Big/\sum_{i=1}^{N} I(X_i - x > 0)$$

$[X_{(N)}, \infty)$ areSi $\widehat{\overset{0}{e}_x}$ Sefaseba gansazRvruli araa (misi mniSvneli iRebs nulis tol mniSvnelobas), amitom zog SemTxvevaSi azri aqvs (7.8.1)-is nacvlad Semdegi garTulebuli modifikaciis gamoyenebas:

$$\widehat{\overset{0}{e}_x} = \sum_{i=1}^{N}(X_i - x)I(X_i - x) > 0 / \sum_{i-1}^{N} S\left(\frac{x - X_i}{a_N}\right).$$

analogiuri msjelobiT $\overset{0}{e}_{x:n]} = \frac{1}{s(x)}\int_0^n s(x+t)dt$, $DT(x)$ da $D\{\min(T(x),n)\}$-sTvis gvaqvs:

$$\widehat{\overset{0}{e}_{x:n]}} = \frac{1}{s_N(x)}\int_0^n \frac{1}{N}\sum_{i=1}^{N} I(X_i > x+t)\,dt =$$

$$= \frac{1}{s_N(x)N}\sum_{i=1}^{N}\int_0^{\min(n,X_i-x)I(X_i-x>0)} dt =$$

$$= \sum_{i=1}^{N}\min(n,X_i-x)I(X_i-x>0)\Big/\sum_{i=1}^{N} I(X_i-x>0) \ ,$$

$$D\,\widehat{\widehat{T}}(x) = \frac{\sum_{i=1}^{N}(X_i-x)^2 I(X_i-x>0)}{\sum_{i=1}^{N} S\left(\frac{x-X_i}{a_N}\right)} - (\hat{\overset{0}{e}})^2,$$

$$Dmin(\widehat{\widehat{T}}(x),n) = \frac{\sum_{i=1}^{N}(min(n,X_i-x))^2 I(X_i-x>0)}{\sum_{i=1}^{N} S\left(\frac{x-X_i}{a_N}\right)} - (\hat{\overset{0}{e}})^2,$$

dasasruls ganvixiloT zemoT ganxiluli Sefasebebis magaliTebis gamoyenebis Taviseburebani koleqtiur dazRvevaSi.

magaliTad, $\overset{0}{e}_{x:x_2}$ Sefasebis saxiT SesaZlebelia aRebuli iyos

$$\widehat{\overset{0}{e}_{x:x_2}} = \frac{1}{s_N(x_1,x_2)}\int_0^{\infty} \hat{P}\{\min(X-x_1,Y-x_2) > t\}dt =$$

$$= \frac{1}{s_N(x_1,x_2)}\int_0^{\infty}\frac{1}{N}\sum_{i=1}^{N} I\{\min(X_i-x_1,Y_i-x_2) > t\}dt =$$

$$= \frac{1}{s_N(x_1,x_2)N}\sum_{i=1}^{N}\int_0^{\min(X_i-x_1,Y_i-x_2)} dt =$$

$$= \frac{1}{s_N(x_1,x_2)N}\sum_{i=1}^{N}\min(X_i-x_1,Y_i-x_2)I\{\min(X_i-x_1,Y_i-x_2) > 0\},$$

sadac $s_N(x_1,x_2) = \frac{1}{N}\sum_{i=1}^{N} I\{X_i - x_1 > 0\}I\{Y_i - x_2 > 0\}$. SevniSnoT, rom moyvanili formulas azri aqvs im SemTxvevaSi, roca X da Y SemTxveviTi sidideebi statistikurad EerTmaneTTan dakavSirebulni arian. cxadia, rom aseTi damokidebuleba aucileblad gasaTvaliswinebelia, magaliTad, naTesavebis, erTi sawarmos TanamSromlebis da a.S. dazRvevisas.

7.9 intensivobis funqciis Sefasebis asimptoturi normaluroba

ramdenadac intensivobis funqciis Sefasebis zRvruli ganawilebis parametrebi gamoisaxeba saimedoobis funqciis da ganawilebis simkvrivis Sefasebebis kovariaciis meSveobiT, amdenad am punqtSi vipoviT am Sefasebebis zRvrul kovariaciul matricebs.

avRniSnoT: $b_N = (b_{1N},\ldots,b_{sN}$ - Sefasebis veqtori,

$$C(b_n) = \begin{bmatrix} cov(b_{1N},b_{1N}) & \cdots & cov(b_{1N},b_{sN}) \\ \ldots & \ldots & \ldots \\ cov(b_{sN},b_{1N}) & \cdots & cov(b_{sN},b_{sN}) \end{bmatrix}.$$

vipoviT simkvrivis da saimedoobis funqciis Sefasebebis zRvruli kovariaciuli matrica.

lema 7.9.1 (f_N, $\widehat{s_N}$ Sefasebebis kovariaciuli matrica). *Tu $f(x) \in \mathcal{N}_{0,1}(R)$, $\sup_{x\in R^{1+}} f(x) < \infty$, birTvi $K(u) \in \mathcal{A}$, Sesrulebulia 7.3.1 lemis pirobebi da $h_N \in \mathcal{H}_2$, maSin*

$$\lim_{N\to\infty} N h_N \mathcal{C}(f_{N(x)}, \widehat{s_N}(x)) = f(x)\begin{bmatrix} \int_{R^1} K^2(u)du & 0 \\ 0 & 0 \end{bmatrix} = \sigma_\lambda. \qquad (7.9.1)$$

damtkiceba. (7.4.1) da (7.6.8)-is Tanaxmad

$cov\left(\widetilde{s_N}(x), \widetilde{s_N}(x)\right) = D\widetilde{s_N}(x) = \frac{1}{N}s(x)\big(1-s(x)\big) + o\left(\frac{1}{N}\right),$

$cov\big(f_N(x), f_N(x)\big) = Df_N(x) = \frac{f(x)}{Nh_N}\int_{-\infty}^{\infty}K^2(u)du + o\left(\frac{1}{Nh_N}\right),$

aqedan gamomdinare

$\lim_{N\to\infty} nh_N cov\left(f_N(x), f_N(x)\right) = f(x)\int_{-\infty}^{\infty}K^2(u)du,$

$\lim_{N\to\infty} Nh_N cov\left(\widetilde{s_N}(x), \widetilde{s_N}(x)\right) = \lim_{N\to\infty} h_N s(x)\big(1-s(x)\big) = 0.$

Semdgom

$$cov\big(f_N(x), \widetilde{s_N}(x)\big) =$$

$$= \frac{1}{Nh_N}\left[\int_0^{\infty} K\left(\frac{x-y}{h_N}\right)S\left(\frac{x-y}{a_N}\right)f(y)dy - \int_0^{\infty} K\left(\frac{x-y}{h_N}\right)f(y)dy\int_0^{\infty} S\left(\frac{x-y}{a_N}\right)f(y)dy\right]$$

ramdenadac $S(.)$ -SezRuduli funqciaa, amdenad $\sup_{x\in R^{1+}} f(x) \le C < \infty$ da amgvarad,

$$I_1 = \frac{1}{Nh_N}\int_0^{\infty} |K\left(\frac{x-y}{h_N}\right)S\left(\frac{x-y}{a_N}\right)f(y)dy \le C\frac{1}{Nh_N}\int_0^{\infty} K\left(\frac{x-y}{h_N}\right)f(y)dy$$

(7.6.3) Tanaxmad gvaqvs $N\to\infty$: $I_1 = O(N^{-1})$ pirobebSi. Semdeg, (7.6.3) da (7.3.2)-is gaTvaliswinebiT miviRebT $N\to\infty$ dros:

$$\frac{1}{Nh_N}\int_0^{\infty} K\left(\frac{x-y}{h_N}\right)f(y)dy\int_0^{\infty} S\left(\frac{x-y}{a_N}\right)f(y)dy = O(N^{-1}),$$

$$cov\big(f_N(x), \widetilde{s_N}(x)(x)\big) = cov\big(\widetilde{s_N}(x)(x), f_N(x)\big) = O(N^{-1}). \qquad (7.9.2)$$

lema 7.9.1. damtkicebulia.

avirCioT saimedoobis funqciis Sefasebis saxiT $\widetilde{s_N}(x)$ (7.3.1) Sefaseba, xolo simkvrivis Sefasebis saxiT - $f_N(x)$ (7.6.2) birTvul Sefaseba da avagoT intensivobis funqciis Canacvlebis Sefaseba

$$\lambda_N(x) = \frac{f_N(x)}{\widetilde{s_N}(x)}. \qquad (7.9.3)$$

(7.9.3) Sefaseba warmoadgens SemTxveviT wilads. cxadia, rom $\lambda_N(x)$ Sefasebis Tvisebebis gamokvleva ufro rTulia, vidre mricxvelis da mniSvnelis Sefaseba. amis gamo $\lambda_N(x)$ Sefasebis zRvruli ganawilebis mosaZebnad gamoviyenebT Teorema 4.1.1-s, romelic moyvenilia me-4 danarTSi rogor Teorema 3p. imisaTvis rom gamoviyenoT Teorema 3p, vipovoT Semdegi organzomilebiani SemTxveviTi veqtoris zRvruli ganawileba

$$\sqrt{Nh_N}\left(f_N(x) - f(x), \widetilde{s_N}(x) - s(x)\right).$$

organzomilebiani SemTxveviTi veqtoris zRvruli ganawilebis mosaZebnad dagvWirdeba centraluri zRvruli Teorema seriebis sqemaSi, romelic Camoyalibebulia organzomilebiani SemTxvevisaTvis da moyvanilia me-4 danarTSi.

SemovitanoT aRniSvnebi: $\{\xi_{j,N}, \eta_{j,N}\}_{j=1,}^{N}$ $N = 1,2,\ldots$ - seriebis sqemaSi erTnairad ganawilebuli damoukidebeli organzomilebiani veqtorebis mimdevroba (ganawileba $(\xi_{j,N}, \eta_{j,N})$ damokidebulia mxolod N-ze); $\sigma_N = NE\left(\xi_{1,N}, \eta_{1,N}\right)^T (\xi_{1,N}, \eta_{1,N})$ aq ^{T}- transponirebis niSania; $\|(\xi,\eta)\| = \sqrt{\xi^2 + \eta^2}$. $m = \overline{0,\nu}$;

Teorema 7.9.1 *vTqvaT 1) simkvrive* $f(z) \in \mathcal{N}_{\nu,1}(R)$; *2)* $\sup_{x \in R^{1+}} |f(x)| < \infty$, *3) birTvi* $K(u) \in \mathcal{A}_\nu$; *4)* $x \to \infty$*-sTvis* $1 - \mathcal{K}(x) = o(x^{-\nu})$; *5) Sesrulebulia lema 7.3.3-is pirobebi; 6)* $h_N \in \mathcal{H}_2$, $\lim_{N\infty} \sqrt{Nh_N}(a_N + h_N^\nu) = 0$. *maSin* $\sqrt{Nh_N}\left(f_N(x) - f(x), \widetilde{s_N}(x) - s(x)\right)$ *veqtors gaaCnia*

$\mathcal{N}_2\{(0,0),\sigma_\lambda\}$ *organzomilebiani zRvruli normaluri ganawileba, sadac* σ_λ *– (6.9.1)-Si gansazRvruli zRvruli kovariaciuli matricaa.*

damtkiceba. warmovadginoT

$$\sqrt{Nh_N}\left(f_N(x)-f(x),\widetilde{s_N}(x)-s(x)\right) =$$

$$= \sqrt{Nh_N}\left(f_N(x)-\mathbf{E}f_{N,}(x),\widetilde{s_N}(x)-\mathbf{E}\widetilde{s_N}(x)\right) + \sqrt{Nh_N}\left(b(f_N(x)),b(\widetilde{s_N}(x))\right).$$

(7.9.4)

(7.9.4)-is marjvena nawilSi meore Sesakrebi $N\to\infty$ sTvis miiswrafis nulovani veqtorisaken:

$$\sqrt{Nh_N}\left(b\big(f_N(x)\big),b\big(\widetilde{s_N}(x)\big)\right) = \sqrt{Nh_N}\left(Oh_N^{\nu}),O(a_N)\right)$$

$$= \left(O\left(\sqrt{Nh_N^{2\nu+1}}\right),\left(\sqrt{Nh_Na_N}\right)\right)\longrightarrow (0,0)$$

vaCvenoT, rom (7.9.4)-is marjvena nawilSi pirveli SesakrebisaTvis sruldeba seriis sqemaSi organzomilebiani centraluri zRvruli Teoremis yvela piroba (ix. danarTi 4, Teorema 6p). vTqvaT,

$$\xi_{j,N} = \frac{1}{\sqrt{Nh_N}}\left[K\left(\frac{x-X_j}{h_N}\right)-h_N\mathbf{E}f_N(x)\right],$$

$$\eta_{j,N} = \frac{\sqrt{h_N}}{\sqrt{N}}\left[\left(\frac{x-X_j}{a_N}\right)-\mathbf{E}S\left(\frac{x-X_j}{a_N}\right)\right].$$

amgvarad,

$$f_N(x)-\mathbf{E}f_N(x) = \frac{1}{\sqrt{Nh_N}}\sum_{j=1}^{N}\xi_{j,N},\quad \widetilde{s_N}(x)-\mathbf{E}\widetilde{s_N}(x) = \frac{1}{\sqrt{Nh_N}}\sum_{j=1}^{N}\eta_{j,N}.$$

aSkaraa, rom $\mathbf{E}\left(\xi_{j,N},\eta_{j,N}\right) = (0,0)$. SevamowmoT Teorema 6p-s 2) pirobis Sesruleba. amisaTvis saWiroa davamtkicoT, rom

$\boldsymbol{E}\left\|\xi_{j,N},\eta_{j,N}\right\|^{2} = \frac{1}{\sqrt{N}h_N}DK\left(\frac{x-X_j}{h_N}\right) + \frac{h_N}{N}DS\left(\frac{x-X_j}{a_N}\right) < \infty,$

$\widetilde{s_N}(x)$ da $f_N(x)$ Sefasebebis dispersiebis Sesaxeb lemebis Sedegebis – (7.4.21), (7.6.8) gaTvaliswinebiT, miviRebT:

$$\mathbf{E}\left\|\xi_{j,N},\eta_{j,N}\right\|^{2} \le \frac{1}{N}C_1\int_{-\infty}^{\infty}K^2(u)du + \frac{h_N}{N}C_2 < \infty$$

aq $0 \le C_1,\ C_2 < \infty$ _ raRac mudmivebia.

7.9.1 lemis mixedviT $\lim_{N\to\infty} NE\left(\xi_{1,N},\eta_{1,N}\right)^T\left(\xi_{1,N},\eta_{1,N}\right) = \overline{\sigma_\lambda}$.

SevamowmoT lindebergis pirobis Sesruleba, romelic warmoadgens seriebis sqemaSi centraluri zRvruli Teoremis gamoyenebadobis sakmaris pirobas. nebismieri $\tau > 0$–sTvis gvaqvs

$$\beta_N = NE\left(\left\|\xi_{1,N},\eta_{1,N}\right\|^2;\left\|\xi_{1,N},\eta_{1,N}\right\| > \tau\right) < \frac{N}{\tau}E\left\|\xi_{1,N},\eta_{1,N}\right\|^3 =$$

$$= \frac{N}{\tau}E\left(\xi_{1,N}^2 + \eta_{1,N}^2\right)^{3/2} \le \frac{CN}{\tau}\left[E\left|\xi_{1,N}\right|^3 + E\left|\eta_{1,N}\right|^3\right] =$$

$$= \frac{CN}{(Nh_N)^{3/2}}\left[E\left|K\left(\frac{x-X_1}{h_N}\right) - EK\left(\frac{x-X_1}{h_N}\right)\right|^3 + h_N^3E\left|S\left(\frac{x-X_1}{h_N}\right) - ES\left(\frac{x-X_1}{h_N}\right)\right|^3\right] \le$$

$$\le \frac{C}{(Nh_N)^{1/2}}\left[\frac{1}{h_N}\left\{E\left|K\left(\frac{x-X_1}{h_N}\right)\right|^3 + h_N^3E^3f_N(x)\right\} + h_N^2\left\{E\left|S\left(\frac{x-X_1}{h_N}\right)\right|^3 + E^3S\left(\frac{x-X_1}{h_N}\right)\right\}\right].$$

Semdgom, ramdenadac $K(.),S(.)$ birTvebi SezRudulebi arian R^1–ze, amdenad $N\to\infty$ sTvis

$\beta_N = \mathrm{O}\left(\frac{1}{\sqrt{\mathrm{Nh_N}}}\right) \to 0,$

e.i. lindebergis piroba sruleba, aqedan gamomdinare Teorema 6p-is Tanaxmad viRebT sasurvel Sedegs.

Teorema 7.9.1 damtkicebulia.

SemoviRoT aRniSvnebi:

$x_N = (x_{N_1}, \dots, x_{N_s})$ – veqtoruli statistika $x_{N_j} = x_{N_j}(x) = x_{N_j}(t; X_1, \dots, X_N)$, $j = \overline{1, s}$ komponentebiT;

funqcia $H(z): R^s \to R^1$;

$H^{(1)}(z) = \nabla H(z) = (H_1(z), \dots, H_s(z))$, $H_j(z) = \partial H(z)/\partial_{z_j}$;

d_N _ dadebiT ricxvTa mimdevroba SeuzRudavad mzardi N zrdiT;

$\phi = \phi(x) = (\phi_1(x), \dots, \phi_s(x))$ – SezRuduli veqtor-funqcia, romelic SesaZlebelia iyos, magaliTad, x_N statistikis maTematikuri lodini.

axla 7.9.1 da 3p Teoremebis Tanaxmad SeiZleba moiZebnos intensivobis funqciis Sefasebis zRvruli ganawileba.

Toerema 7.9.2 (asimptoturi normaluroba λ_N). *Tu sruldeba 7.9.1 Teoremis piroba da* $\lambda(x), s(x) \neq 0$, *maSin*

$$\sqrt{Nh_N}(\lambda_N(x) - \lambda(x)) \Rightarrow \mathcal{N}_1\left\{0, \frac{\lambda(x)\int_{R^1} K^2(u)du}{s(x)}\right\}. \qquad (7.9.5)$$

damtkiceba. Torema 3p-s aRniSvnebSi (ix. danarTi 4) gvaqvs:

$d_N = \sqrt{Nh_N}$, $\lambda_N = H(x_N) = H(x_{N_1}, x_{N_2}) = \frac{x_{N_1}}{x_{N_2}} = \frac{f_N}{\widetilde{s_N}}$,

$\lambda = H(\phi) = H(\phi_1, \phi_2) = \frac{\phi_1}{\phi_2} = \frac{f}{s}$,

$H^{(1)}(\phi) = \left(\frac{1}{\phi_2}, -\frac{\phi_1}{\phi_2^2}\right) = \left(\frac{1}{s}, -\frac{f}{s^2}\right)$, $\mu_1 = \mu_2 = 0$, $\sigma = \overline{\sigma_\lambda}$

($\overline{\sigma_\lambda}$ matrica gansazRvrulia (7.9.1)-Si).

napovni sidideebis $H_j^{(1)}$, μ_j da σ_{jp}(p1)-Si (ix. danarTi 4) Casmis Semdeg mivdivarT Teoremis damtkicebamde.

Toerema 7.9.2 damtkicebulia.

SeniSvna 7.9.1 *7.9.2 Teoremis analogiurad SeiZleba naCvenebi iyos Semdegi saxis intensivobis funqciis araparametruli Sefasebebis asimetriuli normaluroba:* $\lambda_{N,2}(x) = f_n(x)/(1-F_{N,2}(x))$, *sadac* $f_N(x) = f_N(x, h_N)$ *(*$f_N(x)$ *simkvrivis Sefaseba gansazRvrulia (7.6.2)-is Tanaxmad),* $F_{n,2}(x) = \int_0^x f_N(t, a_N)dt$.

7.10 intensivobis funqciis intervaluri Sefasebebi

7.9.2 Teoremis Sedegi saSualebas iZleva napovni iqnas $\lambda_N(x)$ intensivobis funqciis gardaqmnebi, romlebsac gaaCniaT zRvruli standartuli normaluri ganawilebebi.

Teorema p4-Si (ix danarTi 4) naCvenebia, rom Tu $\{\mathcal{T}_N, N = 1,2,\dots\}$ - statistikis mimdevroba iseTia, rom $\sqrt{q_N}[\mathcal{T}_N - \theta] \Longrightarrow \mathcal{N}_1\{0, \sigma^2(\theta)\}$, ricxviTi mimdevroba $q_N \uparrow \infty$, maSin

$$\sqrt{q_N}[g(\mathcal{T}_N) - g(\theta)] \Longrightarrow \mathcal{N}_1\{0, [g'(\theta)\sigma(\theta)]^2\}, \qquad (7.10.1)$$

sadac g aris funqcia, romelsac gaaCnia pirveli warmoebuli $g'(\theta) \neq 0$.

g funqcia ise unda SevarCioT, rom

$$g'(\theta)\sigma(\theta) = c, \qquad (7.10.2)$$

sadac c araa damokidebuli θ-ze. am SemTxvevaSi $g(\mathcal{T}_N)$ statistikis asimptoturi dispersia ar iqneba damokidebuli θ-ze.

g funqcia vipovoT Semdegi gantolebidan $g = \int \frac{cd\theta}{\sigma(\theta)}$. Cven SemTxvevaSi, (7.9.5)-is gaTvaliswinebiT, gvaqvs

$$g = \int \frac{cd\theta}{\sigma(\theta)} = \int \frac{\sqrt{s(x)}d\lambda(x)}{\sqrt{\lambda(x)}\sqrt{\int_{R^1} K^2(u)du}} = \int \frac{cd\lambda(x)}{\sqrt{\lambda(x)}} \quad . \qquad (7.10.3)$$

(7.10.3)-dan gamomdinareobs, rom $g(x) = \sqrt{x}$.

vipovoT intensivobis funqciis Sefasebis gardaqmna, romelsac zRvruli standartuli normaluri ganawileba gaaCnia.

Teorema 7.10.1. 7.9.2 Teoremis pirobebis dros

$$\frac{2\sqrt{Nh_N}\sqrt{s(x)}}{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}\left[\sqrt{\lambda_N(x)}-\sqrt{\lambda(x)}\right] \Rightarrow \mathcal{N}_1\{0,1\}. \qquad (7.10.4)$$

damtkiceba. Teorema p4-is aRniSvnebiT, Tu vivaraudebT, rom

$q_N = Nh_N$, $\mathcal{T}_N = \lambda_N$, $\theta = \lambda g(\lambda) = \sqrt{\lambda}$

$$g'(\lambda_N)\sigma(\lambda_N) = \frac{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}{2\sqrt{s(x)}},$$

miviRebT 7.10.4 mtkicebulebas.

Teorema 7.10.1 damtkicebulia.

imis gaTvaliswinebiT, rom$|b(\widehat{s_N}(x)| = O(a_N)$ (ix lema 7.3.3) da $\lim_{N\to\infty} a_N\sqrt{Nh_N} = 0$ (7.9.1 Teoremis 6) piroba), aviRebT ra $a_N = \left(n^{-1/2}\right)$ (7.10.4) formulaSi marTlzomieria $\sqrt{s(x)}$ SevcvaloT $\sqrt{\widehat{s_N}(x)}$-iT. amgvarad 7.9.2 Teoremis pirobebis Sesrulebis dros

$$\frac{2\sqrt{Nh_N}\sqrt{\widehat{s_N}(x)}}{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}\left[\sqrt{\lambda_N(x)}-\sqrt{\lambda(x)}\right] \Rightarrow \mathcal{N}_1\{0,1\}. \qquad (7.10.5)$$

(7.10.5) mtkicebulebis safuZvelze $\lambda(x)$ intensivobis funqciisaTvis SesaZlebelia aigos mocemuli ndobis $(1-\alpha)$ intervalSi Sefaseba:

$$\frac{2\sqrt{Nh_N}\sqrt{\widehat{s_N}(x)}}{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}\left|\sqrt{\lambda_N(x)}-\sqrt{\lambda(x)}\right| < U_{1-\alpha/2}, \qquad (7.10.6)$$

sadac $U_{1-\alpha/2}$ - standartuli normaluri ganawilebis $1-\frac{\alpha}{2}$ donis kvantilia. amgvarad

$$\left(\sqrt{\lambda_N(x)} - \frac{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}{2\sqrt{Nh_N}\sqrt{\widehat{s_N(x)}}} U_{1-\alpha/2}\right)^2 < \lambda(x) < \left(\sqrt{\lambda_N(x)} + \frac{\sqrt{\int_{-\infty}^{\infty} K^2(u)du}}{2\sqrt{Nh_N}\sqrt{\widehat{s_N(x)}}} U_{1-\alpha/2}\right)^2.$$

(7.10.7)

(6.10.7) intervaluri Sefasebis Rirsebas warmoadgens is, rom is ganisazRvreba cnobili funqciebis saSualebiT.

7.11 intensivobis funqciis saSualokvadtratul SefasebebeSi krebadoba

Sefasebebis zusti maxasiaTeblebidan erT-erT ZiriTads warmoadgens misi saSualokvadratuli gadaxra (skg) WeSmariti mniSvnelobidan. $\lambda_N(x)$ Casmis Sefasebis sgk-s povnisas Cndeba sirTuleebi, romlebic ganpirobebulia maTi SesaZlo SeuzRudavobiT, magaliTad, rodesac Sefasebebis mniSvnelebi nulis tol mniSvnelobebs iReben. am problemis gadawyveta mdgomareobs an $\lambda_N(x)$-s Sefasebis Sekvecili modifikaciis gamoyenebaSi, an maT calobiT-gluv aproqsimaciebSi:

$$\psi(\lambda_N, \delta) = \hat{\psi}(x, \delta) = \frac{\lambda_N(x)}{[1+\delta|\lambda_N(x)|^{\tau}]^{\rho}}, \qquad (7.11.1)$$

sadac $\tau > 0$, $\rho > 0$ $\rho\tau \geq 1$, $\delta > 0$.

aq ganvixilavT (7.11.1) calobiT-gluv aproqsimacias.

SemoviRoT aRniSvna: $M_\nu \| y_N \| = E \| y_N - \phi \|^\nu$ - x wertilSi $\phi(x)$ funqciidan y_N Sefasebis ν rigis normis gadaxris momenti. (τ, k, m) sameulisaTvis , sadac k, m–naturaluri ricxvebia, SemovitanoT simravle $T(m) = \{(\tau, k): \tau \geq \tau(m) = \frac{2K}{m-k-1} > 0, m \geq m_0 = [3, k = 1; 2k, k \geq 2]\}$.

$\psi(\lambda_N^{(r)},\delta_r)$ Sefasebebis skg-s mosaZebnad dagvWirdeba danarTis Teorema 5p-s me-4 Sedegi.

imisaTvis rom visargebloT danarTis (p.2) formuliT $\hat{\psi}(x,\delta)$ calobiT-gluv aproqsimaciis skg-s mTavari nawilis gansasazRvravad, 5p Teoremis 2) pirobis Tanaxmad, unda vicodeT $\widetilde{s_N}(x)$da $f_N(x)$ Sefasebebis gadaxrebis meoTxe momentis nulisaken krebadobis xarisxi. es Sedegebi moyvanilia qvemoT lemebis 7.11.1 da 7.11.2 saxiT.

lema 7.11.1. Tu sruldeba lema 7.3.3 pirobebi da

$a_N = o\left(N^{-1/2}\right)$,

maSin $N \to \infty$-sTvis

$$M_4\|\widetilde{s_N}(x)\| = \mathbf{E}(\widetilde{s_N}(x) - s(x))^4 = O(N^{-2}) \qquad (7.11.2)$$

damtkiceba. warmovadginoT

$$\mathbf{E}(\widetilde{s_N}(x) - s(x))^4 = \mathbf{E}\left|\left(\widetilde{s_N}(x) - \mathbf{E}\widetilde{s_N}(x)\right) + b(\widetilde{s_N(x)})\right|^{\mathbf{4}}$$

$p = 4$ da $m = 2$ sTvis utolobis gamoyenebiT

$$\left(\sum_{i=1}^{m}|a_i|\right)^p \leq m^{p-1}\sum_{i=1}^{m}|a_i|^p, \qquad P > 1,$$

mivRebT,

$$\mathbf{E}(\widetilde{s_N}(x) - s(x))^4 \leq 2^3[\mathbf{E}(\widetilde{s_N}(x) - \mathbf{E}\widetilde{s_N}(x))^4 + b^4(\widetilde{s_N}(x))]^4$$

7.3.3 lemis mixedviT $b\left(\widetilde{s_N}(x)\right) = O(a_N)$, anu $b^4(\widetilde{s_N}(x) = o(N^{-2})$.

iqidan, rom $\mathbf{E}(\widetilde{F_N}(x) - \mathbf{E}\widetilde{F_N}(x))^4 = O(N^{-2})$ gamomdinareobs $\mathbf{E}(\widetilde{s_N}(x) -$ **E***sN x)4=O(N–2),* aqedan dgindeba (7.11.2) Tanafardobis WeSmariteba. lema 7.11.1 damtkicebulia.

vipovoT $f(x)$ sikvdilebis WeSmariti mrudidan $f_N(x)$ Sefasebis gadaxris meoTxe momentis krebadobis xarisxi.

lema 7.11.2. vTqvaT sruldeba Semdegi pirobebi:

1) $f(x) \in \mathcal{N}_{2,1}(R)$; 2) $\sup_{x\in R^1}\left|f^{(m)}(x)\right| < \infty, m = \overline{0,2}$;

3) birTvi $K(u) \in \mathcal{A}_2$; 4) $x \longrightarrow \infty: 1 - \mathcal{K}(x) = o(x^{-2})$; 5) $\lim_{N\to\infty}\Big(h_N +$ $1NhN=0$;

maSin $N \to \infty$-sTvis

$$M_4\|f_N(x)\| = E(f_N(x) - f(x))^4 = O\left(\left[\frac{1}{Nh_N} + h_N^4\right]^2\right). \qquad (7.11.3)$$

damtkiceba. imave msjelobiT, rogorc es lema 7.11.1-is damtkicebisas iyo da imis gaTvaliswinebiT, rom 4.3.3 lemis ZaliT $E(f_N(x) - f(x))^4 = O((Nh_N)^{-2})$, 7.6.4 lemis Tanaxmad roca $\nu = 2$: $b(f_N(x)) = O(h_N^2)$, viRebT (7.11.3)-m gamosaxulebas.

lema 7.11.2 damtkicebulia.

axla gamoviyenoT Teorema 5p da vipovoT $\psi(\lambda_N, \delta)$ calobiT-gluv aproqsimaciis skg

Toerema 7.11.1. *vTqvaT sruldeba Semdegi pirobebi: 1)* $\lambda(x) \neq 0, s(x) \neq 0$*; 2) srulbeba lemebis 7.11.1. da 7.11.2-is pirobebi; 3)* $\delta = \left(\frac{1}{Nh_N} + h_N^4\right)$.

maSin nebismieri $(\tau, 2) \in T(m)$ *-sTvis roca* $N \to \infty$:

$$\left|E|\psi(\lambda_N, \delta) - \lambda(x)|^2 - \frac{\lambda(x)}{s(x)Nh_N}\int_{-\infty}^{\infty}(K(u)^2 du - \frac{\mathcal{T}_2^2 h_N^4}{4}\left(\frac{(f^{(2)}(x))^2}{s^2(x)}\right)\right| = O\left(\left[\frac{1}{Nh_N} +\right.\right.$$

hN432. (7.11.4)

damtkiceba. vaCevenoT (7.11.4) damokidebulebis samarTlianoba. 5p Teoremis aRniSvnebSi (ix. danarTi 4) gvaqvs: $s = 2$, $z = (z_1, z_2)$, $H(z) = {}^{z_1}/_{z_2}$, $y_N = (y_{1N}, y_{2N}) == (f_N, \widehat{s_N})$, $\phi = (\phi_1, \phi_2) = (f(x), s(x))$,

$d_N = O(Nh_N + h_N^{-4})$, $k = 2$. aviRoT $m = m_0 = 4$ da vaCvenoT, rom $M_4\|f_N, \widehat{s_N}\| = O(d_N^{-2})$. es gamomdinareobs (7.11.2), (7.11.3) damokidebu l ebebidan da u t o l obidan $M_4\|f_N,\widehat{s_N}\| \leq 2|M_4\|f_N\| + M_4\|\widehat{s_N}\||$.

Semdgom, ramdenadac $s(x) \neq 0$, xo l o $z_2 \neq 0$-sTvis funqcia

$z_1/z_2 \in \mathcal{N}_{2,2}(f(x), s(x))$,

amdenad $\tau \geq \tau_0 = 4$-is dros $\psi(\lambda_N, \delta) = \psi(\lambda_N, \delta)$-sTvis 5p Teoremis yve l a piroba Sesru l ebu l ia. Toerema 7.11.1 damtkicebu l ia.

7.12 sakont r o l o dava l ebebi

dava l eba 7.11.1

$P_N(A)$ a l baTobis SefasebisaTvis daamtkiceT p. 7.1-dan 1)-6) Tvisebebi.

dava l eba 7.11.2

Camoaya l ibeT hipoteza danarTi 1-dan sikvdi l ebis cxri l iSi q_x a l baTobebis aproqsimaciis miRebis meTodis Sesaxeb.

dava l eba 7.11.3

aRwereT erTgvarovani cdebis seriiebis organizacia Semdegi a l baTobebis Sefasebis misaRebad ${}_tq_x$, ${}_tp_x$, q_x, p_x, ${}_{t|u}q_x$, ${}_{t|}q_x$.

dava l eba 7.11.4

SemTxveviTi sidideebis Tanabari ganawi l ebis cxri l idan aRebu l i 5 ricxvis mixedviT aageT ganawi l ebis empiriu l i funqcia da gadarCenis funqcia. SeadareT Teoriu l mrudebs. am xuTi ricxvis saxiT SegiZ l iaT aiRoT 10, 9, 73, 25, 33.

davaleba 7.11.5

7.11.4 davalebis pirobebiT aageT ganawilebis gluvi empiriuli funqciebi da gadarCenis funqcia. ganawilebis sabaziso funqciad $\mathcal{F}(u)$ SesaZlebelia aRebuli iyos:

ganawilebis Tanabari funqcia

$$\mathcal{F}(u) = \begin{cases} 0, u \le -\frac{1}{2}, \\ \frac{1}{2} + u,\ -\frac{1}{2} < u \le \frac{1}{2}, \\ 1,\quad u > \frac{1}{2}; \end{cases}$$

koSis ganawilebis funqcia

$\mathcal{F}(u) = \frac{1}{2} + \frac{1}{\pi} arctgu,\ -\infty < u < \infty;$

ganawilebis logistikuri funqcia

$\mathcal{F}(u) = \frac{1}{1+e^{-u}},\ -\infty < u < \infty;$

hiperboluri kosinusi

$\mathcal{F}(u) = 1 - \frac{2}{\pi}\ arctge^{-u},\ -\infty < u < \infty;$

ormagi maCvenebliani kanoni

$\mathcal{F}(u) = e^{-e^{-u}},\ -\infty < u < \infty.$

davaleba 7.11.6

moaxdineT ganawilebis da gadarCenis funqciebis yvelaze saukeTeso Canacvlebuli Seafsebebis sinTezireba x wertilSi.

davaleba 7.11.7

aageT ganawilebis simkvrivis birTvuli Sefasebebi gluvi empiriuli ganawilebis funqciis 7.11.5 davalebis birTvebiT 7.11.4 davalebis mixedviT. gamoTvaleT simkvrivis Sefasebis mniSvnelobebi da misi dipersia 0, 50, 100 wertilebSi.

davaleba 7.11.8

7.11.7 davalebis pirobebiT aageT bundoivnobis empiriuli parametrebis moZebnis algoriTmi dasajerobis maqsimumis meTodiT.

Tavi 8. danarTi

სიცოცხლისგაგრძელების ცხრილი (სსრკ 1984-1985წ.)[3,გვ86]

ასაკი	კაცები			ქალები		
x	l_x	q_x	d_x	l_x	q_x	d_x
14	95,438	0,00068	65	96,407	0,00037	36
15	95,373	0,00082	78	96,371	0,00041	40
16	95,295	0,00101	97	96,331	0,00047	45
17	95,198	0,00124	118	96,286	0,00053	51
18	95,080	0,00149	142	96,235	0,00059	57
19	94,938	0,00173	164	96,178	0,00065	62
20	94,774	0,00196	186	96,116	0,00069	66
21	94,588	0,00216	205	96,050	0,00072	69
22	94,383	0,00234	221	95,981	0,00074	71
23	94,162	0,00249	235	95,910	0,00076	73
24	93,927	0,00263	247	95,837	0,00078	75
25	93,680	0,00277	260	95,762	0,00081	77
26	93,420	0,00293	274	95,685	0,00084	80
27	93,146	0,00312	290	95,605	0,00088	84
28	92,856	0,00333	310	95,521	0,00093	89
29	92,546	0,00356	330	95,432	0,00099	95
30	92,216	0,00381	352	95,337	0,00106	101
31	91,864	0,00405	372	95,236	0,00113	108
32	91,492	0,00425	389	95,128	0,00121	116
33	91,103	0,00445	406	95,012	0,00131	125
34	90,697	0,00465	422	94,887	0,00142	135
35	90,275	0,00487	440	94,752	0,00155	147
36	89,835	0,00514	462	94,605	0,00168	159
37	89,373	0,00550	492	94,446	0,00182	172
38	88,881	0,00595	529	94,274	0,00196	185
39	88,352	0,00649	573	94,089	0,00212	199
40	87,779	0,00708	622	93,890	0,00228	214
41	87,157	0,00770	671	93,676	0,00247	231
42	86,486	0,00831	719	93,445	0,00267	249

ასაკი	კაცები			ქალები		
x	l_x	q_x	d_x	l_x	q_x	d_x
43	85,767	0,00888	762	93,196	0,00289	270
44	85,005	0,00943	801	92,926	0,00314	292
45	84,204	0,00997	840	92,634	0,00341	316
46	83,364	0,01057	881	92,318	0,00369	341
47	82,483	0,01126	929	91,977	0,00399	367
48	81,554	0,01208	985	91,610	0,00430	394
49	80,569	0,01303	1,050	91,216	0,00465	424
50	79,519	0,01409	1,121	90,792	0,00506	459
51	78,398	0,01522	1,193	90,333	0,00554	500
52	77,205	0,01637	1,264	89,833	0,00610	548
53	75,941	0,01754	1,332	89,285	0,00673	601
54	74,609	0,01872	1,397	88,684	0,00740	656
55	73,212	0,01997	1,462	88,028	0,00806	709
56	71,750	0,02136	1,532	87,319	0,00866	756
57	70,218	0,02293	1,610	86,563	0,00919	795
58	68,608	0,02470	1,695	85,768	0,00969	831
59	66,913	0,02665	1,783	84,937	0,01023	869
60	65,130	0,02871	1,870	84,068	0,01094	919
61	63,260	0,03080	1,949	83,149	0,01193	992
62	61,311	0,03296	2,021	82,157	0,01318	1,083
63	59,290	0,03523	2,089	81,074	0,01467	1,189
64	57,201	0,03765	2,153	79,885	0,01634	1,305
65	55,048	0,04027	2,217	78,580	0,01819	1,430
66	52,831	0,04310	2,277	77,150	0,02024	1,561
67	50,554	0,04616	2,333	75,589	0,02249	1,700
68	48,221	0,04947	2,385	73,889	0,02497	1,845
69	45,836	0,05304	2,431	72,044	0,02771	1,997
70	43,405	0,05691	2,470	70,043	0,03073	2,153
71	40,935	0,06108	2,500	67,894	0,03406	2,212
72	38,435	0,06558	2,521	65,582	0,03772	2,474
73	35,914	0,07044	2,530	63,108	0,04176	2,635

ასაკი	კაცები			ქალები		
x	l_x	q_x	d_x	l_x	q_x	d_x
74	33,384	0,07568	2,527	60,473	0,04620	2,794
75	30,857	0,08129	2,508	57,679	0,05106	2,945
76	28,349	0,08738	2,477	54,736	0,05642	3,088
77	25,872	0,09393	2,430	51,646	0,06232	3,218
78	23,442	0,10098	2,367	48,428	0,06879	3,331
79	21,075	0,10857	2,288	45,097	0,07589	3,423
80	18,787	0,11672	2,193	41,674	0,08368	3,487
81	16,594	0,12548	2,082	38,187	0,09221	3,521
82	14,512	0,13489	1,957	34,666	0,10155	3,520
83	12,555	0,14497	1,820	31,146	0,11176	3,481
84	10,735	0,15577	1,672	27,665	0,12291	3,400
85	9,063	0,16733	1,517	24,265	0,13507	3,277
86	7,546	0,20000	1,509	20,988	0,20000	4,197
87	6,037	0,40000	2,414	16,791	0,40000	6,716
88	3,623	0,60000	2,174	10,075	0,60000	6,045
89	1,449	0,80000	1,159	4,030	0,80000	3,224
90	290	1,00000	290	806	1,00000	806

x_p დონის p სტანდარტული ნორმალური განაწილების კვანტილები

p	0,90	0,95	0,975	0,99	0,995	0,999	0,9999
x_p	1,282	1,645	1,960	2,326	2,576	3,090	3,291

სიცოცხლის გაგრძელებადობს ცხრილის ფრაგმენტი (აშშ 1979-1981)[1,გვ.55-58]

x	l_x	q_x	d_x	L_x	T_x	$\overset{\circ}{e}_x$
0	100 000	0,01260	1 260	98 973	7 387 758	73,88
1	98 740	0,00093	92	98 694	7288785	73,82
2	98 648	0,00065	64	98 617	7190091	72,89
3	98 584	0,00050	49	98 560	7091474	71,93
⋮	⋮	⋮	⋮	⋮	⋮	⋮
19	97 851	0,00112	110	97796	5518733	56,40
20	97 741	0,00120	118	97682	5420937	55,46
21	97 623	0,00127	124	97561	5323255	54,53
22	97 499	0,00132	129	97435	5225694	53,60
⋮	⋮	⋮	⋮	⋮	⋮	⋮
40	94 926	0,00232	220	94817	3492100	36,79
41	94 706	0,00254	241	94585	3397283	35,87
42	94 465	0,00279	264	94334	3302698	34,96
43	94 201	0,00306	288	94057	3208364	34,06
⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	83 726	0,01368	1145	83153	1676326	20,02
61	82 581	0,01493	1233	81965	1593173	19,29
62	81 348	0,01628	1324	80686	1511208	18,58
63	80 024	0,01767	1415	79316	1430522	17,88
⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	1 150	0,29120	335	983	3137	2,73
101	815	0,30139	245	692	2154	2,64
102	570	0,31089	177	481	1462	2,57
103	393	0,31970	126	330	981	2,50
104	267	0,32786	88	223	651	2,44
105	179	0,33539	60	150	428	2,38
106	119	0,34233	41	99	278	2,33
107	78	0,34870	27	64	179	2,29

mtkicebulebaA 1p. Tu $f(x)$ funqcias $x = x_0 \in X$ wertilSi gaaCnia $f^{(n)}(x_0)$ warmoebuli, maSin is $x = x_0$ wertilis midamoSi uwyvetia da gaCnia $x = x_0$ wertilis midamoSi uwveti warmoebulebi $f'(x)$, $f''(x)$, … , $f^{(n-2)}(x)$ da gaCnia $x = x_0$ wertilis midamoSi uwveti $f^{(n-1)}(x)$ warmoebuli.

mtkicebulebaA 2p. (teiloris formula Ppeanos formaSi naSTiTi wevriT). Tu $f(x)$ funqcia $n-$jer diferencirebadiaa $x = x_0$ wertilis midamoSi, maSin am wertilis romeliRac midamoSi

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + o((x - x_0)^n),$$

$x \to x_0$

mtkicebulebaA 3p. (teiloris formula Plagranjis formaSi naSTiTi wevriT). Tu $f(x)$ funqcias x_0 wertilis romeliRac midamoSi $(n+1)$ rigis warmoebulia, maSin miTiTebuli midamodan nebismieri x wertilisaTvis

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n +$$

$$+ \frac{f^{(n+1)}(x_0+\theta(x-x_0))}{(n+1)!}(x - x_0)^n, \ 0 < \theta < 1.$$

Teorema 1p. (lebegis Teorema integrelis qveS zRvulis gadasvlis anu maJorirebuli krebadobis Sesaxeb). vTqvaT mimdevroba $f_n(x)$ da $\varphi(x)$ funqcia integrebadebia zomad A simravleze, yvela n-sTvis da TiTqmis yvela $x \in A$-sTvis

$|f_n(x)| \leq \varphi(x)$

da $\lim_{n\to\infty} f_n(x) = f(x)$ TiTqmis yvela $x \in A$-sTvis. maSin $\int_A |f(x)|dx < \infty$,

$$\lim_{n\to\infty} \int_A |f(x)|dx = \int_A f(x)dx,$$

$$\lim_{n\to\infty} \int_A |f_n(x) - f(x)|dx = 0.$$

Teorema 2p. (maJorirebuli krebadobis Sesaxeb lebegis Teorema maTematikuri lodinis terminebSi). vTqvaT, $\eta, \xi_1, \xi_2, \ldots$ - iseTi

SemTxveviTi sidideebia, rom $|\xi_n| \le \eta$, $E\eta < \infty$ da $\xi_n \to \xi$ 1-is toli albaTobiT. maSin $E\xi < \infty$,

$E\xi_n \to E\xi$,

$E|\xi_n - \xi| \to 0$, roca $n \to \infty$.

Teorema 3p. (Teorema 2.1.1). Tu $d_n(x_n - \phi) \Rightarrow \mathcal{N}_s\{\mu, \sigma\}$, $H(z) \in \mathcal{N}_{1,s}(\phi) \neq 0$, maSin

$$d_n(H(x_n) - H(\phi)) \Rightarrow \mathcal{N}_1\left\{\sum_{j=1}^{s} H_j(\phi)\mu_j, \sum_{j,p=1}^{s} H_j(\phi)H_p(\phi)\sigma_{jp}\right\} \quad \text{(p.1)}$$

Teorema 4p. vTqvaT $\{\mathcal{T}_n, n = 1,2, \dots\}$ - statistikebis mimdevroba iseTia, rom $\sqrt{d_n}|\mathcal{T}_n - \theta| \Rightarrow \mathcal{N}_1\{0, \sigma^2(\theta)\}$, ricxviTi mimdevroba $d_n \uparrow \infty$. vTqvaT g – aris erTi cvladis funqcia, romelsac gaaCnia pirveli warmoebuli g'. Tu $g'(\theta) \neq 0$, maSin

$$\sqrt{d_n}[g(\mathcal{T}_n) - g(\theta)] \Rightarrow \mathcal{N}_1\{0, |g'(\theta)\sigma(\theta)|^2\}.$$

Tu, garda imisa, g' uwyvetia, maSin

$$\frac{\sqrt{d_n}[g(\mathcal{T}_n) - g(\theta)]}{g'(\mathcal{T}_n)} \Rightarrow \mathcal{N}_1\{0, \sigma^2(\theta)\},$$

xolo TuU $\sigma(\theta)$ uwyvetia, maSin

$$\frac{\sqrt{d_n}[g(\mathcal{T}_n) - g(\theta)]}{g'(\mathcal{T}_n)\sigma(\mathcal{T}_n)} \Rightarrow \mathcal{N}_1\{0,1\}.$$

Teorema 5p. (zemoT moyvanili aRniSvnebis Tanaxmad). vTqvaT sruldeba Semdegi pirobebi:

1) $H(z) \in \mathcal{N}_{2,s}(\phi)$,

2) $M_m\|x_n\| = O(d_n^{-m/2})$ romeliRac $m \geq 3$, $m \in N$,

3) $\delta = \delta_n C d_n^{-1}$,

4) $H(\phi) \neq 0$ an $\tau \in N^+$. maSin nebismieri $(\tau, k) \in T(m)$-sTvis

$$\left|E\left[\widehat{\Phi}(x_n, \delta) - H(\phi)\right]^k - E[\nabla H(\phi)(x_n - \phi)^T]^k\right| = O\left(d_n^{-\frac{(k+1)}{2}}\right), \quad \text{(p.2)}$$

sadac $\widehat{\Phi}(x_n, \delta) = H(x_n)/(1 + \delta|H(x_n|^\tau)^\rho$, $\tau > 0$, $\rho > 0$, $\rho\tau \geq 1$, $\delta > 0$.

Camovaya l iboT ser i is sqemaSi o r ganz omi l ebiani cent r a l u r i zRv r u l i Teo r ema Semdeg aRniSvnebSi: $\{\xi_{j,n}, \eta_{j,n}\}_j^n, n = 1,2,\ldots$ _ ser i is sqemaSi er Tnai r ad ganawi l ebu l i o r ganz omi l ebiani veqt o r ebis mimdev r oba (ganawi l eba $(\xi_{j,n}, \eta_{j,n})$ damokidebu l ia n-ze); $\sigma_n = n\mathbf{E}(\xi_{1,n}, \eta_{1,n})^T(\xi_{1,n}, \eta_{1,n})$, sadac T _ t ransponi rebis niSania; $\|(\xi, \eta)\| = \sqrt{\xi^2 + \eta^2}$.

Teo r ema 6p [9] (o r ganz omi l ebiani cent r a l u r i zRv r u l i Teo r ema ser i is sqemaSi),

xer xi: 1) $\mathbf{E}(\xi_{j,n,}\eta_{j,n}) = (0,0)$, 2) $\mathbf{E}\|\xi_{j,n}, \eta_{j,n}\|^{\mathbf{2}} < \infty$, 3) *Ses r u l ebu l ia l idenbe r gis pi r oba: nebismie r i* τ *-saTvis* $n \to \infty$:

$$n\mathbf{E}\left(\|\xi_{j,n}, \eta_{j,n}\|^2; \|\xi_{j,n}, \eta_{j,n}\| > \tau\right) \longrightarrow 0,$$

4) $\sigma_n \longrightarrow \sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$. maSin, veqt o r i $\sum_{j=1}^{n}(\xi_{j,n}, \eta_{j,n})$-s gaaCnia o r ganz omi l ebiani no r ma l u r i ganawi l eba $\mathcal{N}_2\{(0,0), \sigma\}$.

pirobiTi aRniSvnebi

l_o – axaldabadebulTa jgufi;

l_x - 0 wlis sawyisi sicocxleebis raodenoba, romlebic cocxali iqneba X wlis Semdeg.

d_x - 0 wlis asakis sicocxleebis raodenoba, romlebic daixocebian x da $x+1$ asakis SualedSi.

ω - mokvdavobis (sicocxlis) cxrilis dasrulebis asaki;

e_x - Semoklebuli sicocxlis xangrZlivoba;

$\overset{0}{e}_x$ -sicocxlis sruli xangrZlivoba x asakSi.

p_x niSnavs ${}_1p_x$-s da q_x _ ${}_1q_x$-s.

q_x - x asakSi sikvdilianobis siCqare.

(x) - x asakis pirovneba;

${}_nq_{-n}$ _ sikvdilis albaToba x wlis asakSi.

${}_{n|k}q_x$ _ albaToba imisa, rom (x) mokvdeba $x+n$ da $x+n+k$ asakebs Soris;

| - simbolos aRniSnavs lodinis periods;

$e_{x:\overline{n|}}$ _ Semoklebuli sicocxlis xangrZlivoba;

$\overset{0}{e}_{x:\overline{n|}}$ _ sruli sicocxlis xangrZlivoba.

ν _ sadazRvevo SemTxveva;

$b\nu$ _ sadazRvevo gadaxdebi;

P_n _sadazRvevo premia.

U _kompaniis kapitali an rezervi;

S _ gadaxdebis jami;

$b_i = 1$ _ firmis mimarT wayenebuli sarCeli, Tu wlis ganmavlobaSi i-uri klienti gardaicvleba.

$E\xi$ da $D\xi$ _ ξ SemTxveviTi sididis maTematikuri lodini da dispersia.

X _ sicocxlis xangrZlivoba;

$S(x)$- gadarCenis funqcia;

$\omega -$ zRvruli asaki (demuavris modelSi);

μ_x _mokvdavobis intensivoba;

A - ubeduri SemTxvevebi;

$T(x)$- sicocxlis narCeni dro;

t_x- albaToba imis, romadamiani aRwevs $x+t$ asaks.

P_x albaToba imisa, rom adamiani individi icocxlebs 1 wels mainc;

ET -maTematikuri lodini

DT - dispersia;

$\overset{0}{e}_{x:n]}$ _ nawilobrivi sicocxlis xangrZlivoba;

$K(x)$_ damrgvalebuli sicocxlis narCeni xangrZlivoba;

$e_x = Ek(x)$_ saSualo damrgvalebuli sicocxlis narCeni xangrZlivoba;

$\{X\}$ _ X sididis wiladuri nawili;

τ-sidide, romelic aRwers sikvdilis moments wlis SigniT;

$a(n)$_ τ wiladuri asakis saSualo (sikvdili dgeba n wlis asakSi)

L_x _ wlebis saSualo jamuri ricxvi, romelic ganvles l_0 axalSobilTagan Semdgari jgufidan yvela warmomadgenelma (x,∞) intervalSi.

U: _gaerTianebuli sicocxlis statusi. Ukanasknali cocxlad darCenilis statusi;

$T(U)$_ sakuTari sicocxlis xangrZlivoba;

$F_N(x)$_ ganawilebis empiriuli funqcia;

skg _ standartuli kvadratuli gadaxra;

sxc _ sicocxlis xangrZlivobis cxrili.

♠- Teoremis an lemis damtkicebis dasruleba

gamoyenebuli literatura:

1. Кошкин Г.М., Основы страховой математики, Томск – 2002.
2. Фалин Г.И., Фалин А.II. Введение в актуарную математику. М.: Изд-во МГУ, 1994, gv .86.
3. Фалин Г.11. Математический анализ рисков в страховании. М.:Российский юридический издательский дом, 1994, 130 с.
4. Гербер X. Математика страхования жизни. М., Мир, 1995, gv. 154.
5. Штрауб Э.,Актуарная математика имущественного страхования.М., Мир, 1988, gv. 146 .
6. Колемаев В.А., Калинина В.Н. Теория вероятностей и математическая статистика. М.: Инфра-М, 1997, gv. 302
7. Вааль В.А., Кошкин Г.М. Оценивание функции интенсивности отказов и ее производной в условиях неопределенности/ / Межд. конф. по проблемам управления (29 июня - 2 июля 1999 г.). Тезисыдокл. в трех томах. — М.: Фонд "Проблемы управления". 1999. Т.2. - С.138-140.
8. Konstantopoulos T., Notes on Survival Models, Spring 2006.
9. Rosenblatt M., Remarks on some nonparaletric estimates of a density function// Ann. Math. Statist. 1956. V.27. gv.832-835.
10. Parzen E., On estimation of a probability density and mode// Ann. Math. Statist. 1962. V.33. gv.1065-1076.
11. Gasser T., Muller H.-G. Kernel estimatIon of regression functions//Lect. Notes Math. 1979. V 757. P.23-68
12. Woodrofe M., On choocing adelta-sequence. Ann. Of Math. Stat. 1970. V41.- P.1665-1671.