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“Algorithms of Terminal Control of Spatial Movements of Agricultural Robots”

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General Description of the Work

A rapid development of science and engineering gave the humanity absolutely new means of automation – these are commercial robots used in the most diverse spheres of economy: in industry they are used as flexible systems of complex automation, transporting facilities, technological machines and so on. Among an enormous variety of robotic devices a special place is held by manipulating robots designed to perform all sorts of technological operations such as assembly and erection, painting, welding and many others.

In recent years, manipulating robots have been actively used for agricultural work, in particular, for gathering various fruits and vegetables, and also for various agro-technical operations such as pruning and so on. The use of robots lowers the production cost of final agricultural products, contributes to the improvement of their quality and decreases the share of hard manual labor.

One of the basic scientific and technical tasks connected with the development and implementation of robots designed for fulfilling various technological processes in industry and agriculture is the solution of problems dealing with the control of spatial motions of robots. Among the latter problems, the problem of controlling spatial rotations of multi-joint working components of robots is considered to be the most difficult one. The existing methods of its solution are cumbersome and complicated and hence it becomes necessary to use sophisticated hardware and software, which, in turn, increases the price both of robots themselves and, in the end, of a technological process as a whole.

This dissertation is devoted to:

1. The development of a new method of representation of spatial rotations of mechanical objects;
2. The development of simple adaptive algorithms of control of terminal states of spatial rotations of working organs of agricultural robots.

Topicality of the Theme: Problems of control of spatial motions in general and, in particular, of rotational motions of robots belong to the most topical directions in the complex of high technologies which demand a lot of scientific research. Most of the methods used to solve these problems are optimal methods of programmed control (disconnected methods without feedback). They include the maximum principle, the dynamic programming method, the momentum method and others.

As has been noted, all the listed methods are the programming ones, i.e. demanding the preliminary calculation of the law of control $u(t)$ and not making it possible to correct this law during motion. However practice demands the construction of automatic control systems (ACS) employing the feedback principle, since such systems make it possible to correct the motion trajectory in the course of the process.

Besides, in a majority of cases, for a successful solution of technological problems of robot application it is necessary to provide an exact positioning in the terminal stage of motion and thus, in the case of manipulating robots, the control of their terminal states (terminal control) becomes of special topical interest. An effective solution of such problems will enable us to improve the quality of technological processes, since the quality of these processes depends in many respects on the accuracy of the terminal positioning of the gripping devices of robots.

From the above-said it follows that problems connected with the development of simple adaptive systems of automatic control of terminal states of moving objects are topical and meet the up-to-date requirements of the development of technologies based on scientific research.

The Scientific Novelty consists in the following:

1. Spatial rotations are for the first time described by their spinor representation, which made it possible to obtain simple relations for describing by means of an element of the

controlling orthogonal matrix of the basic representation by the known coordinates of three defining rotation points: central, initial and terminal.

2. Simple formulas are obtained for calculation of controlling Euler angles;
3. The obtained results have enabled us to reduce the actually three-dimensional problem of spatial motion control to the one-dimensional problem;
4. A general variational method is obtained to solve problems of terminal control of spatial rotations;
5. Simple adaptive algorithms are obtained, by means of which various partial problems on the terminal control of acceleration, transfer of the object to a given point, and approach are solved under various terminal conditions.
6. New algorithms of control of spatial rotations of manipulating robots are studied;
7. An optimal control circuit is developed for the work of the electric drive realizing the algorithms of control of spatial rotations of manipulating robots.

Methods of Investigation. The following methods are used in the work: elements of the theory of representation of rotation groups, the spinor theory, variational methods of control of electric drive motion, methods of ordinary differential equations, and methods of programming by Mat-Cad.

The Practical Importance of the work consists in that the developed algorithms can be successfully used in programming robot-manipulators for the solution of practical technological problems, which will lead to the improvement of their terminal positioning and thereby to the perfection of the technological process as a whole. In addition to this, the obtained results can also be used for the solution of the corresponding problems of computer graphics.

Approbation of the Work. The results of the work were announced at an international conference, at the applied mathematics chair of Georgian Technical University (2005) and at the Machine Mechanics Institute of the Georgian Academy of Sciences (2005, 2006).

Published Works. 3 works have been published on the topic of the dissertation.

Structure and Volume. The work includes 122 computer type-set pages and consists of four chapters, a list of references and 37 figures.

Contents of the Thesis. The first Chapter is devoted to the analyses of the state of the problem. The survey of corresponding references was carried out. At the end of the chapter the objectives were formulated.

The second chapter is devoted to the presentation of the spinor method of the solution of the spatial rotations kinematics. The problems related to the control of manipulation robots belong to the most urgent modern scientific and technical problems. The majority of methods used to solve them is represented by program methods (rarely – by adaptive ones) of optimal control based either on the principle of maximum or on boundary problems for ordinary linear differential equations. All of them use to some extent the kinematical relationships of spatial movements of multi-joint mechanisms. The main difficulty here is a problem related to three-dimensional rotations. The given problem for mechanisms with rotatory kinematical pairs in many cases results in essential computational complications, while for spatial mechanisms with spherical kinematics pairs it, apparently, has no solution in general.

Proceeding from the above-said, the purpose of the present work consist of the derivation of new relationships for the description of spatial mechanisms with spherical joints that in future will make a basis for development of a new method of control of their movement.

In works [1,2], on the basis of spinor model of generalized rotations of the three-dimensional Euclid space, were received:

1. Unitary matrix of the second order (spinor matrix) $C = \frac{1}{(|\alpha|^2 + |\beta|^2)}\begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix}$, performing the rotation of three-dimensional space

$$Y = \overline{C}^T XC\,, \tag{1}$$

where $X = \begin{vmatrix} x^3 & x^1 - ix^2 \\ x^1 + ix^2 & -x^3 \end{vmatrix}$ and $Y = \begin{vmatrix} y^3 & y^1 - iy^2 \\ y^1 + iy^2 & -y^3 \end{vmatrix}$ - Hermitian matrices of the spinor representation of the initial $x(x^1, x^2, x^3)$ and final $y(y^1, y^2, y^3)$ rotation points; $\alpha = \alpha_1 + i\alpha_2$ and $\beta = \beta_1 + i\beta_2$ - complex parameters of the spinor matrix, for which the following dependences of the coordinates of initial and final points of rotation were established

$$\operatorname{Re}\beta = \beta_1 = \frac{\alpha_1(x^1 - y^1) + \alpha_2(x^2 + y^2)}{x^3 + y^3}\,; \quad \operatorname{Im}\beta = \beta_2 = \frac{\alpha_2(x^1 + y^1) - \alpha_1(x^2 - y^2)}{x^3 + y^3}\,; \tag{2}$$

2. Simple relationships between elements of a three-dimensional orthogonal matrix A of the basic representation of the three-dimensional rotation group and Euler angles, on the one hand, and coordinates of initial and final rotation points, on the other hand

$$a_1^1 = (\alpha_1^2 - \alpha_2^2) - (\beta_1^2 - \beta_2^2)\,; \quad a_2^1 = 2(\alpha_1\alpha_2 + \beta_1\beta_2)\,; \quad a_3^1 = 2(\alpha_2\beta_2 - \alpha_1\beta_1)\,;$$
$$a_1^2 = 2(\beta_1\beta_2 - \alpha_1\alpha_2)\,; \quad a_2^2 = (\alpha_1^2 - \alpha_2^2) + (\beta_1^2 - \beta_2^2)\,; \quad a_3^2 = 2(\alpha_1\beta_2 + \alpha_2\beta_1)\,;$$
$$a_1^3 = 2(\alpha_1\beta_1 + \alpha_2\beta_2)\,; \quad a_2^3 = 2(\alpha_2\beta_1 - \alpha_1\beta_2)\,; \quad a_3^3 = (\alpha_1^2 + \alpha_2^2) - (\beta_1^2 + \beta_2^2)\,; \tag{3}$$
$$\cos\theta = a_{33}\,; \ \sin\varphi\sin\theta = a_{31} \text{ and } \sin\psi\sin\theta = a_{13}\,, \tag{4}$$

where x^1, x^2, x^3 and y^1, y^2, y^3 - coordinates of initial and final points of rotation, respectively; $\alpha_1; \alpha_2; \beta_1; \beta_2$ parameters of the spinor rotation matrix $\theta; \phi; \psi$. - Euler angles.

Thus, having expressions (2) (3), it is easy to calculate Euler angles ensuring the turn of a point $x(x^1, x^2, x^3)$ into a point $y(y^1, y^2, y^3)$. If we assume that zero Euler angles $x(x^1, x^2, x^3)$ correspond to the initial point, then the control of rotation consists in changes in time of Euler angles $\theta_0 = \phi_0 = \psi_0 = 0$ from initial values $\theta_0; \phi_0; \psi_0$ to final ones $\theta_f; \phi_f; \psi_f$ computed by the formulas (4). In a general form the control process can be presented as functions of change of Euler angles $\theta(t)$; $\phi(t)$; $\psi(t)$ that should satisfy the following conditions:

$$\theta(t_0) = 0\,; \ \phi(t_0) = 0\,; \ \psi(t_0) = 0\,,$$
$$\theta(t_f) = \theta_f\,; \ \phi(t_f) = \phi_f\,; \ \psi(t_f) = \psi_f\,, \tag{5}$$

where t_0 and t_f - initial and final moments of control process.

Based on the above-said the problem of determination of control functions $\theta(t)$; $\phi(t)$; $\psi(t)$ naturally follows, to which the given work is devoted.

It is necessary to point out that the following: dependences $\theta(t)$; $\phi(t)$; $\psi(t)$ have kinematical nature character, as they do not allow for neither moments nor elasticity nor any other dynamic characteristics of process, therefore after their definition the task of synthesis of the dynamic adaptive control on the basis of these functions arises [3]. This problem will be considered in the subsequent works.

In a fig. 1 the fixed vectors $x(x^1, x^2, x^3)$; $y(y^1, y^2, y^3)$ and intermediate rotating vector $\xi(\xi^1; \xi^2; \xi^3)$, which at the initial moment of time $t=t_0$ coincides with an initial vector of rotation $x(x^1, x^2, x^3)$ and at the final one $t=t_f$ - with a final vector $y(y^1, y^2, y^3)$, are represented The current angle γ between vectors $x(x^1, x^2, x^3)$ and $\xi(\xi^1;\xi^2;\xi^3)$ at the moment of time t=t$_0$ is equal to zero, and at the moment t=t$_f$ - $\gamma = \gamma_f$, where $\gamma_f = ar\cos(\frac{(x,y)}{|x|*|y|}) = ar\cos(\frac{(x,y)}{|x|^2})$; $(x,y)-$ scalar product of vectors x and y. It is obvious that the current angle between vectors $y(y^1, y^2, y^3)$ and $\xi(\xi^1;\xi^2;\xi^3)$ is equal to $\gamma_f - \gamma$.

Let us determine the coordinates of a vector $\xi(\xi^1;\xi^2;\xi^3)$ from a condition that it makes angles γ and $\gamma_f - \gamma$ with vectors $x(x^1, x^2, x^3)$ and $y(y^1, y^2, y^3)$ and at the same time is located in their plane. With this purpose we shall enter a vector $r(x^2y^3 - x^3y^2; x^3y^1 - x^1y^3; x^1y^2 - x^2y^1)$, being a vector product of the vectors of vectors x and y.

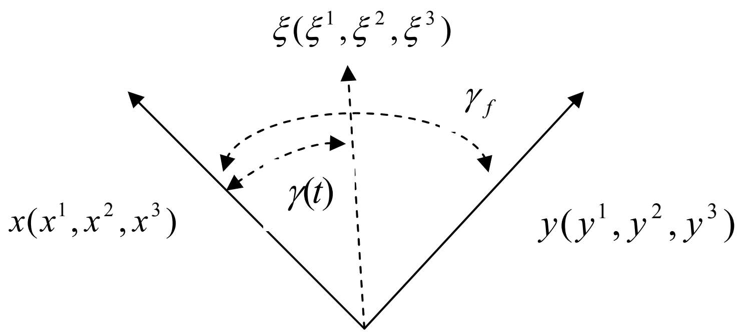


Fig. 1 Initial, final and intermediate vectors of spatial rotation

Then the above conditions can be written down as the following system of linear equations:

$$(\xi, r) = 0;$$
$$(\xi, x) = |x|^2 \cos\gamma;$$
$$(\xi, y) = |x|^2 \cos(\gamma_f - \gamma). \qquad (6)$$

It is easy to see that the vector $\xi(\xi^1;\xi^2;\xi^3)$ determined by the system (6) meets the following conditions:

1. At $\gamma = 0$ $\xi(\xi^1;\xi^2;\xi^3) = x(x^1, x^2, x^3)$, that follows from the second equation of system (4) as in this case $(\xi, x) = |x|^2$ that is possible only under condition of $\xi(\xi^1;\xi^2;\xi^3) = x(x^1, x^2, x^3)$;
2. At $\gamma = \gamma_f$ $\xi(\xi^1;\xi^2;\xi^3) = y(y^1, y^2, y^3)$, that follows from the third equation of system (4) as in this case $(\xi, y) = |x|^2$ that is possible only under condition of $\xi(\xi^1;\xi^2;\xi^3) = y(y^1, y^2, y^3)$;
3. $|\xi| = |x| = |y|$, that follows from the second and third equations of system (6).

Thus, the vector $\xi(\xi^1;\xi^2;\xi^3)$ determined by the system (6) corresponds to a fig. 1, that is, it can be considered as a vector rotating (the condition 3) from a vectors $x(x^1, x^2, x^3)$ (the condition 1) to a vector $y(y^1, y^2, y^3)$ (the condition 2). At that the angle γ changes in limits $0 \le \gamma \le \gamma_f$.

The equations of the system (6) can be presented in the coordinate form

$$\xi^1 r^1 + \xi^2 r^2 + \xi^2 r^2 = 0$$
$$\xi^1 x^1 + \xi^2 x^2 + \xi^2 x^2 = |x|^2 \cos\gamma$$
$$\xi^1 y^1 + \xi^2 y^2 + \xi^2 y^2 = |x|^2 \cos(\gamma_f - \gamma) \qquad (6')$$

It is easy to see that its determinant is equal to

$$\Delta = \begin{vmatrix} r^1 & r^2 & r^3 \\ x^1 & x^2 & x^3 \\ y^1 & y^2 & y^3 \end{vmatrix} = |r|^2 \tag{7}$$

Other determinants of the Cramer's formulas for system (6) will be equal to:

$$\Delta_1 = \begin{vmatrix} 0 & r^2 & r^3 \\ |x|^2 \cos\gamma & x^2 & x^3 \\ |x|^2 \cos(\gamma^f - \gamma) & y^2 & y^3 \end{vmatrix} = |x|^2((\cos(\gamma_f - \gamma)(r^2x^3 - r^3x^2) - \cos\gamma(r^2y^3 - r^3y^2))$$

;

$$\Delta_2 = \begin{vmatrix} r^1 & 0 & r^3 \\ x^1 & |x|^2 \cos\gamma & x^3 \\ y^1 & |x|^2 \cos(\gamma_f - \gamma) & y^3 \end{vmatrix} = |x|^2((\cos(\gamma_f - \gamma)(r^3x^1 - r^1x^3) - \cos\gamma(r^3y^1 - r^1y^3));$$

;

$$\Delta_3 = \begin{vmatrix} r^1 & r^2 & 0 \\ x^1 & x^2 & |x|^2 \cos\gamma \\ y^1 & y^2 & |x|^2 \cos(\gamma_f - \gamma) \end{vmatrix} = |x|^2((\cos(\gamma_f - \gamma)(r^1x^2 - r^2x^1) - \cos\gamma(r^1y^2 - r^2y^1)). \tag{8}$$

(7) and (8) allow coordinates of a required vector $\xi(\xi^1;\xi^2;\xi^3)$ to be obtained in the following form:

$$\xi^1 = \frac{|x|^2}{|r|^2}((\cos(\gamma_f - \gamma)(r^2x^3 - r^3x^2) - \cos\gamma(r^2y^3 - r^3y^2))$$

$$\xi^2 = \frac{|x|^2}{|r|^2}((\cos(\gamma_f - \gamma)(r^3x^1 - r^1x^3) - \cos\gamma(r^3y^1 - r^1y^3))$$

$$\xi^3 = \frac{|x|^2}{|r|^2}((\cos(\gamma_f - \gamma)(r^1x^2 - r^2x^1) - \cos\gamma(r^1y^2 - r^2y^1)). \tag{9}$$

In these expressions an independent variable is the angle γ which can be considered as a function of time, and it means that the coordinates of a vector $\xi(\xi^1;\xi^2;\xi^3)$ also are functions of time. Additionally we assume that $\gamma(t)$ is enough smooth and meets the following conditions: $\gamma(t = t_o) = 0\ u\ \gamma(t = t_f) = \gamma_f$. We emphasize that the problem of synthesis of spatial movement control is reduced in this way to the definition of a concrete kind of function $\gamma(t)$, that is related to the dynamics of rotation process and that will be stated in future works. In the given work we suppose that $\gamma(t)$ is any function satisfying the above conditions. For certainty let us assume

$$\gamma(t) = \omega t, \tag{10}$$

where $\omega = 2\pi f$ - constant angular frequency.

If we introduce new vectors $r_x = (r^2x^3 - r^3x^2; r^3x^1 - r^1x^3; r^1x^2 - r^2x^1)$ and $r_y = (r^2y^3 - r^3y^2; r^3y^1 - r^1y^3; r^1y^2 - r^2y^1)$ equal to the vector products [r×x] and [r×y], respectively, we shall obtain simple expressions for required coordinates as functions of time

$$\xi^1(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t)r_x^1 - \cos\omega t r_y^1);$$

$$\xi^2(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t) r_x^2 - \cos\omega t r_y^2);$$

$$\xi^3(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t) r_x^3 - \cos\omega t r_y^3). \tag{11}$$

As it was already said, the vector $\xi(\xi^1;\xi^2;\xi^3)$ is a rotating vector, therefore at each moment of time it can be considered as a final vector of the current moment of rotation process. Having substituted in the formulas (2) instead of coordinates of the point $y(y^1,y^2,y^3)$ expressions (11), we obtain the representations of parameters of the spinor matrix C, orthogonal matrix A (the formula (2)) and Euler angles (3) as functions of time. Thus, we obtain a time-dependent (kinematical) representation of rotation of a point $x(x^1,x^2,x^3)$ into a point $y(y^1,y^2,y^3)$. Before this , however, we should redefine a matrix C in such a manner that at the initial moment of time the spinor equation of rotation (1) would look like $X = \overline{C}^T XC$, that, obviously, is possible only when C is an unitary matrix. It can be made by the appropriate selection of parameters α_1 and α_2.

Indeed, having put $\alpha_1 = 1;\ \alpha_2 = 0$, we obtain that

$$\operatorname{Re}\beta = \beta_1 = \frac{x^1 - y^1}{x^3 + y^3}; \qquad \operatorname{Im}\beta = \beta_2 = \frac{y^2 - x^2}{x^3 + y^3}. \tag{2 '}$$

We substitute further in (2') instead of coordinates of the vector y the coordinates of a vector ξ from (11), then the matrix C will depend on time and will have the following form:

$$C(t) = \frac{1}{1+|\beta|^2}\begin{vmatrix} 1 & \dfrac{((\xi^1(t) - x^1) + i(\xi^2(t) - x^2))}{x^3 + \xi^3(t)} \\ \dfrac{((x^1 - \xi^1(t)) - i(x^2 - \xi^2(t)))}{x^3 + \xi^3(t)} & 1 \end{vmatrix}, \tag{12}$$

where $|\beta|^2 = \dfrac{(x^1 - \xi^1(t))^2 + (x^2 - \xi^2(t))^2}{(x^3 + \xi^3(t))^2}$; $\xi^1(t),\ \xi^2(t),\ \xi^3(t)$ - the functions of time determined in (11).

It is obvious that at the initial moment of time t_0 the matrix $C(t = t_o) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, as in this case

$\gamma(t_0) = 0$ and $\xi(\xi^1;\xi^2;\xi^3) = x(x^1,x^2,x^3)$.

For $t = t_f$ we have $\gamma(t_f) = \gamma_f$, $\xi(\xi^1;\xi^2;\xi^3) = y(y^1,y^2,y^3)$ and, respectively,

$$C(t = t_f) = \frac{1}{1+|\beta|^2}\begin{vmatrix} 1 & \dfrac{((y^1 - x^1) + i(y^2 - x^2))}{x^3 + y^3} \\ \dfrac{((x^1 - y^1) - i(x^2 - y^2))}{x^3 + y^3} & 1 \end{vmatrix}.$$

From the above-said follows that the spinor matrix of rotation (12) is determined correctly. But in such case the Euler angles (4) are correctly determined also which turn out to be the functions of time that it is easy to see from (3), (4) and (11)

$$\theta(t) = \arccos\left(\frac{(x^3 + \xi^3(t))^2 - (x^1 - \xi^1(t))^2 - (x^2 - \xi^2(t))^2}{(x^3 + \xi^3(t))^2}\right)$$

$$\varphi(t) = \arcsin\left(\frac{2(x^1 - \xi^1(t))}{(x^3 + \xi^3(t))\sin\theta(t)}\right)$$

$$\psi(t) = \arcsin(\frac{2(\xi^1(t) - x^1)}{(x^3 + \xi^3(t))\sin\theta(t)}). \qquad (13)$$

The obtained expressions (13) solve the formulated problem of determination of the control kinematical functions $\theta(t)$; $\phi(t)$; $\psi(t)$. On the other hand, it is necessary to note that the offered theory allows the problem of spatial movement control to be reduced to one-dimensional one. Indeed, it is enough to synthesize in any way a function $\gamma(t)$ meeting the appropriate boundary conditions, then, obviously, the control process will be completely determined by means of the spinor matrix of rotation (12) and functions of Euler angles (13).

The third chapter is devoted to the elaboration of solution and creation of algorithms of terminal control of moving objects. Problems related to the control of moving mechanical objects belong to the class of sufficiently well studied problems, many of which for a long time have been regarded as classical ones [1÷4]. In the first place, they include such methods as the principle of maximum, dynamic programming, the momentum method and others directly connected with the classical methods of variational calculus. These methods are rather difficult for application, since the eventual control algorithms obtained with their aid are actually of programming character, i.e. explicitly depending on time. Therefore it is impossible to carry out the current correction of a phase trajectory, though such a correction is absolutely necessary because a moving objected is influenced by perturbing environmental factors (both systematic and random). A change of controlling forces brings about a change of uncontrolled forces too. All forces (uncontrolled +controlled) acting on the controlled object generate the object motion acceleration $\dot{V}$. It is obvious that $\dot{V}$ can be easily measured directly and therefore we should pose the problem on the synthesis of a controlling function in the form of acceleration $\ddot{\psi}(t)$. Then the control process reduces to the fulfillment of the equality

$$\dot{V} = \ddot{\gamma}(t), \qquad (14)$$

where $\dot{V}$ is the measured acceleration of the object and $\ddot{\gamma}(t)$ is the given (synthesized) acceleration of the object.

The synthesis of a control algorithm can be reduced to some variational problem in a phase space: Given two points $(\gamma_0;\dot{\gamma}_0)$ and $(\gamma_f;\dot{\gamma}_f)$ in a two-dimensional phase space, it is required to derive the equation of a curve of this phase space that connects $(\gamma_0;\dot{\gamma}_0)$ and $(\gamma_f;\dot{\gamma}_f)$ delivers a minimum to the next functional

$$J_F = \frac{1}{T}\int_0^T f^2(t,\gamma,\dot{\gamma},\alpha(t))dt. \qquad (A)$$

Functional (3.8) belongs to the type of functionals containing derivatives of second order and therefore its corresponding Euler equation can be written in the form

$$\frac{d^2\ddot{\gamma}}{dt^2} = 0. \qquad (15)$$

Solution (15) is a third order polynomial

$$\gamma = C_0 + C_1 t + C_2\frac{t^2}{2} + C_3\frac{t^3}{6}. \qquad (16)$$

The boundary conditions are equal:

$$t = 0; \quad \gamma = \gamma_0; \quad \dot{\gamma} = \dot{\gamma}_0, \qquad (17)$$

$$t = T; \quad \gamma = \gamma_f; \quad \dot{\gamma} = \dot{\gamma}_f. \qquad (18)$$

These four conditions are sufficient for defining four constants Ci (i = 0, 1, 2, 3) contained in (18), which completely defines an optimal trajectory.

This approach is quite general, so it gives possibility to solve different terminal control problems: reduction, acceleration and approachment. Having limited text space we present only the results connected with the last one, but we have to mention that all full solution of all the problems are completely presented in thesis.

The approachment problem employs four boundary conditions (17) and (18) which allow us to calculate immediately the coefficients Ci (i = 0, 1, 2, 3) in the controlling function (16):

$$C_0 = \gamma_0;\ C_1 = \dot{\gamma}_0;\ C_2 = \frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(\dot{\gamma}_f + 2\dot{\gamma}_0);\ C_3 = \frac{12}{T^3}(\gamma_0 - \gamma_f) + \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0)\,. \quad (19)$$

The last gives possibility to get the controlling function for the problem of approachment

$$\ddot{\gamma}(t) = \left(\frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0)\right) + \left(\frac{12}{T^3}(\gamma_0 - \gamma_f) - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0)\right)t\,. \quad (20)$$

In order to obtain an adaptive control algorithm we proceed as follows: since now the object is all the time at the initial point of time, it is assumed that t = 0 and the initial velocity and coordinate values are replaced by the respective current values, and the moment of time T is replaced by the difference T – t. In the case the adaptive controlling function are obtained

$$\ddot{\gamma}(t) = \frac{6\gamma_f}{(\Delta T)^2} - \frac{6\gamma}{(\Delta T)^2} - \frac{4\dot{\gamma}}{(\Delta T)} - \frac{2\dot{\gamma}_f}{(\Delta T)}\,. \quad (21)$$

The forced and transitional components of controlling process are as follows:

$$\gamma_{fr} = \frac{\Delta T^2}{6}\left[K_0 - \frac{2}{3}\Delta T K_1 + \frac{5}{9}\Delta T^2 K_2 - \frac{4}{9}\Delta T^3 K_3 + \left(K_1 - \frac{4}{3}\Delta T K_2 + \frac{5}{3}\Delta T^2 K_3\right)t + \left(K_2 - 2\Delta T K_3\right)t^2 + K_3 t^3\right]$$

where

$$A = \gamma_{10} - \frac{1}{6}\Delta T^2 K_0 + \frac{1}{9}\Delta T^3 K_1 - \frac{5}{54}\Delta T^4 K^2 + \frac{4}{54}\Delta T^5 K_3;$$

$$B = \sqrt{2}\,\gamma_{10} + \frac{\sqrt{2}}{2}\dot{\gamma}_{10}\Delta T - \frac{\sqrt{2}}{6}\Delta T^2 K_0 + \frac{\sqrt{2}}{36}\Delta T^3 K_1 + \frac{5\sqrt{2}}{270}\Delta T^4 K_2 - \frac{7\sqrt{2}}{108}\Delta T^5 K_3\,.$$

It should be emphasized that in the above expressions the initial values γ_{10} *and* $\dot{\gamma}_{10}$ are not equal to the initial values given (17) and (18) and thus there arises the transitional process (21) which gets damped with time (in this case the time constant is equal to $\frac{\Delta T}{2}$), i.e. the object moves to the forced trajectory, which leads to a complete solution of the approachment problem.

Frequently, it is not enough to have four boundary conditions (17) and (18) of the approachment problem to solve applied problems of terminal control. For example, in the case deceleration it is not enough to assume that the terminal velocity is equal to zero: for a complete stop it is necessary that the terminal acceleration, too, be equal to zero. Thus there arise an additional boundary condition (the fifth one) related to acceleration:

$$\begin{array}{llll} t = 0; & \gamma = \gamma_0; & \dot{\gamma} = \dot{\gamma}_0, & \\ t = T; & \gamma = \gamma_f; & \dot{\gamma} = \dot{\gamma}_f; & \ddot{\gamma} = \ddot{\gamma}_f\,. \end{array} \quad (22)$$

Omitting complicate and long details we are giving below final solution of the five boundary problems for the approachment problem

$$\ddot{\gamma}(t) = \frac{12}{(T-t)^2}\left(\gamma_f - \gamma\right) - \frac{6}{(T-t)}\left(\dot{\gamma}_f + \dot{\gamma}\right). \quad (23)$$

Here we assume that $\ddot{\gamma}_f = 0$ what is naturally for braking process.

Chapter four: The spinor model of the kinematics of spatial rotations developed on the basis of spinor representation of generalized spatial rotations (Chapter II) and the methods of the control theory of terminal states of motion of mechanical objects (Chapter III) made it possible to create simple methods of controlling terminal states of spatial rotations of robot-manipulators. All the below results were modeled by means of Mat-Cad system.

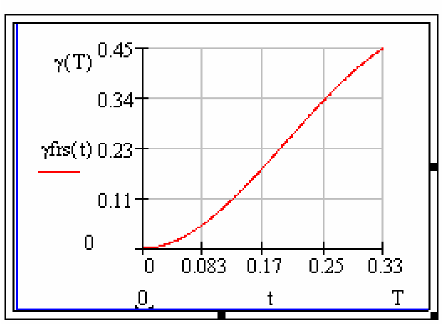
The theory developed in Chapter II has enabled us to reduce the three-dimensional problem of spatial motion control to the one-dimensional problem because we have defined the coordinates of the rotating vector (11) as functions of one rotation angle lying in the rotation plane[1]. It is obvious that the trajectories corresponding to this kind of rotations consist of three natural stages: acceleration, uniform rotation[2] and deceleration for the control of which we will use the results of Chapter III.

From the standpoint of dynamics, the initial process of rotation means that the object of control which is at rest must be accelerated to the desired velocity $\dot{\gamma}_f$. It might seem from this definition that in this case we should use the results of the solution of the acceleration control problem, but the matter is that if we want to finish the initial stage of motion in the right-hand end of the segment $\lfloor 0;\alpha_1\gamma_f \rfloor$, then, certainly, we should use the methods of the approach problem which take into account all boundary conditions.

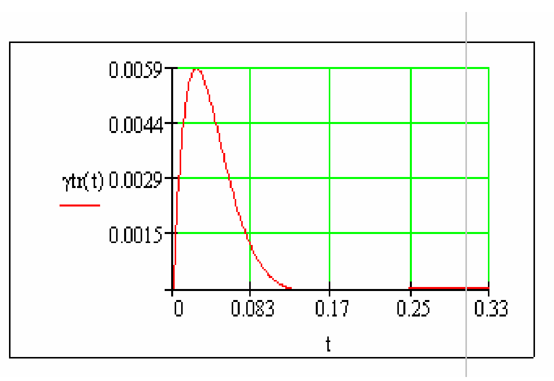
As the initial and terminal vector we took x(10, -45, 30) and y(1, 20, 51.225). It is obvious that the angle between them is equal to $\gamma_f = ar\cos(\frac{(x,y)}{|x|^2}) = 77.65^0$.The angle of rotation $\gamma_f = 77.650$ was divided into three equal angles $\gamma_f/3$ = 25.880; $2\gamma_f/3$= 51.770 and $\gamma_f = 77.650$, i.e. in that case $\alpha_1 = \frac{1}{3}$ *and* $\alpha_2 = \frac{2}{3}$. In what follows we will use the angle values expressed in terms of radians; therefore $\gamma_f/3$ = 0.452; $2\gamma_f/3$ = 0.904 and γ_f = 1.355. Let us assume that the angular velocity is equal to ω = 1 and the rotation time is also T = 1 sec. We also assume that all three rotation stages are of equal duration, i.e. $T_1 = \frac{T}{3} = 0.333$ sec. Initial conditions for the first stage are

t=0; $\gamma_0 = 0$; $\dot{\gamma}_0 = 0$, t=T_1; $\gamma_f = 0.452$; $\dot{\gamma}_f = \omega_f = 1$.

Below the dynamic characteristics of control process are given



a)



b)

[1] The rotation plane is defined by three principal points of each rotation: central, initial and terminal.

[2] The segment corresponding to uniform rotation may be zero.

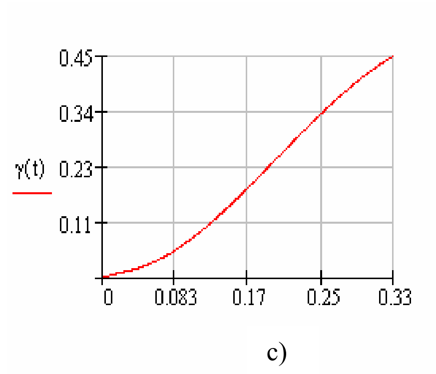


Fig.2. The initial motion segment: rotation angle values as functions of time:
a) the forced component; b) the transient component; c) the phase trajectory

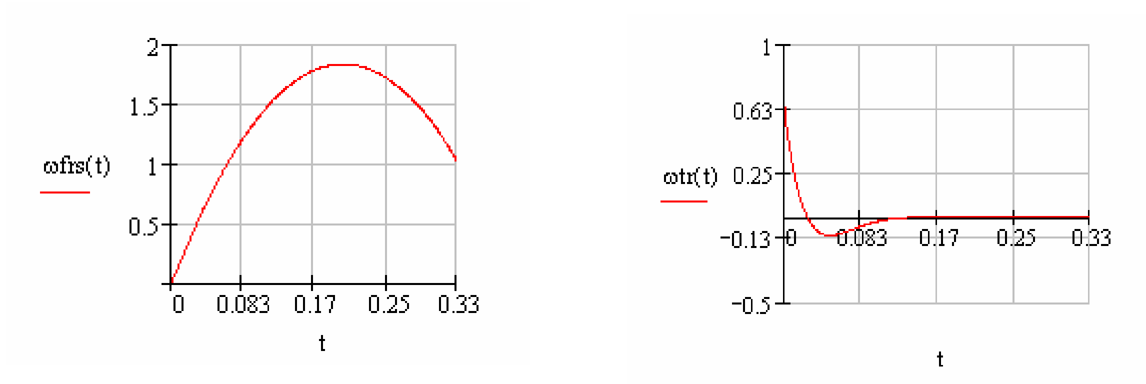


Fig.3. The initial motion segment: the angular velocity value as a function of time:
a) the transient component;
b) the complete solution: the sum of the forced and transient components

Finally, let us comment on the character of the curves. Fig. 2,b shows the presence of a transient process, but it is two orders weaker that the forced component (Fig. 2,a) and soon damps down. The transient component of the angular velocity also damps down soon (Fig. 3,a), but its order is comparable with the order of the forced component. Weak deflections of the phase trajectory (Fig.2,c) and the total velocity (the sum of the transient and forced components), (Fig.3,b)) is a result of the transient process.

For Uniform rotation stage control is the same, only initial conditions are changed

$$t = 0;\ \gamma_0 = 0.452;\ \ \dot{\gamma}_0 = 1, t = T_1;\ \gamma_f = 0.904;\ \ \dot{\gamma}_f = \omega_f = 1.$$

Figs. 4÷5 show the dynamic characteristics of the control process on the uniform rotation segment. Again we clearly see that the control satisfies the boundary conditions: at the end of the control period T = 0.33 sec. the controlled object really has the given angular coordinate $\gamma_f = 0.904$ and the velocity $\dot{\gamma}_f = 1$. Though the transient process takes place, the transient component for the angular coordinate function is insignificant (Fig.5,b), while the velocity function (Fig..6,b) is comparable with the forced (Fig. 6,a) and total components.

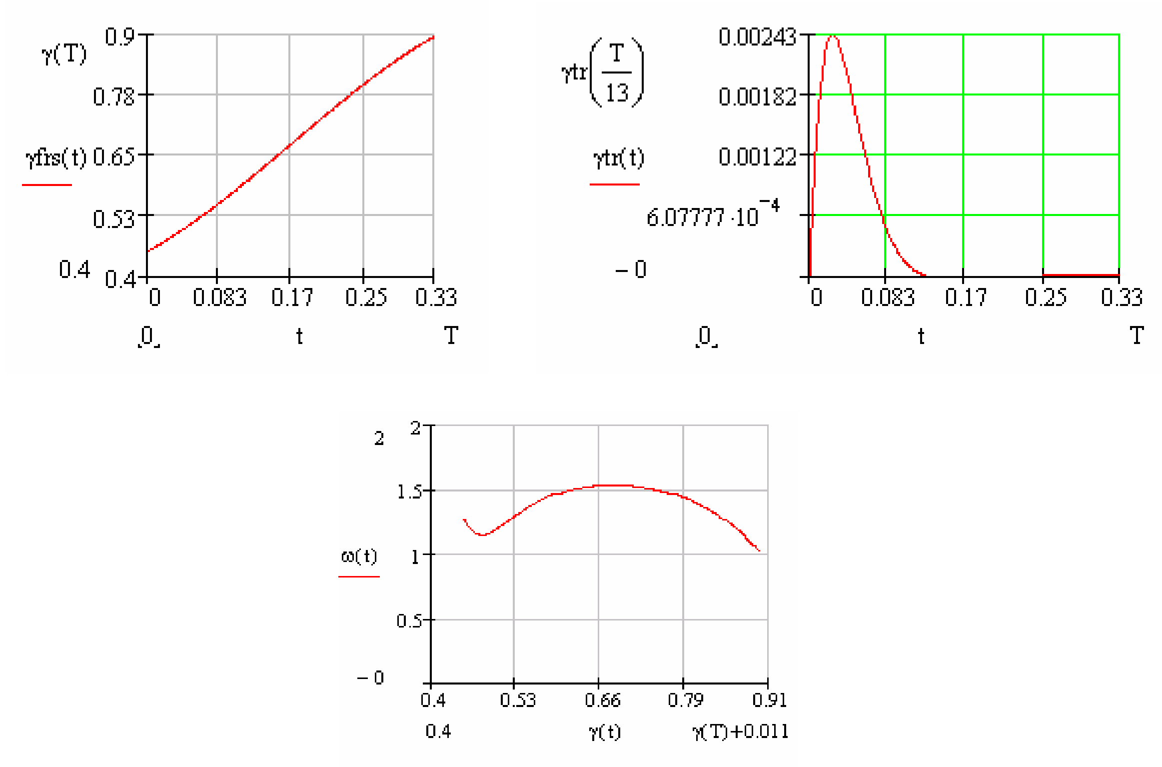


Fig.4. The uniform motion segment: rotation angle values as functions of time:
a) the forced component;
b) the transient component;
c) the phase trajectory

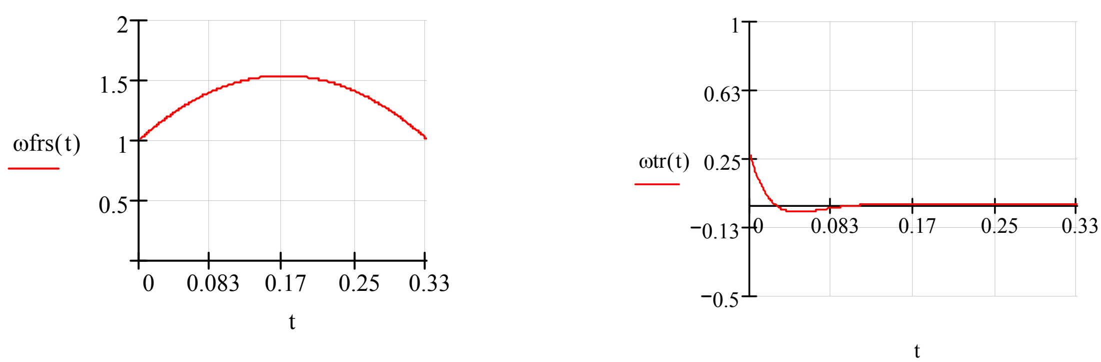


Fig.5. The uniform motion segment: the angular velocity value as a function of time:
a) the transient component;
b) the complete solution: the sum of the forced and transient components

For the deceleration process ending in a complete stop we need to used the problem with five conditions since it is clear that at the end of the rotation process the acceleration must be equal to zero. Therefore the boundary conditions take the following form:

$$t = 0;\ \gamma = 0.904;\ \ \dot{\gamma} = 1,$$
$$t = T;\ \gamma = 1.355;\ \ \dot{\gamma} = 0;\ \ddot{\gamma} = 0.$$

Figs. 6 show the dynamic characteristics when $\gamma_{10} = \gamma_0 = 0$ *and* $\dot{\gamma}_{10} = \dot{\gamma}_0 = 0$. In this case, as seen from Figs. 7 there exists a transient process. As different from the preceding motion stages, in this case the intensity of transient processes is quite comparable with stationary functions though these transient processes damp down soon. Nevertheless the control again satisfies the boundary conditions – this fact also follows from trajectory functions for $t = T_1$, which gives for the deceleration stage the values $\gamma(T_1) = \gamma_f = 1.355$ and $|x| = |\xi(0)| = |\xi(T_1)| = 55$.

After we have obtained the algorithms of an adaptive terminal control of spatial rotations of robot-manipulators, there arises a problem on the development of an optimal control of the electric drive of these systems. In this case, too, we have used the variational methods connected with power losses. It should be said that these methods have found quite a wide application for the solution of problems of this kind.

It is required to find functions v(τ) and i(τ) that reduce the functional $W = \int_0^T i^2 d\tau$ to a minimum.

The boundary conditions are given in the form υ(0) = 0, v(T) = 0. In addition to this, the isoperimetric condition $\alpha = \int_0^T v d\tau$ is given, where α is the rotation angle. Then we consider the synthesis of the control of electric drive of manipulator shown in fig. 7

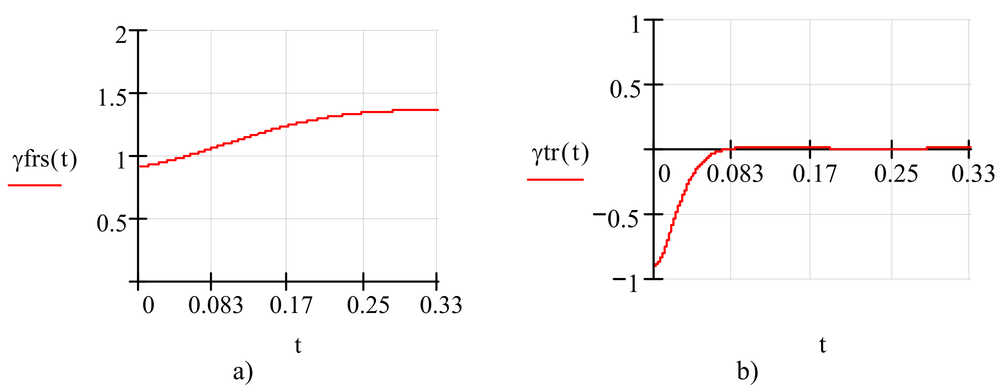


Fig. 6 The deceleration segment: the rotation angle value as a function of time:
a) the forced component; b) the transient component

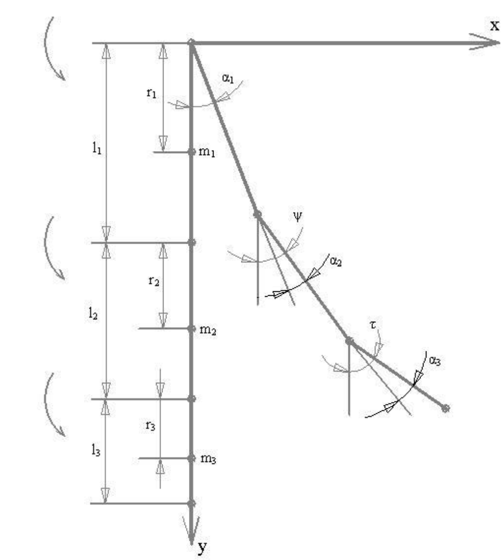


Fig. 7 The diagram of the manipulator with three joints

Let us minimize $W = \int_0^T v d\tau$ for the given ,

$$\int_0^{\Delta t} M_{Дi}^2 dt = \Delta t M_{ДiH}^2; \quad \int_0^{\Delta t} \dot{\alpha}_3 dt = \Delta\alpha_3; \quad \alpha_i(0) = \alpha_{i0};$$

$$\dot{\alpha}_i(0) = \dot{\alpha}_{i0}; \qquad Q_i = M_{Дi}\dot{J}_i - J'_{Дi}\dot{\Omega}_{\Pi i},$$

conditions, where Mдiн — moment of the electric drive, $\dot{\Omega}_{\Pi i}$ —acceleration of the electric drive shaft, the friction force is not taken into account.
For the third drive we minimize the functional

$$W_3 = \int_0^{\Delta t} \left[1 + \lambda \frac{1}{j_3^2} \left(\ddot{\alpha}_3 m_3 r_3^2 + q_3 + J'_{Д3}\Omega_{\Pi 3}^{(1)} \right)^2 \right] dt.,$$

what finally gives control function.

$$\alpha_3(t) = \alpha_{30} + \dot{\alpha}_{30} t + \frac{\pm j_3 M_{Д3H} - q_3 - J'_{Д3}\Omega_{\Pi 3}^{(1)}}{2 m_3 r_3^2} t^2 + \frac{\Delta\ddot{\alpha}3}{6\Delta t} t^3..$$

Analogically, we got for the second and the first joints (drive)

$$\alpha_2(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3,$$

were $C_1 = \alpha_{20}$, $C_2 = \dot{\alpha}_{20}$, $C_4 = \frac{\Delta\ddot{\alpha}_2}{6}$,

$$\frac{1}{j_2^2} \left\{ 2C_3 \left[m_2 r_2^2 + m_3 \left(l_2^2 + r_3^2 + 2 l_2 r_3 \cos\alpha_3 \right) \right] + q_2 + J'_{Д2}\Omega_{\Pi 2}^{(1)} \right\}^2 = M_{Д2H}^2$$

C3 can be determined from the equation

$$C_3 = \frac{\pm j_2 M_{Д2H} - q_2 - J'_{Д2}\Omega_{\Pi 2}^{(1)}}{m_2 r_2^2 + m_3 \left(l_2^2 + r_3^2 + 2 l_2 r_3 \cos\alpha_3 \right)},$$

the first joint (drive)

$$\alpha_1(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3,$$

where $C_1 = \alpha_{10}$; $C_2 = \dot{\alpha}_{10}$; $C_4 = \frac{\Delta\ddot{\alpha}_1}{6\Delta t}$;

$$C_3 = \frac{\pm j_1 M_{Д1H} - q_1 - J'_{Д1}\Omega_{\Pi 1}^{(1)}}{\left[m_1 r_1^2 + m_2 \left(l_1^2 + r_2^2 + 2 l_1 r_2 \cos\alpha_2 \right) + m_3 \left(\begin{matrix} l_1^2 + l_2^2 + r_3^2 + 2 l_1 r_2 \cos\alpha_2 + \\ 2 l_1 r_3 \cos\alpha_3 + 2 l_2 r_3 + 2 l_1 r_2 \cos(\alpha_2 + \alpha_3) \end{matrix} \right) \right]}.$$

Fig. 8 shows the circuit realizing the control of the manipulator motors. The circuit inputs receive information on the velocities $\dot{\alpha}1, \dot{\alpha}2, \dot{\alpha}3$ accelerations $\dot{\alpha}1, \dot{\alpha}2, \dot{\alpha}3$ moments of force Q1 Q2, Q3 calculated or measured on the preceding time interval and recorded in the memory within the given time interval Δt. This information is used to calculate q1, q2, q3 and, after that, the control for the next time interval which begins when new memorized potentials are delivered to the circuit inputs; in that case, the signal values at the integrator outputs are set to zero by closing the discharge loops of the capacitors.

The control realized by the circuit shown in Fig. 4.19 is cancelled by the programming device in the initial and terminal parts of the arm trajectory when the inequalities β1 i< α3i-αi < β2i are fulfilled.

This ensures the smoothness of the start and stop. Here α3i is the given stepwise displacement of the i-th joint set in time by the programming device.

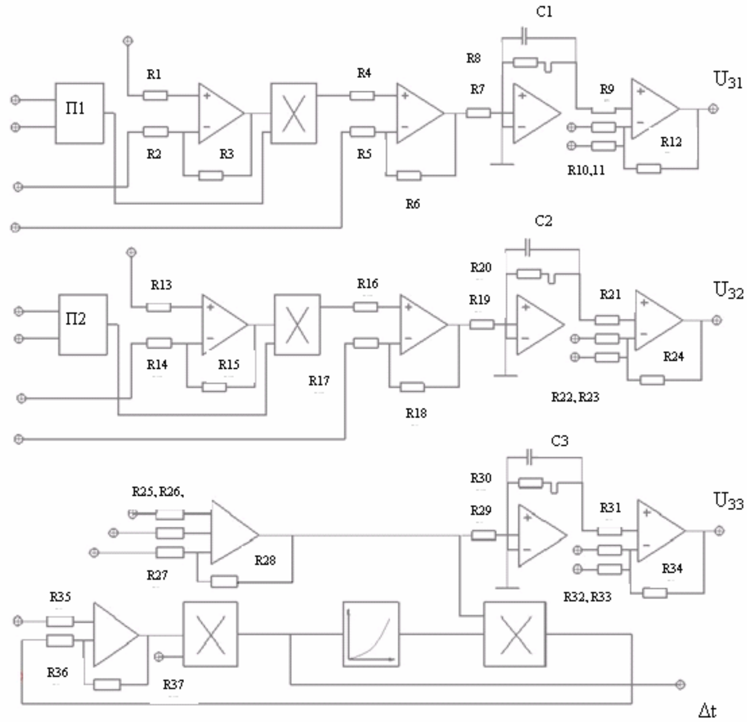


Fig. 8 The Circuit Realizing the Control of the Manipulator Motors.

Conclusions

1. Spatial rotations are for the first time described by their spinor representation, which made it possible to obtain simple relations for describing by means of an element of the controlling orthogonal matrix of the basic representation by the known coordinates of three defining rotation points: central, initial and terminal.
2. Simple formulas are obtained for calculation of controlling Euler angles;
3. The obtained results have enabled us to reduce the actually three-dimensional problem of spatial motion control to the one-dimensional problem;
4. A general variational method is obtained to solve problems of terminal control of spatial rotations;
5. Simple adaptive algorithms are obtained, by means of which various partial problems on the terminal control of acceleration, transfer of the object to a given point, and approach are solved under various terminal conditions.
6. New algorithms of control of spatial rotations of manipulating robots are studied;
7. An optimal control circuit is developed for the work of the electric drive realizing the algorithms of control of spatial rotations of manipulating robots.

List of Articles

1. Erguven J., Milnikov A.A.,. Rodonaia I.D., Suladze A.S. Moving Mechanical Objects Terminal Adaptive Control Problems // Problems of Applied Mechanics, Tbilisi, 2004 №2(15), p.54-58
2. Milnikov A.A., Onal H., Erguven J., Rodonaia I. A spinor Method of Solution of Manipulators Inverse Kinematic Problem// Problems of Mechanics, Tbilisi, 2006 №1, p.41-48
3. Milnikov A.A., Onal H., Erguven J., Rodonaia I. A Kinematics of Spatial Rotations and Euler's Angles// Problems of Mechanics, Tbilisi, 2006 №1, p.102-108