INSTITUTE OF MECHANICS OF MACHINES NAMED AFTER R.DVALI

Cabir Erguven

"ALGORITHMS OF TERMINAL CONTROL OF SPATIAL MOVEMENTS OF AGRICULTURAL ROBOTS"

Thesis submitted in Fulfillment of the Requirements for
the Degree of Candidate of Science (Ph.D)

05.13.16- Using of Computers, Computer Modelling and
Mathematical Methods for Scientific Researches

Supervisor: Doctor of Science, Professor Dr. Alexander Milnikov

Tbilisi - 2006

CONTENTS

General Description of the Work

A rapid development of science and engineering gave the humanity absolutely new means of automation – these are commercial robots used in the most diverse spheres of economy: in industry they are used as flexible systems of complex automation, transporting facilities, technological machines and so on. Among an enormous variety of robotic devices a special place is held by manipulating robots designed to perform all sorts of technological operations such as assembly and erection, painting, welding and many others.

In recent years, manipulating robots have been actively used for agricultural work, in particular, for gathering various fruits and vegetables, and also for various agro-technical operations such as pruning and so on. The use of robots lowers the production cost of final agricultural products, contributes to the improvement of their quality and decreases the share of hard manual labor.

One of the basic scientific and technical tasks connected with the development and implementation of robots designed for fulfilling various technological processes in industry and agriculture is the solution of problems dealing with the control of spatial motions of robots. Among the latter problems, the problem of controlling spatial rotations of multi-joint working components of robots is considered to be the most difficult one. The existing methods of its solution are cumbersome and complicated and hence it becomes necessary to use sophisticated hardware and software, which, in turn, increases the price both of robots themselves and, in the end, of a technological process as a whole.

This dissertation is devoted to:

1. The development of a new method of representation of spatial rotations of mechanical objects;
2. The development of simple adaptive algorithms of control of terminal states of spatial rotations of working organs of agricultural robots.

Topicality of the Theme: Problems of control of spatial motions in general and, in particular, of rotational motions of robots belong to the most topical directions in the complex of high technologies which demand a lot of scientific research. Most of the methods used to solve these problems are optimal methods of programmed control (disconnected methods without feedback). They include the maximum principle, the dynamic programming method, the momentum method and others.

As has been noted, all the listed methods are the programming ones, i.e. demanding the preliminary calculation of the law of control $u(t)$ and not making it possible to correct this law during motion. However practice demands the construction of automatic control systems (ACS) employing the feedback principle, since such systems make it possible to correct the motion trajectory in the course of the process.

Besides, in a majority of cases, for a successful solution of technological problems of robot application it is necessary to provide an exact positioning in the terminal stage of motion and thus, in the case of manipulating robots, the control of their terminal states (terminal control) becomes of special topical interest. An effective solution of such problems will enable us to improve the quality of technological processes, since the quality of these processes depends in many respects on the accuracy of the terminal positioning of the gripping devices of robots.

From the above-said it follows that problems connected with the development of simple adaptive systems of automatic control of terminal states of moving objects are topical and meet the up-to-date requirements of the development of technologies based on scientific research.

The Scientific Novelty consists in the following:

1. Spatial rotations are for the first time described by their spinor representation, which made it possible to obtain simple relations for describing by means of an element of the controlling orthogonal matrix of the basic representation by the known coordinates of three defining rotation points: central, initial and terminal.
2. Simple formulas are obtained for calculation of controlling Euler angles;
3. The obtained results have enabled us to reduce the actually three-dimensional problem of spatial motion control to the one-dimensional problem;
4. A general variational method is obtained to solve problems of terminal control of spatial rotations;
5. Simple adaptive algorithms are obtained, by means of which various partial problems on the terminal control of acceleration, transfer of the object to a given point, and approach are solved under various terminal conditions.
6. New algorithms of control of spatial rotations of manipulating robots are studied;
7. An optimal control circuit is developed for the work of the electric drive realizing the algorithms of control of spatial rotations of manipulating robots.

Methods of Investigation. The following methods are used in the work: elements of the theory of representation of rotation groups, the spinor theory, variational methods of control of electric drive motion, methods of ordinary differential equations, methods of programming by Mat-Cad.

The Practical Importance of the work consists in that the developed algorithms can be successfully used in programming robot-manipulators for the solution of practical technological problems, which will lead to the improvement of their terminal positioning and thereby to the perfection of the technological process as a whole. In addition to this, the obtained results can also be used for the solution of the corresponding problems of computer graphics.

Approbation of the Work. The results of the work were announced at an international conference, at the applied mathematics chair of Georgian Technical University (2005) and at the Machine Mechanics Institute of the Georgian Academy of Sciences (2005, 2006).

Published works. 3 works have been published on the topic of the dissertation.

Structure and volume. The work includes 122 computer type-set pages and consists of four chapters, a list of references and 37 figures.

1. Analysis of Literature Sources

Robotics is one of the fastest growing engineering fields of today. Millions of dollars have been spent in the developments of robots to be used in all sorts of field. The use of robots is more common today than ever before and it is no longer exclusively used by the heavy production industries. Robots are designed to remove the human factor from labor intensive or dangerous work. The computer is the brain of the robot which receives data from various sources to control the movement of the robot in order to accomplish a task.

Robotics is usually associated with the manufacturing industry, where it has had a history which is patchy, to say the least. In agriculture, the opportunities for robot-enhanced productivity are immense and the robots are appearing on farms in various guises and in increasing numbers.

The essential 'robotic' blending of intelligent sensing with mechanical actuation can be found in vision-guided tractors, product grading systems, planters and harvesters, applicators for fertilizers and pest control. Robot manipulators can divide plant material for micro-propagation in sterile conditions; others can skin fruit for canning.

All the ingredients of robotics are there. Sensing is important in all aspects. These range from simple transducers to measure actuator positions to vision for guidance and grading, time-series analysis of cutter vibration, flow rates for yield monitoring and GPS for precision agriculture and many more which have not yet been thought of. Actuation, software for intelligent control, kinematics and communication all have a party to play in this rapidly growing art.

1.1 Types of Robots

Robots can be found in the manufacturing industry, the military, space exploration, transportation, and medical applications. Below are just some of the uses for robots.

Typical industrial robots do jobs that are difficult, dangerous or dull. They lift heavy objects, paint, handle chemicals, and perform assembly work [1,7]. They perform the same job hour after hour, day after day with precision. They don't get tired and they don't make errors associated with fatigue and so are ideally suited to performing repetitive tasks. The major categories of industrial robots by mechanical structure are [2÷5]:

Cartesian robot /Gantry robot: Used for pick and place work, application of sealant, assembly operations, handling machine tools and arc welding. It's a robot whose arm has three prismatic joints, whose axes are coincident with a Cartesian coordinator.

Cylindrical robot: Used for assembly operations, handling at machine tools, spot welding, and handling at die-casting machines. It's a robot whose axes form a cylindrical coordinate system.

Spherical/Polar robot: Used for handling at machine tools, spot welding, die-casting, fettling machines, gas welding and arc welding. It's a robot whose axes form a polar coordinate system.

SCARA robot: Used for pick and place work, application of sealant, assembly operations and handling machine tools. It's a robot which has two parallel rotary joints to provide compliance in a plane.

Articulated robot: Used for assembly operations, die-casting, fettling machines, gas welding, arc welding and spray painting. It's a robot whose arm has at least three rotary joints.

Parallel robot: One use is a mobile platform handling cockpit flight simulators. It's a robot whose arms have concurrent prismatic or rotary joints.

Industrial robots are found in a variety of locations including the automobile and manufacturing industries. Robots cut and shape fabricated parts, assemble machinery and inspect manufactured parts. Some types of jobs robots do: load bricks, die cast, drill, fasten, forge, make glass, grind, heat treat, load/unload machines, machine parts, handle parts, measure, monitor radiation, run nuts, sort parts, clean parts, profile objects, perform quality control, rivet, sand blast, change tools and weld.

Outside the manufacturing world robots perform other important jobs. They can be found in hazardous duty service, CAD/CAM design and prototyping, maintenance jobs, fighting fires, medical applications, military warfare and on the farm.

Farmers drive over a billion slow tractor miles every year on the same ground. Their land is generally gentle, and proven robot navigation techniques can be applied to this environment. A robot agricultural harvester named Demeter is a model for commercializing mobile robotics technology. The Demeter harvester contains controllers, positioners, safeguards, and task software specialized to the needs commercial agriculture [6].

Some robots are used to investigate hazardous and dangerous environments. The Pioneer robot is a remote reconnaissance system for structural analysis of the Chornobyl Unit 4 reactor building. Its major components are a teleoperated mobile robot for deploying sensor and sampling payloads, a mapper for creating photorealistic 3D models of the building interior, a coreborer for cutting and retrieving samples of structural materials, and a suite of radiation and other environmental sensors [8].

An eight-legged, tethered, robot named Dante II descended into the active crater of Mt. Spurr, an Alaskan volcano 90 miles west of Anchorage. Dante II's mission was to rappel and walk autonomously over rough terrain in a harsh environment; receive instructions from remote operators; demonstrate sophisticated communications and control software; and determine how much carbon dioxide, hydrogen sulfide, and sulfur dioxide exist in the steamy gas emanating from fumaroles in the crater. Via satellite, Dante II sent back visual information and other data, as well as received instruction from human operators at control stations in Anchorage, Washington D.C., and the NASA Ames Research Center near San Francisco. Dante II saves volcanologists from having to enter the craters of active volcanoes. It also demonstrates the technology necessary for a robot to explore the surface of the moon or planets. That is, the robot must be able to walk on rough terrain in a harsh environment, receive instructions from remote operators about where to go next, and reach those commanded goals autonomously [9,10].

Robotic underwater rovers are used explore and gather information about many facets of our marine environment. One example of underwater exploration is Project Jeremy, collaboration between NASA and Santa Clara University. Scientists sent an underwater telepresence remotely operated vehicle (TROV) into the freezing Arctic Ocean waters to investigate the remains of a whaling fleet lost in 1871. The TROV was tethered to the surface boat Polar Star by a cable that carried power and instructions down to the robot and the robot returned video images up to the

Polar Star. The TROV located two ships which it documented using stereoscopic video cameras and control mechanisms like the ones on the Mars Pathfinder. In addition to pictures, the TROV can also collect artifacts and gather information about the water conditions. By learning how to study extreme environments on earth, scientists will be better prepared to study environments on other planets.

Space-based robotic technology at NASA falls within three specific mission areas: exploration robotics, science payload maintenance, and on-orbit servicing. Related elements are terrestrial/commercial applications which transfer technologies generated from space telerobotics to the commercial sector and component technology which encompasses the development of joint designs, muscle wire, exoskeletons and sensor technology.

Today, two important devices exist which are proven space robots. One is the Remotely Operated Vehicle (ROV) and the other is the Remote Manipulator System (RMS). An ROV can be an unmanned spacecraft that remains in flight, a Lander that makes contact with an extraterrestrial body and operates from a stationary position, or a rover that can move over terrain once it has landed. It is difficult to say exactly when early spacecraft evolved from simple automatons to robot explorers or ROVs. Even the earliest and simplest spacecraft operated with some preprogrammed functions monitored closely from Earth. One of the best known ROV's is the Sojourner rover that was deployed by the Mars Pathfinder spacecraft. Several NASA centers are involved in developing planetary explorers and space-based robots.

The most common type of existing robotic device is the robot arm often used in industry and manufacturing. The mechanical arm recreates many of the movements of the human arm, having not only side-to-side and up-and-down motion, but also a full 360-degree circular motion at the wrist, which humans do not have. Robot arms are of two types. One is computer-operated and programmed for a specific function. The other requires a human to actually control the strength and movement of the arm to perform the task. To date, the NASA Remote Manipulator System (RMS) robot arm has performed a number of tasks on many space-missions serving as a grappler, a remote assembly device, and also as a positioning and anchoring device for astronauts working in space.

1.1.1 Robots in Agriculture

Agricultural robots and precision farming have a great potentiality not only to change the agriculture, but also to solve many problems even for global issues [11]. The idea of applying robotics technology in agriculture is very new. The main area of application of robots in agriculture is at the harvesting stage. Fruit picking robot and sheep shearing robot are designed to replace human labor. The agricultural industry is behind other industries in using robots because the sort of jobs involved in agriculture are not straight forward and many repetitive tasks are not exactly the same every time. In most cases, a lot of factors have to be considered (i.e.: size and color of the fruit to be picked) before the commencement of a task [12÷17].

In the field of agriculture, various operations for handling heavy material are performed. For example, in vegetable cropping, workers should handle heavy vegetables in the harvest season. Additionally, in organic farming, which is fast gaining popularity, workers should handle heavy compost bags in the fertilizing season. These operations are dull, repetitive, or require strength and skill for the workers. In the 1980’s many agricultural robots were started for research and development. Kawamura and co-workers developed the fruit harvesting robot in orchard. Grand and co-workers developed the apple harvesting robot .They have been followed by many other works including our previous works. Many of the works focus on structure systems design (e.g., mechanical systems design) of the robots and report realization of the basic actions in actual open fields. However, many of the robots are not in the stages of diffusion but still in the stages of research and development. It is important to find rooms to achieve higher performance and lower cost of the robots [18÷20].

There are many robots used in agriculture and food industry. Some of them are as follows [21÷24]:

1.1.1.1 Fruit Picking Robot

The principles of fruit picking robots have been developed since the early 1980's. These principles have opened up new approaches to the harvesting of crops. To start with, the fruit picking robots need to pick ripe fruit without damaging the branches or leaves of the tree. Mobility is a priority, and the robots must be able to access all areas of the tree being harvested. It goes then without saying that the robots must be 'intelligent', and have a human-like interaction with their surroundings through senses of touch, sight, and image processing. The fundamental blocks of these robots are shown in the diagram here [25÷33].

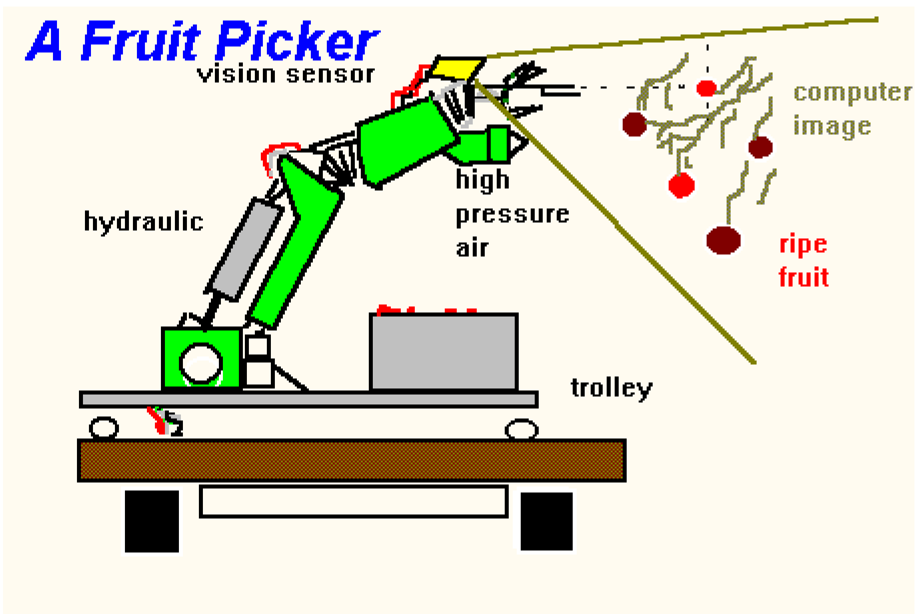


Fig 1.1 Fruit Picking Robot

The robot can distinguish between fruit and leaves by using video image capturing. The camera is mounted on the robot arm, and the colours detected are compared with properties stored in memory. If a match is obtained, the fruit is picked. If fruit is hidden by leaves, an air jet can be used to blow leaves out the way so a clearer view and access can be obtained.

The robot arm itself is coated in rubber to minimise any damage to the tree. It has 5 degrees of freedom, allowing it to move, in, out, up, down, and in cylindrical and spherical motion patterns. The pressure applied to the fruit is sufficient for removal from the tree, but not enough to crush the fruit. This is accomplished by a feedback process from the gripper mechanism, which is driven by motors, hydraulics, or a pneumatic system. The shape of the gripper depends on the fruit being picked, as some fruits, such as plums, crush very easily, while others, like oranges are not so susceptible to bruising.

1.1.1.2 Tomato and Cherry Tomato Harvesting Robot

Fig. 1.2 shows a tomato and cherry tomato harvesting robot. This robot consists of 4 components; manipulator, end-effector, visual sensor, and traveling device. Here, those components are described. A seven degrees of freedom manipulator was used to harvest larger size tomato and cherry tomato fruits as shown in Fig.1.2. This manipulator could have high manipulatability when it had a harvesting posture. The manipulator consisted of two prismatic joints and five rotational joints. The lengths of upper arm and fore arm were 250 mm and 200 mm, while strokes of the prismatic joints are 200 mm in horizontal direction and 300 mm in vertical direction [25].

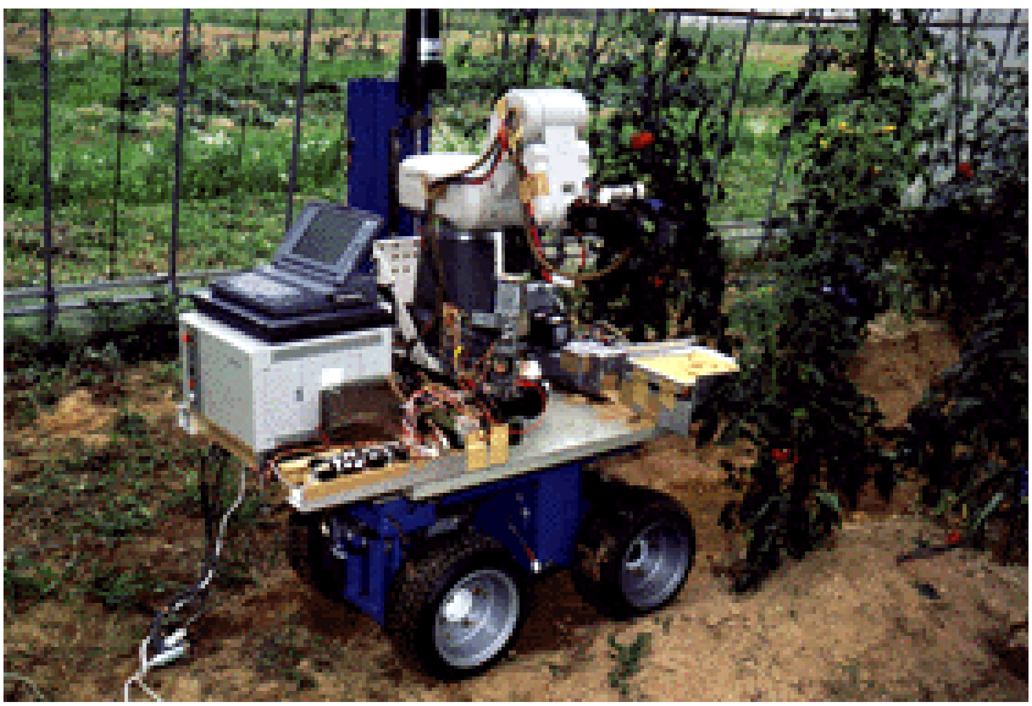


Fig.1.2 Tomato and Cherry tomato harvesting robot

1.1.1.3 Strawberry Harvesting Robots

Two types of strawberry harvesting robots **[19]** have been developed. One is for hydroponic system (Fig.1.3) and the other is for soil system. Since the plant training systems are different each other, two different types of robot are required. These robots also have similar components. The former robot is also cooperated with an agricultural machinery company.

Fig. 1.3 Strawberry Harvesting Robot for Hydroponic System.

1.1.1.4 Cucumber Harvesting Robot

Fig. 1.4 shows a cucumber harvesting robot designed in 1996. As its manipulator, 6 DOF articulate manipulator was prepared to be able to work in the inclined trellis training system, which was developed for the robotic harvesting system. The training system made fruits hang down from its trellis to be able to detect fruits easily. Its visual sensor should be able to discriminate green fruit from green leaves and stems, since immature fruit is usually harvested. In this robot, therefore, a monochrome TV camera with 850 nm wavelength interference optical filter was used to discriminate the fruit based on its spectral reflectance. In its end-effector, a peduncle detector, a cutter and fingers were installed, because it is difficult to detect peduncle position by the visual sensor and it is necessary to cut the peduncle. This project is collaborated with an agricultural machinery company.

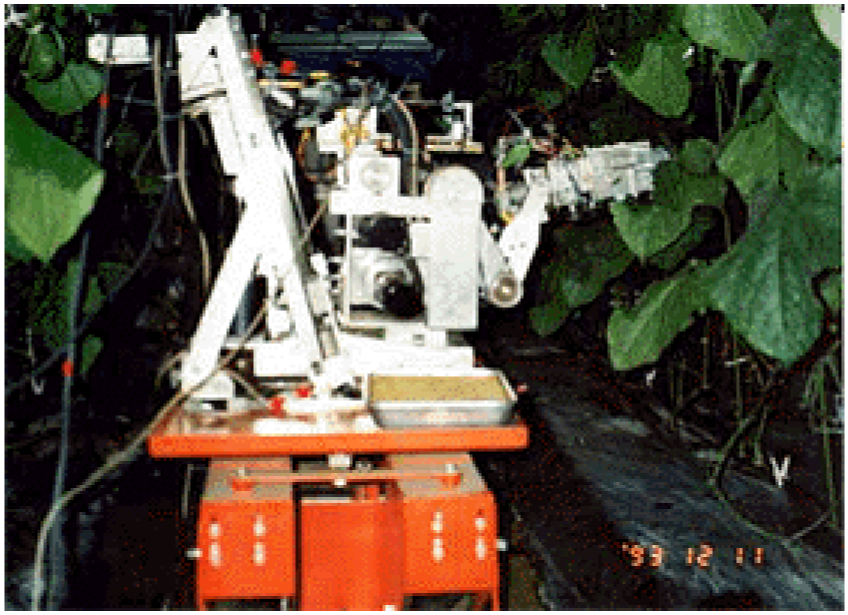


Fig. 1.4 Cucumber Harvesting Robot.

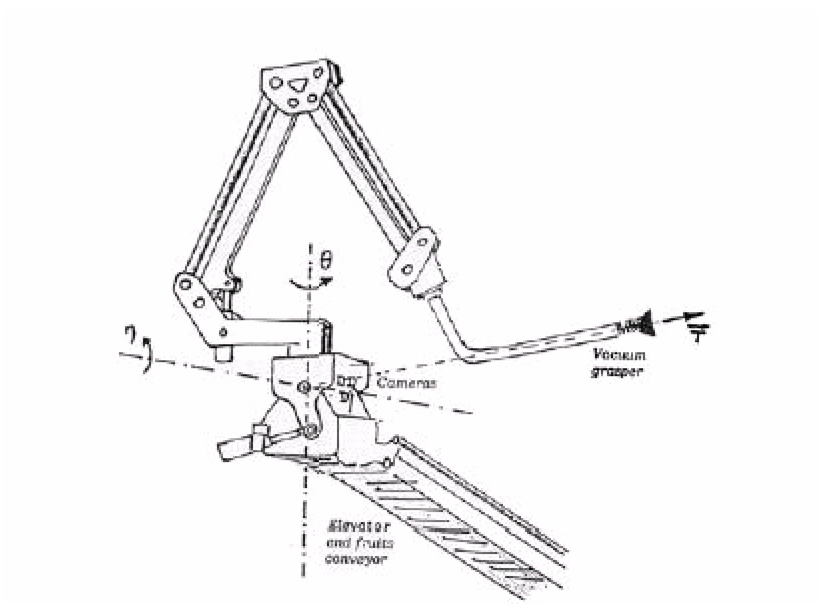


Fig. 1.5 MAGALI Apple Picking Arm ,1987

1.1.1.5 Multi-Operation Robot for Grapevine

Fig.1.6 shows a robot to work in vineyard. In open field, harvesting period is so short that a harvesting robot may not be efficient if it is not able to do other operation. To make working period of the robot longer, several end-effectors were developed. This robot has a polar coordinate manipulator with five degrees of freedom. The manipulator end could be moved on horizontal plane below the trellis at a constant speed under CP control.

The length of the arm was 1.6 m, and the stroke was 1 m. In our laboratory, a harvesting end-effector (Fig.1.7), a berry thinning end-effector, a bagging end-effector, and a spray nozzle were attached to the manipulator end and experimented.

Fig.1.6 Multi-Operation Robot for Grapevine (Bagging operation).

Fig.1.7 Harvesting End-effector.

1.1.1.6 Chrysanthemum Cutting Sticking Robot

Fig.1.8 shows a robotic cutting sticking system. The system mainly consists of three sections; a cutting providing system, a leaf removing device (Fig.1.9) and a sticking device (Fig.1.10). First, a bundle of cutting is put into a water tank. The cuttings are spread out on the water by vibration of the tank. The cuttings are picked by a manipulator based on information of cutting position from a TV camera. Secondly, another TV camera recognizes the shape of cutting transferred from the water tank by the manipulator and detects its position and direction. The TV camera indicates the grasping position of the cutting for another manipulator. Thirdly, the manipulator moves the cutting to the leaf removing device and to the sticking device. Finally, the cuttings are stuck into a plug tray by the planting device [9,10].

Fig. 1.8 Cutting providing system

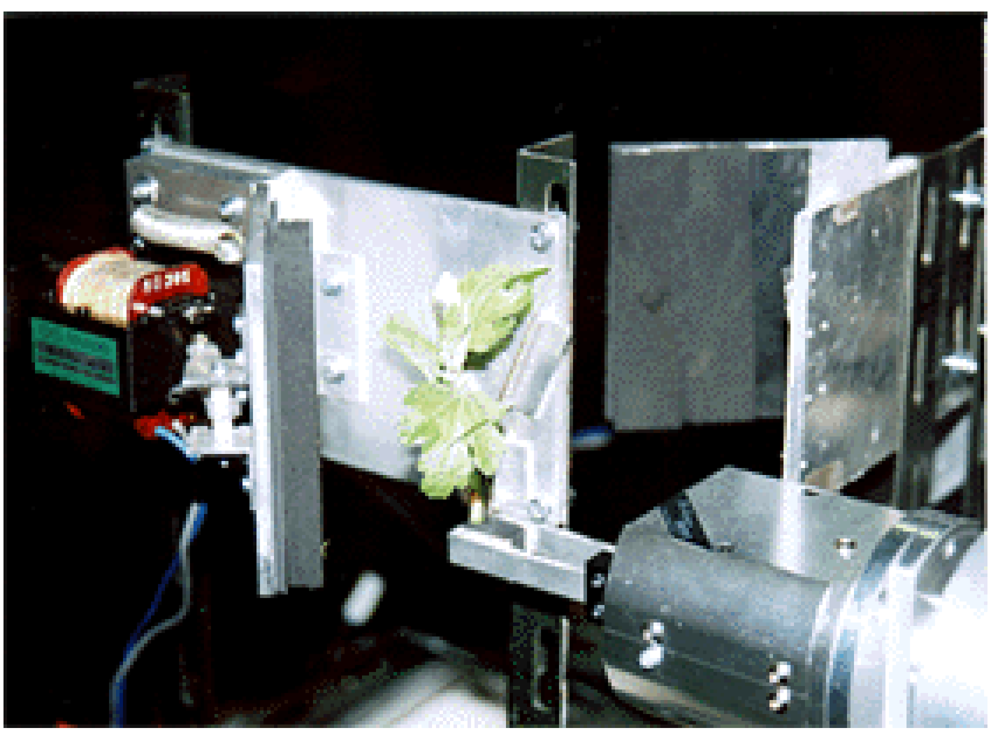


Fig. 1.9 Leaf removing device

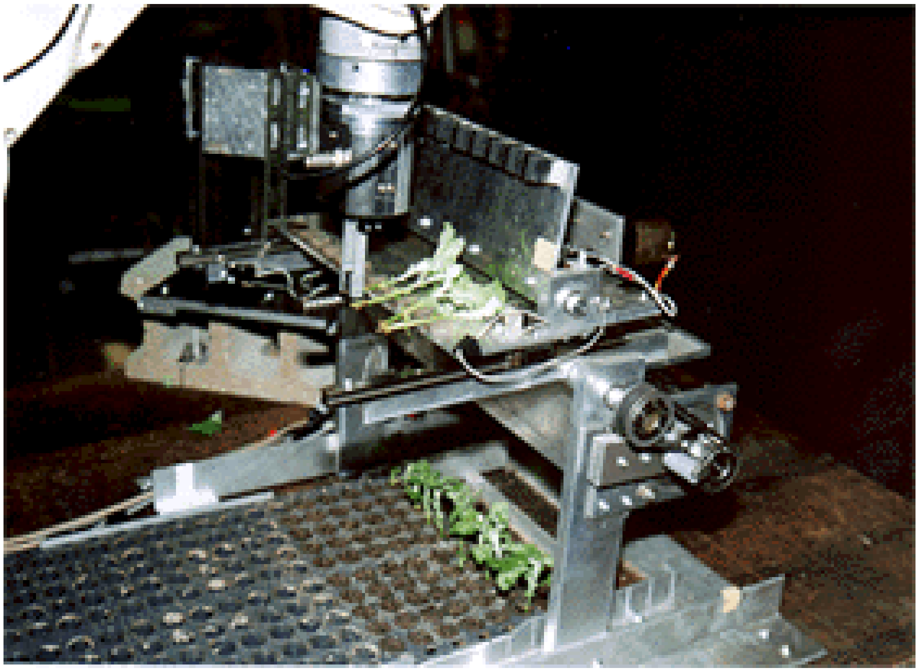


Fig 1.10 Sticking Device

1.1.2 Stock Raising

1.1.2.1 Sheep Sheering Robot

The Oracle robot is a Sheep Sheering robot and it was developed at the University Of Western Australia in 1979. Many of the Oracle's concepts formed the basis for its successor, the Shear Magic Robot. The Oracle is pictured in Fig.1.11

During shearing operations, the sheep is restrained firmly by holding its legs and its head in a manipulator, called the ARAMP. The shearing arm is directed by complex motion control algorithms which maintain the cutter at a predefined height above the sheep's skin. It should be noted that not all sheep are the same shape, and they sometimes object very strongly to being shorn.

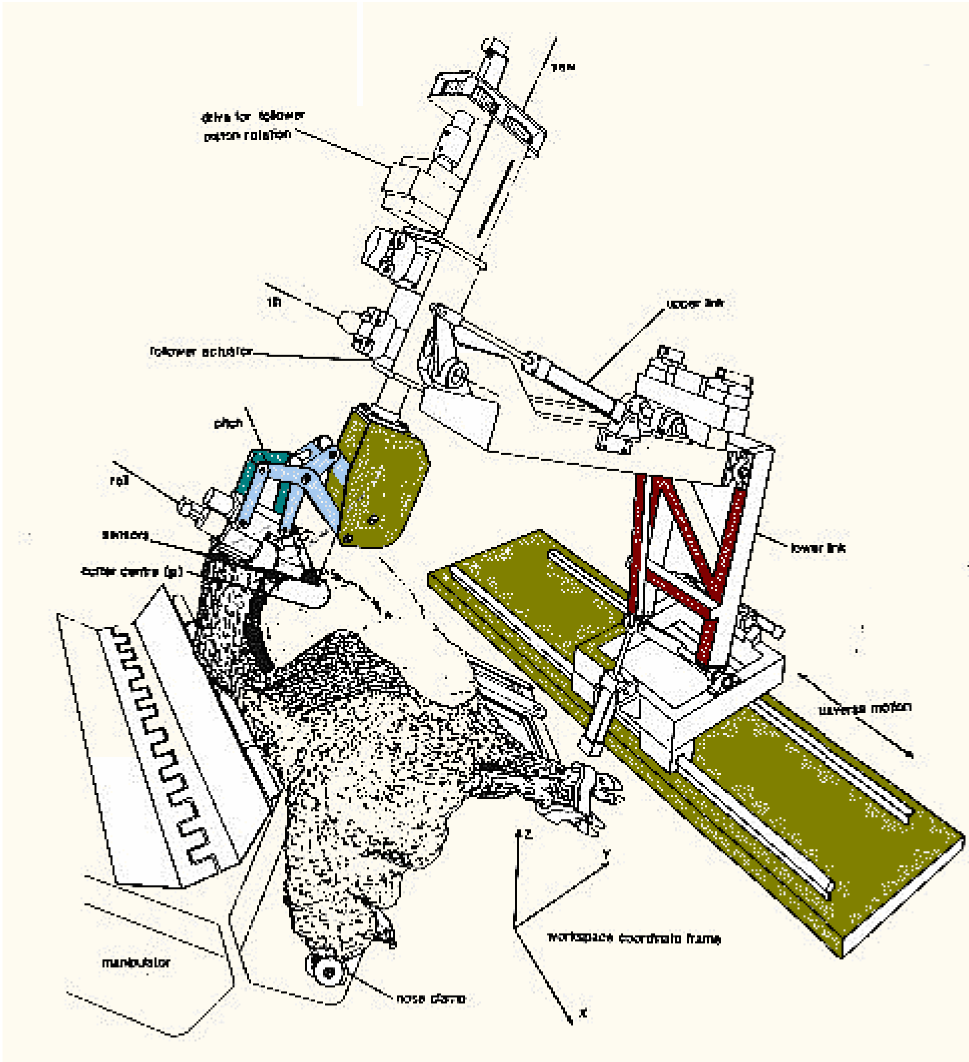


Fig. 1.11: Sheep Shearer

The robotics arm which holds the clippers can be manoeuvred in six directions, and is powered by a series of hydraulic actuators using proportional analogue servo controls. These actuators are controlled by a minicomputer through a conventional D/A interface. The Hewlett Packard 21MX-E is utilised to control Oracle. To maintain such a critical distance from the sheeps' skin, many sensors are used to feedback the actual position of the shearing arm and the relative position of the sheeps' skin. Difficulties are associated with such sensing, as wool has a tendency to conduct electricity, and this characteristic can vary with moisture, humidity and other environmental factors, so sensors can give erroneous readings. However, a combination of several types of sensors are used to keep the clippers at a safe distance from the skin whilst still ensuring a reliable cut to the wool.

1.1.3 Robots for the Food Industry

The benefits of robotic technology have been demonstrated in many aspects of engineering and manufacturing industry. There are also many potential benefits for the food sector although these have yet to be commercially realized. Many business reasons are cited for the introduction of robotics including improved product quality and reductions in unit production costs. For the food industry the extra benefits of robotic automation are improved quality, in terms of hygiene and repeatability of processing, and reduced labor costs. Introducing robots is not however easy. Food products vary tremendously in size, shape, texture, flexibility, etc. and intelligence is needed to optimize processing. Relatively advanced sensors such as machine vision are commonly required to assess product variation and often artificial intelligence is required to make correct processing decisions. However, some robot systems have been used in the food sector. The majority of these are for applications where the product being handled is of uniform size and shape such as boxes or packaged components. There are some systems in development that attempt to go further and cope with the inherent flexibility required to successfully process food materials directly. [34-35].

Robots are grasping a larger foothold in manufacturing, especially in the food industry. In 1995, about 400 robots valued at approximately $20 million were shipped to food and beverage processors only in the U.S. This made the food industry the fifth largest consumer of robotics behind automotive, machinery, electronics and fabricated metal products.

Food processors are turning to robotics for their flexibility, ruggedness and repeatability. Categorized as flexible automation, robots can be programmed to do multiple tasks. And, they can be reprogrammed as the task changes. Changing the end-of-arm tooling or end effectors, for example, from a vacuum to a clamp gripping device, further increases the versatility of the machine. Robots can be built to operate in a constant freezing (32 [degrees] F) or sub-freezing (-10 [degrees] F) environment.

1.1.3.1 The Intelligent Integrated Belt Manipulator (IIBM)

The new IIBM is a hybrid of pneumatics and electro-servo drives. Two pneumatic axes and two electro-servo axes allow motion in four different directions: up and down, parallel with the conveyor belt, perpendicular across the conveyor belt, and a 90-degree rotational pivot.

In automotive and electronics industries, parts are consistently shaped and easy for robots to handle. Yet in the poultry business, products vary considerably in size and shape, making grasping demands another challenge for the IIBM. Physical dimensions of the tray pack remain constant, but the poultry pieces inside vary the contours of the package's top by as much as two inches, causing weight and center of gravity to shift.

Fig.1.12 Belt Manipulator picks up packaged chicken from a conveyor belt.

1.1.3.2 Food Cutting Systems

Most food cutting systems are empirically designed and operated, with little fundamental understanding of the separation processes involved. Many systems use technology developed in other industries, e.g. metal guillotining, and often require the food to be pre-processed (tempered) to give it the required properties. This added pre-processing is costly and time consuming, and if not carried out correctly can result in serious reductions in quality, yield and throughput of cut product. In addition to this, the cutting equipment often causes further problems, such as creation of debris (bone splinters, crumbs etc.), separation of food components, loss of yield and has high maintenance and operating costs.

Fig.1.13 Food cutting system

1.1.3.3 Vision-Based Object Handling

Some industrial applications of a visually-guided system for robot grasping using an inexpensive two-finger gripper have been developed. In all cases, the robot uses visual information as input and is able to reason about the shapes of the objects in the scene in order to decide the best stable grasp online. The first version of this system was able to grasp rectangular parts in arbitrary positions in the scene, and was successfully deployed. New applications in industry have been addressed, that have to cope with the cost, time and reliability requirements imposed by the industrial process. Our results show that the capabilities of the underlying methodology make it feasible to deal with more complex shapes, even a priori unknown, opening new possibilities within industrial domains that have traditionally not been fully automated, such as the food industry, due to a large shape variability of the objects to be handled.

Fig.1.14 Vision-Based Object Handling

1.2 Survey of Robot Manipulators Movement Control Methods

The demands of practice and, in the first place, the needs of modern engineering stimulated the development of controlled systems and gave rise to a multitude of problems which became the subject of the mathematical theory of controlled processes. An essential place in this theory is held by optimal control problems. In a general formulation, the problem consists in the following: an object (a mechanical system, an electric circuit and so on) subjected to a controlling action is considered under given parameters (for instance, the initial and the terminal state of an object) of the desired motion. Requirements for the process quality are also stated. These requirements usually include the condition of minimum or maximum and may also include the condition of minimax or maximin of some performance characteristic of a system. A typical example is the condition of minimum of electric power consumption. It is required to find a law that defines forces realizing the desired motion.

The theory of optimal control unites a great number of various problems. The study of this theory is somewhat difficult because of the absence of a generally recognized classification of problems which are investigated by mathematicians and mechanical researchers, physicists and engineers, biologists and sociologists who may have different aims and who use different methods when studying analogous problems. For individual branches of the control theory that have recently been actively developed there exists vast and rather mixed up bibliography, which makes it difficult to survey the results.

Consideration is given only to finite-dimensional objects whose current state and controlling action can be described by the finite-dimensional vectors

$x = \{x_1,...,x_n\}$, $u = \{u_1,...,u_2\}$, the motion $\{x_i(t)\}$ being defined by the system of ordinary differential equations

$$\dot{x}_i = f_i(t, x_1,...,x_n, u_1,...,u_r), \qquad (i = 1,...,n)., \tag{1.1}$$

where t is the time.

The main attention is focused on the problem of bringing an object to a given state. This situation frequently occurs in problems of mechanical motion control.

There are two well-known basic aspects of the general control problem.

I. The problem on *programmed* control, where the initial information on the initial state of the object is given (by the initial time $t = t_\alpha$) and it is required to find an action in the form of a time function $u = u(t)$, $t \geq t_\alpha$ such that by the process termination $t = t_\beta$ the system would be in the given state. Also, as has been indicated above, it is required to provide the desired quality of the process.

An example of such a problem is the following problem of the limiting programmed speed of action: Given the initial state $t = t_\alpha$, $x(t_\alpha) = x^\alpha$ of the object and the position $x(t_\beta) = x^\beta$ to which the object should be moved, it is required to find an action $u = u^0(t)$ satisfying the condition $\|u^0(t)\| \leq \mu$ and putting the object in the state $x(t_\beta) = x^\beta$ in the shortest possible time $T = t_\beta - t_\alpha$. (The symbol ||u|| denotes the norm of the vector $\left(u_2^1 + ... + u_r^2\right)^{1/2}$).

For problems of this type it is typical that the additional information, which might be delivered in the course of the process, is not used to correct the motion in order to improve the result, i.e. the motion is realized by the rigid program $u=u(t)$ prepared beforehand. This restricts the role of respective results and makes it necessary to consider the second aspect of the problem

II. The problem on the system *synthesis* with feedback. Here the best law of control is sought for in the form of equations connecting the force u with certain values $\{y_1(t), ..., y_m(t)\}$ delivering information on the current states $x(t)$ of the object. In the particular case, where all coordinates $x_i(t)$ of the vector $x(t)$ can be defined quickly and quite accurately, the controlling actions u_j are usually defined in the form of functions $u_j = u_j[t, x_1(t),...., x_n(t)]$.

An example is the problem of pursuance. In this problem, two objects (which can certainly be interpreted as two parts of one composite object) are given. They are described by the equations

$$\dot{x}^{(1)} = f^{(1)}\left[x_1^{(1)},...,x_k^{(1)},u_1,...,u_r\right], \tag{1.2}$$

$$\dot{x}^{(2)} = f^{(2)}\left[x_1^{(2)},...,x_k^{(2)},v_1,...,v_r\right], \tag{1.3}$$

and can therefore be mapped in some k-dimensional space by the points $x^{(1)}(t) = \{x_i^{(1)}(t)\}$, $x^{(2)}(t) = \{x_i^{(2)}(t)\}$, respectively. It is assumed that object (1.2) pursues object (1.3) and the aim of this pursuance is to make the point $x^{(1)}(t)$ coincide *with* the point $x^{(2)}(t)$; while object (1.3), on the contrary, tries to avoid the coincidence of these points. Thus the choice of controlling forces u_j is dictated by the wish to hasten the time $t = t_\beta$ of the coincidence of the points, while the choice of V_J is dictated by the opposite wish to postpone this moment. If it is assumed that at each moment of time t both partners know the realized values $x^{(1}(t)$ and $x^{(2)}(t)$, then one can pose a game problem on choosing optimal controls $u^0\left[x^{(1)},x^{(2)}\right]$ and $v^0\left[x^{(1)},x^{(2)}\right]$ which are restricted by the conditions $\|u\| \le \mu$, $\|v\| \le \nu$, calculated at each moment of time t by means of the actually realized values $x^{(1)}(t)$ and $x^{(2)}(t)$, .i.e. in the form $u^0 = u^0\left[x^{(1)}(t),x^{(2)}(t)\right]$, $v^0 = v^0\left[x^{(1)}(t),x^{(2)}(t)\right]$ and provide *minimax* for the time t_β when the coincidence of the points $x^{(1)}(t_\beta) = x^{(2)}(t_\beta)$ takes place for the first time.

The investigation of problems on system synthesis with feedback naturally includes the problem of defining the current coordinates $x_i(t)$ of the controlled object by means of the values $y_j(t)$ that are accessible for observation. The latter problem is known the *problem on observation* of a dynamic system. Here special importance is attached to questions related to the best concordance of observation and control in terms of the optimality of the final results of the process.

Let us briefly discuss some main trends in the theory of optimal processes described by ordinary differential equations (1.1).

Exhaustive investigations and final results are the necessary criteria of optimality for the programmed control problem with the condition that the integral be minimal:

$$I = \int_{t_\alpha}^{t_\beta} \omega[t, x(t), u(t)]dt. \tag{1.4}$$

The theory of such necessary conditions is based on the classical ideas of variational calculus [36÷41] and on their development by the new methods that have been elaborated in the last few decades. The principle of maximum [42] is a profound and strictly substantiated criterion of optimality whose form makes it convenient for applications. This principle corresponds to the classical Weierstrass' variational principle and to the method of canonical Hamilton equations [43].

Another approach to control problems, which is considered to be suitable for problems of synthesis of optimal systems with feedback, develops in the direction called *the method of dynamic programming* [44]. This method corresponds to the notions well– known in variational calculus – of excitation propagation and leads to equations of the type of Hamilton-Jacobi partial equations [44,45].

One of the difficult and insufficiently studied problems remains *the boundary value problem* connected with the necessity of bringing the controlled object to a given terminal state. At the present time, this boundary value problem frequently becomes the stumbling block on the path of concrete calculation of controlling forces. The matter is that the known optimality criteria are related mainly to the internal properties of optimal motions and describe their *local* behavior in the neighborhood of each point on a given trajectory. By virtue of these properties, each optimal motion develops in time in the absolutely definite way. However the direction in the space *{x}*, in which an optimal trajectory may deviate from the given initial state $x(t_\alpha) = x^\alpha$, is defined by a set of some parameters $l_1, \ldots, l_n$. The difficulty consists in choosing such parameters that direct the trajectory to the desired point $x(t_\beta) = x^\beta$. The above-mentioned aiming problem still has no general effective solution.

Thus, for the theory of controlled systems and its applications an important role belongs to the problem of construction a controlling force u which brings the object to the given state. It is expedient to investigate this control problem first even without taking into account the optimality requirement for this or another parameter. In particular, this is explained by the fact that in a number of numerical methods, optimal motions are found by a descent from some initial motions satisfying the given boundary conditions. It has already been mentioned that in the general case of nonlinear equations (1.1), the boundary value problem has no general working theory. However, for systems described by equations (1.1) whose right-hand parts are linear with respect to x_i and u_j, the considered control problem becomes essentially simplified and can be investigated by the methods of linear analysis. Satisfactory theories have been constructed for this problem and despite its partial character it has quite a vast bibliography. The control problem for linear objects is the main subject of this work.

1.3 Differential Equations of Motion

We will consider the controlled objects whose state at each moment of time t is characterized by the values $x_1(t)$, ,.. ..., $x_n(t)$ which are the parameters connected with motion, for instance, velocity coordinates or some coordinate and velocity functions. The values x_i can be interpreted as components of the n-dimensional vector $x = \{x_i\}$. Let us assume that the object is subjected to the action of controlling forces $u_1..., u_r$ which are interpreted as components of the r-dimensional vector $u = \{u_j\}$. The variables x_i or u_j can indeed be used in the sense of components of real physical vectors. For instance, the numbers u_1 u_2 u_3 can be projections on the coordinate axis of the three-dimensional vector of the force u applied to some mechanical system. However it is not always that the values x_i or u_j are the components of the real vectors $x = \{x_i\}$ $(i=1,..., n)$, $u = \{u_j\}$ $(j=1,...,r)$. For instance, there may occur a situation with $x_i = q_i$, $x_{i+m} = g_i$, where q_t and q_i $(i = l,...,m)$ are respectively generalized curvilinear coordinates and generalized velocities of a mechanical system. In such situations the interpretation of unions $\{x_1,.., x_n\}$ or $\{u_1,..., u_n\}$ in the form of vectors should be regarded as a convenient mathematical technique. The vector u is called the control.

We assume that the time-dependent variation of values $x_i(t)$ is described by a system of ordinary differential equations that can be reduced to the normal form

$$\dot{x}_i = f_i\left(t,\ x_1,...x_n, u_1,...u_r\right) \qquad (i = 1,..., n) \qquad (1.5)$$

We call the vector-function $u(t) = \{U_J\ (t)\}$ a possible control (on the considered time interval $t_\alpha \le t \le t_\beta$) if the components $u_j(t)$ are piecewise-continuous functions admitting only discontinuities of first kind for individual isolated values $t = t_*$.

After substituting some possible control $u(t) = \{u_j(t)\}$ into equation (1.5), the right-hand parts of these equations transform to the functions of t and x_i. We assume that on the considered time interval these functions f_i $(t, x_1..., x_n, u_1\ (t),...,u_r(t))$ satisfy the existence and uniqueness conditions for solutions $x(t) = \{x_i(t)\}$ under all the initial data

$$t = t_0, \quad x_i(t_0) = x_i^0 \quad (t_\alpha \le t \le t_\beta ; \; i = 1,..., n), \qquad (1.6)$$

which can be encountered in the problem. The motion $x(t)=\{x_i\ (t)\}$ generated by the initial condition (1.2) is denoted by the symbol $x\ (t_1,\ t_0,\ x^0),$. where therefore $x^0 = \{x_i^0\}$. If it is necessary to emphasize that the motion $x(t)$ is generated by some fixed control $u=u(t)$, then we will write $x\ (t,\ t_0,\ x^0;\ u)$.

So, if we give the initial conditions and choose a certain possible motion $u(t)$, we will thereby uniquely define the continuous motion $x(t)$. The vector x is called *a phase vector* of the object which is defined as follows.

Definition 1.1. *Any vector $x=\{x_i\}$ $(i = 1,... , n)$ that possesses the properties given below is called a phase vector of the object:*

1. *The components $x_i\ (t)$ characterize the state of the object.*
2. *For the chosen possible control $u(t)$ each initial state $x(t_0) = x^0$ uniquely defines the values $x\ (t) = x\ (t,\ t_0,\ x^0)$ for all considered moments of time t. In this case the equalities $x\ (t,\ т,\ x^T) = x\ (t,\ t_0,\ x^0)$ will be valid if and only if $ж^т = x\ (т,\ t_0,\ x^0)$ no matter what t, т and t_0 from the segment $\lfloor t_{\alpha,} t_\beta \rfloor$ are.*

Components $x_i (i=1,...,n)$ are called *phase coordinates* of the object.

Let, for instance, the controlled object be a holonomic mechanical system having κ degrees of freedom and described by the generalized coordinates $q_1,...,q_2$. As a phase vector we can choose a $2k$-dimensional vector $x = \{q_1,...,q_k, \dot q_1,...,\dot q_k\}$. Indeed, as is known, the motion of such an object can be described by a system of κ differential equations of second order which can be reduced to a system of 2κ equations of form (1.5). In this case, if at some moment of time $t = t_0$ we give all the components $q_i(t_0),\ \dot q(t_0)$ of the vector $x(t_0)$ for the known law of variation of external forces $u(t)=\{u_j(t)\}$, then we will thus define uniquely the motion of the system. Simultaneously, we define the values $\mathrm{q_i\ (t), \dot q_i\ (t)}$. Conversely, a k-dimensional vector $q=\{q_{1,...}\ q_k\}$ is not a phase vector of the considered system since the values $\mathrm{q_i\ (t_0),..., q_k\ (t_0)}$. do not define uniquely the values $q_1(t)$ $(i=1,.., \kappa)$.

Note that in individual situations one and the same object may have several phase vectors of varying dimension. So, in the simplest case of the point m moving along the straight line ξ

according to the equation $m\ddot{\xi} = u,$ as a phase vector x we can choose the two-dimensional vector $x=\{x_1, x_2\}$ = {ξ, ξ). At the same time, if we are interested only in the variation of the velocity ξ of this point and not in its coordinate, then its suffices to consider the one-dimensional phase vector $x = x_1 = \dot{\xi}$. Generally speaking, the choice of this or another phase component depends on the concrete conditions of the problem.

As a rule, the equations which describe control are linear:

$$\dot{x}_i = \sum_{k=1}^{n} a_{ik} x_k + \sum_{j=1}^{r} b_{ij} u_j + w_i \qquad (i = 1,\dots,n). \tag{1.7}$$

where a_{ik}, b_{ij}, w_i are constant values or variable functions of the time t which are assumed to be continuous. The values a_{tk}, b_{ij} and w_t are defined by the parameters of the controlled system (and perhaps by the external forces applied to the object in addition to the controlling forces).

In the sequel, for the sake of brevity, we will frequently use the matrix form of the notation. The prime in the superscript will mean transposition. Thus, for instance, the symbol h' will mean the vector row. Therefore the symbol $h'.x$ means the product of the vector row h' by the column vector x, i. e. the scalar product $\alpha = h'.x = h_1x_1 + \dots + h_nx_n$ of the vectors h and x.

The system of equations in the matrix form looks like

$$\begin{bmatrix} \dot{x}_1 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{Vmatrix} a_{11} & \dots & a_{1n} \\ \cdot & \dots & \cdot \\ \cdot & \dots & \cdot \\ \cdot & \dots & \cdot \\ a_{n1} & \dots & a_{nn} \end{Vmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{Vmatrix} b_{11} & \dots & b_{1r} \\ \cdot & \dots & \cdot \\ \cdot & \dots & \cdot \\ \cdot & \dots & \cdot \\ b_{n1} & \dots & b_{nr} \end{Vmatrix} \begin{bmatrix} u_1 \\ \cdot \\ \cdot \\ \cdot \\ u_r \end{bmatrix} + \begin{bmatrix} w_1 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{bmatrix} \tag{1.4}$$

or, shortly,

$$\dot{x} = Ax + Bu + w.$$

Formulation of the Control Problem.

One of the basic control problems is formulated as follows.

Problem 1.1. *Given the equations of motion* (1.1), *the time interval* $\lfloor t_\alpha, t_\beta \rfloor$, *the initial and the terminal value* $x^\alpha = \{x_i^\alpha\}$, $x^\beta = \{x_i^\beta\}$ *of a phase vector of the controlled object, it is required to find a possible control u(t) that takes system* (1.1) *from the state* $x(t_\alpha) = x^\alpha$ *and brings it to the state* $x(t_\beta) = x^\beta$.[44]

It is therefore required to find piecewise-continuous functions $u_j(t)$ $(j=1,\ldots,r;\ t_\alpha \le t \le t_\beta)$, such that if they are substituted into equations (1.1), the latter will have the solution $x(t,t_\alpha,x^\alpha;\ u)$ satisfying the boundary condition $x(t_\beta,t_\alpha,x^\alpha,u)=x^\beta$.

The above formulated problem can be exemplified by the problem on shaft oscillation damping. Let the rotation angles of flywheels $q_i\ (t_\alpha)$ and their angular velocities $\dot{q}_i(t_\alpha)$ be known at the initial moment of time $t = t_\alpha$, and also let it be assumed that during a time interval $t_\alpha \le t \le t_\beta$ the shaft is subjected to the action of the periodic disturbing moment $v(t)=\sin(\gamma t+\delta)$ vanishig at $t = t_\beta$. Then problem 1.1 consists in choosing controlling moments $u_1(t)$ and $u_2(t)$ which must work during the time $t_\alpha \le t \le t_\beta$ and, by the time $t = t_\beta$, must bring the shaft to the state of uniform rotation and и damp the stress, i.e. it is required to bring the object to the state $q_i(t_\beta)=0,\ \dot{q}_i(t_\beta)=0(i=1,2,3)$.

In concrete problems, as a rule we choose some value characterizing the expenditures of resources on the control process realization. Usually, it is required to achieve the desired result so that this value would not exceed a certain given limit or so that it would be minimal.

We call this value *the intensity* of control and denote by the symbol $x[u]$. It is assumed that the value $x\,[u]$ is meaningful and nonnegative for any possible control $u\,(t)$ $t_\alpha \le t \le t_\beta$.

Let us now formulate the general problem on optimal control of minimal intensity.

Problem 1.2. *Given the equations of motion* (2.1), *the time interval* $[t_\alpha,\ t_\beta]$, *the initial and the terminal value* $x^\alpha=\{x_i^\alpha\}$ *and* $x^\beta=\{x_i^\beta\}$ *of the phase vector*, and *the intensity* $x[u]$ *which is used as a control estimate*. *Among possible controls and* (t) *it is required to find an optimal control* $u^0(t)$ *that transfers the system from the state* $x(t_\alpha) = x^\alpha$ *to the state* $x(t_\beta) = x^\beta$ *and has the smallest possible intensity* $x[u]$.

Therefore Problem 1.2 is the problem of an optimal control defined by the following two conditions:

1. An optimal control $u^0(t)$ is a possible one.
2. The control $u^0(t)$ solves Problem 4.1 and possesses the property

$$x[u^0] \le x[u] \tag{1.9}$$

no matter what other possible control *u(t)* also solving Problem 1.1 could be.

1.4 Terminal Control Problems

In the last 10-15 years, new objects emerged and new problems arose, which preconditioned the development of the automatic control theory. Space vehicles, for instance, require minimal fuel consumption or minimal heating during the descent from the orbit and passage through the atmosphere. These requirements led to a rapid development of various methods of control optimization. Such problems as air traffic control over large airports, soft landing of space satellites, vertical take-off planes and helicopters, the mating of space flying objects and planes for the fuel refilling, control of a row of moving objects and many other problems made it imperative to focus attention to the methods of terminal control of objects, since these methods allow us to achieve a given phase state of the object at a given moment of time. In other words, we can, for instance, to move the object to a chosen point of the space with a given velocity vector within the desired time.

The above-mentioned problems are so important that they have always received a great deal of attention. Thus, the 4th All-Union Conference on the Control of Moving Objects was held in Tbilisi in September 5—October 5, 1968, and the 6th Symposium of IFAK dedicated to the control of objects in space was held in Tsahkadzor in 1974.

Besides, a number of fundamental works dealing with general theoretical questions have been published in the recent years in the sphere of optimal terminal control [46÷50]. However, a majority of the obtained results are related to the methods of programmed control which is calculated in the form $u=u(t)$, while practice demands the construction of automatic control systems in which the feedback principle is used, i.e. there exists a need in the synthesis of controls which are functions of the current phase state of the object.

To solve the problem of synthesis and technical realization of terminal controls it is necessary to overcome a number of difficulties indicated by the authors of the above-mentioned works.

«One of the difficult and little studied problems remains the boundary value problem connected with a necessity to bring the controlled object to a given terminal state. At the present time this boundary value problem frequently becomes the stumbling block on the path of concrete calculation of controlling forces» (N.N. Krasnovski) [48].

«The boom of the control theory prepared by the entire prehistory of its development is closely connected with the appearance of electronic computer facilities, owing to which it became worthwhile to create complex control algorithms» (N.N. Moiseyev) [49].

A.M. Letov also notes in the chapter «Terminal Control» [50] that at the present time there are no sufficiently simple methods of solution of terminal control problems.

As we see, the authors emphasize that algorithms of calculation of optimal controls are difficult and demand the use of computing facilities.

The analysis of the literature and materials of the above-mentioned two conferences has led to the following conclusion: indeed, to this day there are no simple methods of terminal control with feedback. Here simplicity is understood as the simplicity of calculation of a controlling function and the simplicity of its technical realization in automatic control systems.

Despite a great number of works on the theory of terminal state control and a vast area of important applications of this theory, the latter theory remains in the state far from satisfying the requirements of engineering practice. Though scientists have passed from general theoretical questions of optimization to the statement and solution of concrete problems of control, yet many aspects of this theory have been worked out insufficiently.

1.5 Aims and Objectives of the Investigation

As follows from the above survey, manipulating robots have found an active application in various spheres of agricultural production. However the existing methods of control of their spatial motions are cumbersome and complicated, which leads to a necessity to use sophisticated software and hardware, which in turn leads to an increase of the price of manipulating robots themselves and, in the end, to an increase of the cost of a technological process as a whole. Simple and reliable methods of optimal control have not found a wide application for the above-mentioned problems.

Proceeding from the above-said, we have formulated the following aims and objectives of the investigation:

1. To develop a new method of representation of spatial rotations of the working organs of agricultural robots;

2. To develop simple and reliable algorithms of adaptive control of terminal states of spatial rotations of the working organs of agricultural robots.

2. The Spinor Model of Spatial Rotation Kinematics [42,44]

Before we proceed to the development and synthesis of algorithms of terminal control of moving mechanical objects, it is necessary to consider a number of problems connected with the kinematics of spatial rotations, since the control of spatial rotations of mechanical systems (manipulators, joint mechanisms and so on) demands the knowledge of Euler angles that realize a given rotation. The latter problem leads in many cases to essential computational difficulties which eventually decrease the accuracy of the estimation of Euler angles, worsen the control quality of the process on the whole and, in particular, the accuracy of the final positioning of the object to be controlled. The reason for such a situation is evidently the widely used method of representation of three-dimensional rotations (the so-called basic representation of a group of three-dimensional rotations) by means of orthogonal real matrices of third order, the elements of which are trigonometric functions of Euler angles. This method, firstly, is intended for the description of individual concrete rotations with zero center (located at the origin) and, secondly, does not allow one to express Euler angles as functions of the coordinates of three points – central, initial and terminal – which define the considered rotation. It is the latter fact that creates the above-mentioned difficulties.

The solution of this problem is proposed in the works [51,52,53]. It is based on the representation of a group of three-dimensional rotations by means of unitary matrices of second order in two-dimensional complex spaces, i.e. on the so-called spinor representation. Since our further discussion is wholly based on these results, below we will give their brief description following the works we have referred to above.

2.1 Spinor Representation of Generalized Three-Dimensional Rotations

Generalized rotations are understood as a set of all possible rotations with both zero and nonzero centers which transform the initial three-dimensional point to the terminal point. The first problem that arises in this context can be formulated as follows: Given two three-dimensional points $\mathbf{x}(x^1,x^2,x^3)$ and $\mathbf{y}(y^1,y^2,y^3)$ which are the initial and terminal points of some rotation, it is required to express the dependence of the coordinates of a set of all possible centers of rotations of $\mathbf{z}(z^1,z^2,z^3)$ on the fixed coordinates of the points $\mathbf{x}(x^1,x^2,x^3)$ and $\mathbf{y}(y^1,y^2,y^3)$ and to find a set of corresponding transformations of the rotation which ensure the transformation of the point $\mathbf{x}(x^1,x^2,x^3)$ to the point $\mathbf{y}(y^1,y^2,y^3)$.

It has been mentioned above that when we speak of such transformations, we mean such representations of the rotation of a three-dimensional space that are different from the basic representation [54÷56]. If the basic representation is realized by means of real orthogonal matrices of third order which act in a three-dimensional Euclidean space, the spinor representation used in this paper is based on complex unitary matrices of second order which act in a two-dimensional linear space over the field of complex numbers.

Let L^3 be the linear Euclidean space with the orthonormalized basis $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$. To each vector $\boldsymbol{x} = x^1\boldsymbol{e}_1 + x^2\boldsymbol{e}_2 + x^3\boldsymbol{e}_3$ of the space L^3 we assign the traceless Hermitian matrix

$$X = \begin{vmatrix} x^3 & x^1 - ix^2 \\ x^1 + ix^2 & -x^3 \end{vmatrix}, \tag{2.1}$$

the elements of which are the spinor components of the vector $\boldsymbol{x}$. The replacement of the usual Euclidean components of the vector $\boldsymbol{x}$ to the spinor ones means the identification of the vector $\boldsymbol{x}$ with Hermitian functionals on the two-dimensional linear space C^2 over the field of complex numbers C. We denote by $L(C^2)$ the set of all Hermitian functionals on C^2, which is a linear three-dimensional space over the field of real numbers provided that we take Pauli matrices as basis elements. In this case, for each matrix of form (1) the expansion [57÷59]

$$X = x^1\sigma_1 + x^2\sigma_2 + x^3\sigma_3, \tag{2.2}$$

is valid, where $\sigma_1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \sigma_2 = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}, \sigma_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ are Pauli matrices.

Expansion (2.2) allows us to say that the set $L(C^2)$ is a linear three-dimensional space over the field of real numbers and therefore we can identify it with L^3. Note that to each basis of the two-dimensional space C^2 there corresponds a basis σ_1, σ_2, σ_3 of the space $L(C^2)$ (and also an orthonormalized basis $\boldsymbol{e}_1, \boldsymbol{e}_2, e_3$ due to the identification of L^3 and $L(C)$): each of the matrices σ_i is represented as some linear combination of tensor products of basis vectors of the space C^2. The said means that for any matrix $C \in C^2$, which is the transformation matrix between two bases of the space C^2, we can also define the transformation matrix between the corresponding orthonormalized bases of the space L^3.

It is important to note here that, as proved in [51], the transformation matrix of bases in C^2 is unitary.

The initially posed problem can now be reformulated in terms of the spinor space C^2 as follows [60,61]: Given two traceless matrices of Hermitial functionals

$X = \begin{vmatrix} x^3 & x^1 - ix^2 \\ x^1 + ix^2 & -x^3 \end{vmatrix}$ and $Y = \begin{vmatrix} y^3 & y^1 - iy^2 \\ y^1 + iy^2 & -y^3 \end{vmatrix}$, it is required to define

1) a family of unitary matrices $C = \begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix}$ that satisfy the equality [62÷67]

$$Y = \overline{C}^T XC; \tag{2.3}$$

2) one-dimensional subspaces that are invariant with respect to transformations given by matrices C (a set of the corresponding rotation centers).

This problem is easily extended to the case where instead of two points we consider two finite sets of points $\boldsymbol{x}_\mathbf{i}(x_i^1, x_i^2, x_i^3)$ and $\boldsymbol{y}_\mathbf{i}(y_i^1, y_i^2, y_i^3)$ i= 1,2, …, m, which correspond to rotations of a solid body.

The formulation of the second problem is now obvious: it is required to find the dependence of three Euler angles φ, ψ and θ on nine numbers $x^1, x^2, x^3, y^1, y^2, y^3, z^1, z^2$ and z^3.

Substituting the matrix C and the matrices of Hermitian functionals X and Y into the matrix equality (2.3), we obtain the following system of linear homogeneous equations with respect to the unknowns α and β:

$$\begin{aligned} x^3\alpha + \gamma\beta &= y^3\alpha - \overline{\delta\beta}, \\ \overline{\gamma}\alpha - x^3\beta &= y^3\beta + \overline{\delta}\overline{\alpha}, \end{aligned} \tag{2.4}$$

where $\gamma = x^1 + ix^2$ and $\delta = y^1 + iy^2$.

For an arbitrary α, the solution of (2.4) is

$$\beta = \frac{\overline{\gamma}\alpha - \overline{\delta}\overline{\alpha}}{x^3 + y^3} \ . \tag{2.5}$$

From (2.5) it follows that

$$\operatorname{Re}\beta = \beta_1 = \frac{\alpha_1(x^1 - y^1) + \alpha_2(x^2 + y^3)}{x^3 + y^3} \text{ and } \operatorname{Im}\beta = \beta_2 = \frac{\alpha_2(x^1 + y^1) - \alpha_1(x^2 - y^2)}{x^3 + y^3}. \tag{2.6}$$

From the condition that the matrix C ($\alpha_1^2 + \alpha_2^2 + \beta_1^2 + \beta_2^2 = 1$) is unitary we can define either $\alpha_1 = \operatorname{Re}\alpha$ or $\alpha_2 = \operatorname{Im}\alpha$. We can also act in a different way: giving α_1 and α_2 arbitrary values, we multiply the matrix C by the value $\dfrac{1}{\sqrt{|\alpha_1^2|+|\alpha_2^2|}}$, which, as is easy to see, is equivalent to the first way and is actually the normalization of the matrix C.

Thus, (2.6) defines the rotation for $\alpha \neq 0$ and $x^3 + y^3 \neq 0$.

2.2 Calculation of Euler Angles [51,52]

We can establish the correspondence between the elements of the transformation matrix $C = \begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix}$ acting in C^2 and the elements of the orthogonal real rotation matrix A in L^3, which, eventually, will allow us to solve the second problem of defining the dependence of Euler angles on the coordinates of the center, initial and terminal points of the considered rotation.

The matrix A is, by definition, the transformation matrix between two orthonormalized bases of the space L^3, and its rows are expansions of the vectors of the new basis with respect to the vectors of the previous basis. Therefore, due to the identification of the spaces $L(C^2)$ and L^3, we have

$$\overline{C}^T \sigma_i C = a_i^{i'} \sigma_{i'} \quad (i, i' = 1,2,3), \tag{2.7}$$

where σ_i are the Pauli matrices corresponding to the previous basis, $\sigma_{i'}$ are the Pauli matrices corresponding to the new basis, $\alpha_i^{i'}$ are elements of the matrix A^{-1}.

(2.7) can be represented in the expanded form as three matrix equalities

$$a_1^1 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + a_2^1 \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} + a_3^1 \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{vmatrix},$$

$$a_1^2 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + a_2^2 \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} + a_3^2 \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix} \cdot \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \cdot \begin{vmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{vmatrix},$$

$$a_1^3 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + a_2^3 \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} + a_3^3 \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \overline{\alpha} & -\beta \\ \overline{\beta} & \alpha \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \cdot \begin{vmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{vmatrix},$$

from which we easily obtain the expressions for calculating the elements of the matrix A by the elements of the matrix C, which in their turn depend, by virtue of formulas (2.6) and the condition that the matrix C is unitary, on x^1, x^2, x^3, y^1, y^2, y^3, z^1, z^2 and z^3:

$$a_1^1 = (\alpha_1^2 - \alpha_2^2) - (\beta_1^2 - \beta_2^2)\,; \;\; a_2^1 = 2(\alpha_1\alpha_2 + \beta_1\beta_2)\,; \;\; a_3^1 = 2(\alpha_2\beta_2 - \alpha_1\beta_1)\,;$$

$$a_1^2 = 2(\beta_1\beta_2 - \alpha_1\alpha_2)\,; \;\; a_2^2 = (\alpha_1^2 - \alpha_2^2) + (\beta_1^2 - \beta_2^2)\,; \;\; a_3^2 = 2(\alpha_1\beta_2 + \alpha_2\beta_1)\,;$$

$$a_1^3 = 2(\alpha_1\beta_1 + \alpha_2\beta_2)\,; \;\; a_2^3 = 2(\alpha_2\beta_1 - \alpha_1\beta_2)\,; \;\; a_3^3 = (\alpha_1^2 + \alpha_2^2) - (\beta_1^2 + \beta_2^2)\,. \qquad (2.8)$$

Expressions (2.8) make it possible to calculate the elements of the matrix A^{-1} (due to the orthogonality of $A^{-1} = A^{T}$) through the given values of the coordinates of three points – initial, terminal and center – which define the rotation.

Since, on the other hand, the matrix A can be written in the form

$$A = \begin{Vmatrix} \cos\varphi\cos\psi - \cos\theta\sin\varphi\sin\psi & -\cos\varphi\sin\psi - \cos\theta\sin\varphi\sin\psi & \sin\varphi\sin\theta \\ \sin\varphi\cos\psi + \cos\theta\cos\varphi\sin\psi & -\sin\varphi\sin\psi + \cos\theta\cos\varphi\cos\psi & -\cos\varphi\sin\theta \\ \sin\psi\sin\theta & \cos\psi\sin\theta & \cos\theta \end{Vmatrix}, \qquad (2.9)$$

where $-\pi < \varphi \leq \pi$, $0 \leq \theta \leq \pi$ and $-\pi < \psi \leq \pi$ are Euler angles, it is easy to see that, using expressions (2.8) and the corresponding elements of matrix (2.9), we can write three equations for defining the Euler angles (it should al be borne in mind that expressions (2.8) give the transposed matrix with respect to A)

$$\cos\theta = a_{33}\,; \;\; \sin\varphi\sin\theta = a_{31} \text{ and } \sin\psi\sin\theta = a_{13}\,. \qquad (2.10)$$

A Numerical Example

We will give a numerical example of the calculation of Euler angles.

Assume that we are given two arbitrary vectors of equal length $x(100; –30; 10)$ and $\boldsymbol{y}(–12; 2; 104{,}73)$ with zero rotation center. For an arbitrarily taken $\alpha = –5 + 8i$ we calculate by (6) that $\beta = –6.867 + 4.765i$. Hence, after normalization, we obtain the transformation matrix

$$C = \begin{vmatrix} -0{,}397 + 0{,}635i & -0{,}545 + 0{,}378i \\ 0{,}545 + 0{,}378i & -0{,}397 - 0{,}635i \end{vmatrix}.$$

Representing the vectors x and y by the spinor matrices

$$X = \begin{vmatrix} 10 & 100 + 30i \\ 100 - 30i & -10 \end{vmatrix} \text{ and } Y = \begin{vmatrix} 104{,}173 & -12 - 2i \\ -12 + 2i & -104{,}173 \end{vmatrix},$$

we verify the validity of equality (2.3)

$$\begin{vmatrix} -0{,}397 + 0{,}653i & -0{,}545 + 0{,}378i \\ 0{,}545 + 0{,}378i & -0{,}397 - 0{,}653i \end{vmatrix} \cdot \begin{vmatrix} 10 & 100 + 30i \\ 100 - 30i & 10 \end{vmatrix} \cdot \begin{vmatrix} -0{,}397 - 0{,}653i & 0{,}545 - 0{,}378i \\ -0{,}545 - 0{,}378i & -0{,}397 + 0{,}653i \end{vmatrix}$$

$$=\begin{vmatrix} 104{,}173 & -12-2i \\ -12+2i & -104{,}173 \end{vmatrix},$$ i.e. the thus defined transformation matrix C really realizes the rotation we want to define.

Using formulas (2.8) matrix calculated

$$A=\begin{vmatrix} -0{,}399 & -0{,}916 & 0{,}048 \\ 0{,}092 & -0{,}092 & -0{,}992 \\ 0{,}912 & -0{,}392 & 0{,}121 \end{vmatrix}.$$

It is not difficult to see that the determinant of the matrix A is equal to one and that $A^{-1}=A^T$, i.e. that is really an orthogonal matrix.

Further, we check the equality $A\mathbf{x}=\mathbf{y}$

$$\begin{vmatrix} -0{,}399 & -0{,}916 & 0{,}048 \\ 0{,}092 & -0{,}092 & -0{,}992 \\ 0{,}912 & -0{,}392 & 0{,}121 \end{vmatrix} . \begin{vmatrix} 100 \\ -30 \\ 10 \end{vmatrix} = \begin{vmatrix} -12 \\ 2 \\ 104{,}73 \end{vmatrix}.$$

Thus the matrix A performs the same rotation in L^3 as the complex matrix C does in C^2.

2.3. Kinematics Expressions for Euler Angles [53]

Having expressions (2.6) and (2.8), it is easy to calculate the Euler angles which ensure rotation of the point $x(x^1,x^2,x^3)$ to the point $y(y^1,y^2,y^3)$. If it is assumed that to the initial point $x(x^1,x^2,x^3)$ there correspond the zero Euler angles $\theta_0=\phi_0=\psi_0=0$, then the control of rotation consists in making a time-dependent change of the Euler angles from the initial values $\theta_0;\phi_0;\psi_0$ to the terminal values $\theta_f;\phi_f;\psi_f$ calculated by formulas (2.10).

In a general form, the control process can be represented as change functions of the Euler angles $\theta(t)$; $\phi(t)$; $\psi(t)$ which must satisfy the conditions

$$\theta(t_0)=0;\ \phi(t_0)=0;\ \psi(t_0)=0,$$
$$\theta(t_f)=\theta_f;\ \phi(t_f)=\phi_f;\ \psi(t_f)=\psi_f, \tag{2.11}$$

where t_0 and t_f are the initial and terminal moments of time.

The above-said naturally implies the problem on defining the control functions $\theta(t)$; $\phi(t)$; $\psi(t)$.

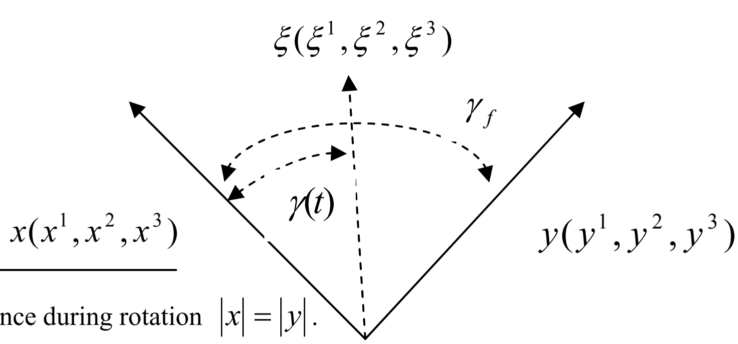
It should be emphasized that dependences $\theta(t)$; $\phi(t)$; $\psi(t)$ have a kinematics character, since they take into account neither moments, nor elastities nor any other dynamic characteristics of the process and therefore, after defining them, there arises a problem of synthesizing – on the basis of these functions – the dynamic adaptive control. This issue will be discussed below.

Fig. 2.1 shows the fixed vectors $x(x^1,x^2,x^3)$; $y(y^1,y^2,y^3)$ and the intermediate rotating vector $\xi(\xi^1;\xi^2;\xi^3)$ which at the initial moment of time $t = t_0$ coincides with the initial rotation vector $x(x^1,x^2,x^3)$ and, at the terminal moment of time $t = t_f$, with the terminal vector $y(y^1,y^2,y^3)$. The moving angle γ between the vectors $x(x^1,x^2,x^3)$ and $\xi(\xi^1;\xi^2;\xi^3)$ is equal, at the initial moment of time $t = t_{0,}$, to zero and, at the moment of time $t = t_f$, to $\gamma = \gamma_f$, where

$\gamma_f = ar\cos(\frac{(x,y)}{|x|*|y|}) = ar\cos(\frac{(x,y)}{|x|^2})$ [1]; (x,y) is the scalar product of the vectors x and y. It is obvious that the moving angle between the vectors $y(y^1,y^2,y^3)$ and $\xi(\xi^1;\xi^2;\xi^3)$ is equal to $\gamma_f - \gamma$.

Let us define the coordinates of the vector $\xi(\xi^1;\xi^2;\xi^3)$ assuming that it forms the angles γ and $\gamma_f - \gamma$ with the vectors $x(x^1,x^2,x^3)$ and $y(y^1,y^2,y^3)$ and is located in their plane. To this end, we introduce the vector $r(x^2y^3 - x^3y^2; x^3y^1 - x^1y^3; x^1y^2 - x^2y^1)$ which is the vector product of the vectors x and y. Then the above conditions can be written in the form of the following system of linear equations:

$$(\xi, r) = 0;$$
$$(\xi, x) = |x|^2 \cos\gamma;$$
$$(\xi, y) = |x|^2 \cos(\gamma_f - \gamma). \qquad (2.12)$$



[1] $|x|*|y| = |x|^2$, since during rotation $|x| = |y|$.

Fig. 2.1 The initial, terminal and intermediate vectors of spatial rotation

which is cross product of vectors x and y, then above mentioned conditions can be written as a following system of linear equation.

$$(\xi, r) = 0;$$

$$(\xi, x) = |x|^2 \cos\gamma;$$

$$(\xi, y) = |x|^2 \cos(\gamma_f - \gamma). \qquad (2.12)$$

It is not difficult to see that the vector $\xi(\xi^1;\xi^2;\xi^3)$ defined from system (2.12) satisfies the following conditions:

1. for $\gamma = 0$, $\xi(\xi^1;\xi^2;\xi^3) = x(x^1, x^2, x^3)$, which follows from the second equation of system (2.12), since in this case $(\xi, x) = |x|^2$, which is possible only provided that $\xi(\xi^1;\xi^2;\xi^3) = x(x^1, x^2, x^3)$;
2. for $\gamma = \gamma_f$, $\xi(\xi^1;\xi^2;\xi^3) = y(y^1, y^2, y^3)$, which follows from the third condition of system (2.12), since in this case $(\xi, y) = |x|^2$, which is possible only provided that $\xi(\xi^1;\xi^2;\xi^3) = y(y^1, y^2, y^3)$;
3. $|\xi| = |x| = |y|$, which follows from the second and third equations of system (2.12).

Therefore the vector $\xi(\xi^1;\xi^2;\xi^3)$ defined from system (2.12) corresponds to Fig. 2.1, i.e. it can actually be regarded as the vector rotating (condition 3) from the vector $x(x^1, x^2, x^3)$ (condition 1) to the vector $y(y^1, y^2, y^3)$ (condition 2). Note that in this case the angle γ changes in within $0 \le \gamma \le \gamma_f$.

The equations of system (12) can be written in the coordinate form as follows:

$$\xi^1 r^1 + \xi^2 r^2 + \xi^2 r^2 = 0$$

$$\xi^1 x^1 + \xi^2 x^2 + \xi^2 x^2 = |x|^2 \cos\gamma$$

$$\xi^1 y^1 + \xi^2 y^2 + \xi^2 y^2 = |x|^2 \cos(\gamma_f - \gamma)\,. \tag{2.12}$$

It is not difficult to see that its determinant is equal to

$$\Delta = \begin{vmatrix} r^1 & r^2 & r^3 \\ x^1 & x^2 & x^3 \\ y^1 & y^2 & y^3 \end{vmatrix} = |r|^2 . \tag{2.13}$$

Other determinants of Kramer's formulas for system (2.12) will be equal to

$$\Delta_1 = \begin{vmatrix} 0 & r^2 & r^3 \\ |x|^2 \cos\gamma & x^2 & x^3 \\ |x|^2 \cos(\gamma^f - \gamma) & y^2 & y^3 \end{vmatrix} = |x|^2 ((\cos(\gamma_f - \gamma)(r^2 x^3 - r^3 x^2) - \cos\gamma (r^2 y^3 - r^3 y^2))\,;$$

$$\Delta_2 = \begin{vmatrix} r^1 & 0 & r^3 \\ x^1 & |x|^2 \cos\gamma & x^3 \\ y^1 & |x|^2 \cos(\gamma_f - \gamma) & y^3 \end{vmatrix} = |x|^2 ((\cos(\gamma_f - \gamma)(r^3 x^1 - r^1 x^3) - \cos\gamma (r^3 y^1 - r^1 y^3))\,;$$

$$\Delta_3 = \begin{vmatrix} r^1 & r^2 & 0 \\ x^1 & x^2 & |x|^2 \cos\gamma \\ y^1 & y^2 & |x|^2 \cos(\gamma_f - \gamma) \end{vmatrix} = |x|^2 ((\cos(\gamma_f - \gamma)(r^1 x^2 - r^2 x^1) - \cos\gamma (r^1 y^2 - r^2 y^1))\,, \tag{2.14}$$

which allows us to obtain the coordinates of the desired vector in the following form:

$$\xi^1 = \frac{|x|^2}{|r|^2} ((\cos(\gamma_f - \gamma)(r^2 x^3 - r^3 x^2) - \cos\gamma (r^2 y^3 - r^3 y^2))$$

$$\xi^2 = \frac{|x|^2}{|r|^2} ((\cos(\gamma_f - \gamma)(r^3 x^1 - r^1 x^3) - \cos\gamma (r^3 y^1 - r^1 y^3))$$

$$\xi^3 = \frac{|x|^2}{|r|^2} ((\cos(\gamma_f - \gamma)(r^1 x^2 - r^2 x^1) - \cos\gamma (r^1 y^2 - r^2 y^1))\,. \tag{2.15}$$

In these expressions, the angle γ is an independent variable and can be treated as time function, which means that the coordinates of the vector $\xi(\xi^1;\xi^2;\xi^3)$ are also time functions. Here we assume that $\gamma(t)$ is sufficiently smooth and satisfies the conditions $\gamma(t = t_o) = 0$ u $\gamma(t = t_f) = \gamma_f$. We would like to emphasize that the problem of synthesis of spatial motion control thus reduces to defining a function $\gamma(t)$ of the concrete form, which is connected with the rotation process dynamics and will be discussed in future works. Here we assume that $\gamma(t)$ is an arbitrary function satisfying the above-given conditions. For definiteness, we assume that

$$\gamma(t) = \omega t\,, \tag{2.16}$$

where $\omega = 2\pi f$ is the constant angular velocity.

If we introduce the new vectors $r_x = (r^2x^3 - r^3x^2; r^3x^1 - r^1x^3; r^1x^2 - r^2x^1)$ and $r_y = (r^2y^3 - r^3y^2; r^3y^1 - r^1y^3; r^1y^2 - r^2y^1)$ which equal to the vector products [r×x] and [r×y], respectively, then we obtain simple expressions for the unknown coordinates as time functions

$$\xi^1(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t)r_x^1 - \cos\omega t r_y^1)\,;$$

$$\xi^2(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t)r_x^2 - \cos\omega t r_y^2)\,;$$

$$\xi^3(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - \omega t)r_x^3 - \cos\omega t r_y^3)\,. \tag{2.17}$$

As has already been noted, the vector $\xi(\xi^1;\xi^2;\xi^3)$ is a rotating vector and therefore at each moment of time it can be considered as a terminal vector of the current moment of the rotation process. If in formulas (2.12) we replace the coordinates of the point $y(y^1,y^2,y^3)$ by expressions (2.17), then we obtain representations of the parameters of the spinor matrix *C*, the orthogonal matrix *A* (formulas (2.9)) and Euler angles (2.10) in terms of time functions. Thus, we obtain a time-dependent (kinematics) representation of the rotation of the point $x(x^1,x^2,x^3)$ to the point $y(y^1,y^2,y^3)$. However first we should predetermine the matrix *C* so that at the initial moment of time the spinor equation of rotation (2.3) would have the form $X = \overline{C}^T XC$, which is evidently possible only if *C* is a unit matrix. This can be done by an appropriate choice of the parameters α_1 and α_2.

Indeed, setting $\alpha_1 = 1;\ \alpha_2 = 0$ we obtain

$$\operatorname{Re}\beta = \beta_1 = \frac{x^1 - y^1}{x^3 + y^3};\ \operatorname{Im}\beta = \beta_2 = \frac{y^2 - x^2}{x^3 + y^3}\,. \tag{2.6'}$$

Further, if in (2.6') the coordinates of the vector *y* are replaced by the coordinates of the vector ξ from (2.17), then the matrix *C* will depend on time and take the following form:

$$C(t)=\frac{1}{1+|\beta|^2}\begin{vmatrix} 1 & \frac{((\xi^1(t)-x^1)+i(\xi^2(t)-x^2))}{x^3+\xi^3(t)} \\ \frac{((x^1-\xi^1(t))-i(x^2-\xi^2(t)))}{x^3+\xi^3(t)} & 1 \end{vmatrix}, \tag{2.18}$$

where $|\beta|^2=\frac{(x^1-\xi^1(t))^2+(x^2-\xi^2(t))^2}{(x^3+\xi^3(t))^2}$; $\xi^1(t),\ \xi^2(t),\ \xi^3(t)$ are the functions of time defined in (2.17).

It is obvious that at the initial moment of time t_0 the matrix $C(t=t_o)=\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, since in that case $\gamma(t_0)=0$ and $\xi(\xi^1;\xi^2;\xi^3)=x(x^1,x^2,x^3)$.

For $t=t_f$ we have $\gamma(t_f)=\gamma_f$, $\xi(\xi^1;\xi^2;\xi^3)=y(y^1,y^2,y^3)$ and accordingly

$$C(t=t_f)=\frac{1}{1+|\beta|^2}\begin{vmatrix} 1 & \frac{((y^1-x^1)+i(y^2-x^2))}{x^3+y^3} \\ \frac{((x^1-y^1)-i(x^2-y^2))}{x^3+y^3} & 1 \end{vmatrix}. \tag{2.19}$$

From the above-said it follows that the spinor matrix of rotation (2.18) is defined correctly. But in that case the Euler angles (2.10), too, are defined correctly. They also turn out to be the functions of time, which can be easily established by (2.9), (2.10) and (2.17)

$$\theta(t)=\arccos\left(\frac{(x^3+\xi^3(t))^2-(x^1-\xi^1(t))^2-(x^2-\xi^2(t))^2}{(x^3+\xi^3(t))^2}\right);$$

$$\varphi(t)=\arcsin\left(\frac{2(x^1-\xi^1(t))}{(x^3+\xi^3(t))\sin\theta(t)}\right):$$

$$\psi(t)=\arcsin\left(\frac{2(\xi^1(t)-x^1)}{(x^3+\xi^3(t))\sin\theta(t)}\right). \tag{2.20}$$

Expressions (2.20) solve the problem we have formulated on defining the kinematics functions $\theta(t)$; $\phi(t)$; $\psi(t)$. On the other hand, it should be noted that the proposed theory allows one to reduce an actually three-dimensional problem of spatial motion control to a one-dimensional problem. Indeed, for this it is sufficient to synthesize in one way or another function $\gamma(t)$ satisfying the corresponding boundary conditions. Then it is obvious that the control process is completely defined

by the spinor matrix of rotation (2.18) and the Euler angle functions (2.20).

A Numerical Example

Let us consider a numerical example illustrating the above reasoning.

Assume that the initial vector $x(10, -45, 30)$ and the terminal vector $y(1, 20, 51.225)$ are given arbitrarily. The angle between them is equal to $\gamma_f = ar\cos(\frac{(x,y)}{|x|^2}) = 77.65^0$. Assuming for the sake of simplicity that $\omega = 1$ with a step equal to $\frac{\gamma_f}{3}$, lets us calculate in five different ways five intermediate positions of the rotating vector $\xi(\xi^1;\xi^2;\xi^3)$. Using formulas (2.17), we obtain the following coordinates of the rotating vector for three angle values (Table 2.1)

Table 2.1 Coordinates of the Rotating Vector $\xi(\xi^1;\xi^2;\xi^3)$

Angle $\gamma^0{}_f$	ξ^1	ξ^2	ξ^3	$\lvert\xi\rvert$
1	**2**	**3**	**4**	**5**
$\gamma_f/3$ **=25.88^0**	**8.48**	**–27.25**	**47.02**	**55**
$2\gamma_f/3$ **=51.77^0**	**5.2**	**–4.03**	**54.60**	**55**
γ_f **=77.65^0**	**1**	**20**	**51.23**	**55**

The procedure of verifying whether the Euler angles have been calculated consists in the following: using the obtained coordinates of the intermediate positions of the vector $\xi(\xi^1;\xi^2;\xi^3)$, for each of five angle values from Table 2.1 we should calculate the Euler angles by formulas (2.20) and the three-dimensional orthogonal matrix A of the basic representation (2.9) and then again the intermediate coordinates of the rotating vector by the formula $\xi = Ax$, where x is the initial rotation vector. The obtained values should coincide with those given in the table. The results of the corresponding calculations are presented in Table 2.2.

The matrix A was calculated for the Euler angle values calculated by formulas (2.20) and given in column 2 of Table 2.2. The coordinate values of the rotating vector $\xi(\xi^1;\xi^2;\xi^3)$ were calculated by multiplying matrix (2.9) by the initial rotation vector
$x(10, -45,30)$: $\xi = Ax$. From columns 4÷6 of Table 2.2 we see that the coordinates of the rotating vector coincide with the coordinates calculated by formulas (2.17) (Table 2.1).

Angle $\gamma^0{}_f$	Euler angles	Orthogonal matrix A	ξ^1	ξ^2	ξ^3
1	**2**	**3**	4	5	6
$\gamma_f/3$=25.88⁰	$\theta = 26.05^0$ $\varphi = 4.87^0$ $\psi = -4.87^0$	$A_1 = \begin{pmatrix} 0.999 & 8.59\times 10^{-3} & 0.037 \\ 8.59\times 10^{-3} & 0.899 & -0.438 \\ -0.037 & 0.438 & 0.898 \end{pmatrix}$	8.48	-27.25	47.02
$2\gamma_f/3$=51.77⁰	$\theta = 56.98^0$ $\varphi = 6.58^0$ $\psi = -6.58^0$	$A_2 = \begin{pmatrix} 0.995 & 0.044 & 0.09 \\ 0.044 & 0.621 & -0.783 \\ -0.09 & 0.783 & 0.616 \end{pmatrix}$	5.27	-4.03	54.60
γ_f=77.65⁰	$\theta = 77.87^0$ $\varphi = 7.88^0$ $\psi = -7.88^0$	$A_3 = \begin{pmatrix} 0.985 & 0.107 & 0.134 \\ 0.107 & 0.225 & -0.968 \\ -0.134 & 0.968 & 0.21 \end{pmatrix}$	1	20	51.23

Table 2.2. Calculation of the coordinates of the rotating vector $\xi(\xi^1;\xi^2;\xi^3)$ by means of the orthogonal matrix A (2.9)

3. Problems of Terminal State Control for Controlled Objects

3.1. General

In Chapter I, it has been shown that many motion control problems can be reduced to problems of control of terminal states of controlled objects. The main purpose of this investigation has also been stated there: to work out a simple adaptive method of terminal state control. The discussion below enables us to accomplish this task [68,69].

For simplicity, let us consider one-dimensional motion of a controlled object, the coordinate of which is γ. It is obvious that its motion is described by the following system of differential equations

$$\dot{V} = \frac{1}{m}\left(\sum_{i=1}^{n} F_i + \sum_{j=1}^{k} f_j\right);$$

$$V = \dot{\gamma}, \tag{3.1}$$

where V is the motion velocity of the controlled object under consideration; F_i (i=1, 2, …, n) are the projections of uncontrolled forces on the direction of motion, i.e. on the γ-axis; f_j (j = 1, 2, …, k) are the projections of controlled forces on the direction of motion, i.e. on the γ-axis; m is the object mass.

Uncontrolled forces may include, for example, all perturbations generated by the environment in which the motion takes place.

The terminal state control problem is formulated as follows: Given the initial phase state of the object ($\gamma_0;\dot{\gamma}_0$), it is required to transfer it – within time T -- to the terminal state ($\gamma_f;\dot{\gamma}_f$). Uncontrolled forces are functions of time t, the coordinate γ and velocity γ - $F_i = F_i(t,\gamma,\dot{\gamma})$, while controlled forces, in addition to being all these functions, are also functions of the controlling parameter α:[2] $f_j = f_j(t,\gamma,\dot{\gamma},\alpha)$. Note that the parameter α is frequently the position of the controlling element and may be a function of time. The traditional approach to the solution of the above-stated motion control problems consists in finding the functions $f_j = f_j(t,\gamma,\dot{\gamma},\alpha)$ for which solutions of system (3.1) satisfy, on the time interval [0; T], the corresponding boundary conditions. As has been said, the uniqueness of a solution is obtained by using an additional condition that solutions must supply an extremum to some specially chosen functional. Such an additional condition is frequently the requirement for a control time minimum (quick action maximum) or an energy minimum of controlling forces. There are also other kinds of functionals. Solutions obtained in this manner are of program character (the control system is open), which leads to the instability of the realized motion because of the unforeseen influence of uncontrolled forces. The development of an adaptive method demands a different approach: it is necessary to keep a continuous control over the current state of the controlled object and these demands to take respective measurements.

Let us discuss this issue in more detail. Let an optimal function $\varphi(t)$ of controlling forces be defined in some manner. Then it is obvious that the controlling parameter function $\alpha(t)$ can be defined as a solution of some differential equation, the right-hand side of which depends on a difference between the given optimal function $\varphi(t)$ of controlling forces and the current measured value of the resultant of these forces $f = f(t,\gamma,\dot{\gamma},\alpha(t))$. Assume that this differential equation has the form

$$\dot{\alpha}(t) = k_c(\varphi(t) - f(t,\gamma,\dot{\gamma},\alpha(t))\ ^3. \qquad (3.2)$$

Assume that a relation between the controlling parameter $\alpha(t)$ and the value of the current (measured) force $f = f(t,\gamma,\dot{\gamma},\alpha(t))$ can be written in the form of an inertia element of first order

[2] For example, in the case of jet engines the throttle quadrant may play the role of a controlling parameter.
[3] This control law gives astatism of third order with respect to external perturbing forces.

$$\dot{f} = (k_f \alpha(t) - f). \tag{3.3}$$

The device described by equation (3.3) is a regulator, i.e. a power unit generating the controlling force $f = f(t, \gamma, \dot{\gamma}, \alpha(t))$.

The control process is therefore described by means of the system of differential equations (3.1)÷(3.3). Knowing the synthesized function of controlling forces $\varphi(t)$, we can transfer the object from the initial state $\gamma(t_0); \dot{\gamma}(t_0)$ to the terminal state $\gamma(t_f); \dot{\gamma}(t_f)$. However here we encounter a difficulty caused by the necessity to measure controlling forces. This, obviously, can be done if these forces are separated from controlled forces during the object motion. From the practical standpoint, the latter is an unsolvable problem and this circumstance impedes the development of adaptive methods which could be applicable to problems of terminal state control.

The problem we consider here can be solved by taking a different approach [69].

A change of controlling forces brings about a change of uncontrolled forces too. All forces (uncontrolled +controlled) acting on the controlled object generate the object motion acceleration $\dot{V}$. It is obvious that $\dot{V}$ can be easily measured directly and therefore we should pose the problem on the synthesis of a controlling function in the form of acceleration $\ddot{\psi}(t)$. Then the control process reduces to the fulfillment of the equality

$$\dot{V} = \ddot{\gamma}(t), \tag{3.4}$$

where $\dot{V}$ is the measured acceleration of the object and $\ddot{\gamma}(t)$ is the given (synthesized) acceleration of the object.

Note that (3.4) is actually the equation of motion of the controlled object under the action of the controlling function $\ddot{\gamma}(t)$ and is equivalent to (3.1). This is explained by the fact that the measured acceleration of the object $\dot{V}$ takes into account changes of both uncontrolled and controlled forces. We will make an essential use of this fact in the sequel. It is not difficult to realize equality (3.4) physically if the regulator (power unit) described by the equation of an inertia element (3.3) is sufficiently powerful. In that case it becomes possible to compensate uncontrolled forces by controlled ones and to fulfill equality (3.4).

Let us assume that the relation between the given acceleration $\ddot{\gamma}(t)$ and controlling forces $f = f(t, \gamma, \dot{\gamma}, \alpha(t))$ is

$$\ddot{\gamma} = kf(t, \gamma, \dot{\gamma}, \alpha(t)) \tag{3.5}$$

where k is the proportionality coefficient.

The synthesis of a control algorithm can be reduced to some variational problem in a phase space: Given two points ($\gamma_0;\dot{\gamma}_0$) and ($\gamma_f;\dot{\gamma}_f$) in a two-dimensional phase space, it is required to derive the equation of a curve of this phase space that connects ($\gamma_0;\dot{\gamma}_0$) and ($\gamma_f;\dot{\gamma}_f$) and delivers a minimum to the next functional

$$J_F = \frac{1}{T}\int_0^T f^2(t,\gamma,\dot{\gamma},\alpha(t))dt. \tag{A}$$

The equation of the curve we want to define can be written parametrically as $\gamma = \gamma(t)\ u\dot{\gamma} = \dot{\gamma}(t)$. Then it is obvious that to the phase curve defined in this manner there corresponds the motion trajectory from the point γ_0 to the point γ_f. The initial velocity at the initial moment of time $t = t_0$ is equal to $\dot{\gamma}_0$ and at the terminal moment of time $t = T$ -- to $\dot{\gamma}_f$.

From (A) it follows that the trajectory $\gamma = \gamma(t)\ u\dot{\gamma} = \dot{\gamma}(t)$ delivering a minimum to (A) is optimal in the sense that it minimizes energetic controlling actions.

The acceleration along the optimal trajectory is the function of phase coordinates

$$\ddot{\gamma} = \varphi(\gamma,\dot{\gamma}). \tag{3.6}$$

From (3.4) and (3.5) we have

$$kf(t,\gamma,\dot{\gamma},\alpha(t)) = \varphi(\gamma,\dot{\gamma}). \tag{3.7}$$

Substituting (3.7) into (A) we obtain

$$J = \frac{1}{T}\int_0^T k_1[\varphi(\gamma,\dot{\gamma})]^2\,dt = \frac{1}{T}\int_0^T [k_1\ddot{\gamma}]^2\,dt\,, \tag{3.8}$$

where $k_1 = \dfrac{1}{k}$.

Functional (3.8) belongs to the type of functionals containing derivatives of second order and therefore its corresponding Euler equation can be written in the form

$$\frac{d^2\ddot{\gamma}}{dt^2} = 0. \tag{3.9}$$

Solution (3.9) is a third order polynomial

$$\gamma = C_0 + C_1 t + C_2\frac{t^2}{2} + C_3\frac{t^3}{6}. \tag{3.10}$$

The boundary conditions are equal:

$$t = 0;\ \gamma = \gamma_0;\ \ \dot{\gamma} = \dot{\gamma}_0, \tag{3.11}$$

$$t = T;\ \gamma = \gamma_f;\ \ \dot{\gamma} = \dot{\gamma}_f. \tag{3.12}$$

These four conditions are sufficient for defining four constants C_i (i = 0, 1, 2, 3) contained in (3.10), which completely defines an optimal trajectory.

Below we will consider some particular cases defined by various values of the boundary conditions (3.11) and (3.12).

3.2. Reduction Problem [69]

3.2.1 Controlling Function Synthesis

The reduction problem is defined by the following boundary conditions:

$$t = 0; \quad \gamma = \gamma_0; \quad \dot{\gamma} = \dot{\gamma}_0, \tag{3.13}$$

$$t = T; \quad \gamma = \gamma_f; \tag{3.14}$$

Conditions (3.13) and (3.14) mean that the object should be transferred from the initial state $\gamma = \gamma_0$ and $\dot{\gamma} = \dot{\gamma}_o$ to the state $\gamma = \gamma_f$ and at that its motion velocity should be arbitrary. In terms of variational calculus, this is a problem with moving ends.

For problems of this kind, the given boundary conditions (3.13) and (3.14) are supplemented by the so-called natural boundary condition which in our case looks like [41,70,71]

$$G_{\dot{\gamma}} - \frac{d}{dt} G_{\ddot{\gamma}} = 0, \tag{3.15}$$

where $G = k\ddot{\gamma}^2$.

Clearly,

$$G_{\dot{\gamma}} = 0 \tag{3.16}$$

and

$$G_{\ddot{\gamma}} = 2\ddot{\gamma}. \tag{3.17}$$

Condition (3.15) is reduced to the form

$$2\dddot{\gamma} = 0. \tag{3.18}$$

Differentiating (3.10) thrice, taking into account the boundary conditions (3.13), (3.14) and the natural condition (3.18), we can define C_i (i = 0, 1, 2, 3) as follows:

$$C_3 = 0; C_2 = \frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\gamma_0}{T}; C_1 = \dot{\gamma}_0; C_0 = \gamma_0. \tag{3.19}$$

Substituting (3.19) into the first and the second derivative of (3.10), we obtain the following expressions for an optimal trajectory in the phase space:

$$\gamma = \left(\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T}\right)\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0, \tag{3.20}$$

$$\dot{\gamma} = \left(\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T}\right)t + \dot{\gamma}_0. \tag{3.21}$$

The acceleration (the second derivative in (3.20)) takes the form

$$\ddot{\gamma} = \frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T}. \tag{3.22}$$

This is the law of control for the reduction problem. It means that if the acceleration of the controlled object on the time interval $[0;T]$ is assumed to be constant and equal to (3.22), then at the moment of time $t = T$ its state will satisfy the boundary conditions (3.12). However this is an open (program) law of control, i.e. the control law without feedback. Due to the possibility of direct measurements of the acceleration of a controlled object, (3.21) can be transformed to the control law with feedback. For this it suffices to assume the initial phase state to be the current one, i.e. to assume $\gamma = \gamma_0 \, u \dot{\gamma} = \dot{\gamma}_0$. In that case, the task fulfillment time should be assumed to be equal to the remaining time $T—t$. Then (3.22) takes the form

$$\ddot{\gamma} = \frac{2(\gamma_f - \gamma)}{(T-t)^2} - \frac{2\dot{\gamma}}{(T-t)}. \tag{3.23}$$

From (3.23) we see that in this case the acceleration acting on the controlled object stops to be constant and becomes dependent on the current velocity and coordinate values of the controlled object, i.e. we have the realization of control with feedback. The block-diagram of the realization of control with feedback is presented in Fig. 3.1. The measured coordinates of the current state $(\gamma; \dot{\gamma})$ are delivered to the automatic control unit (ACU), where the required value of the influencing acceleration (3.23) is computed.

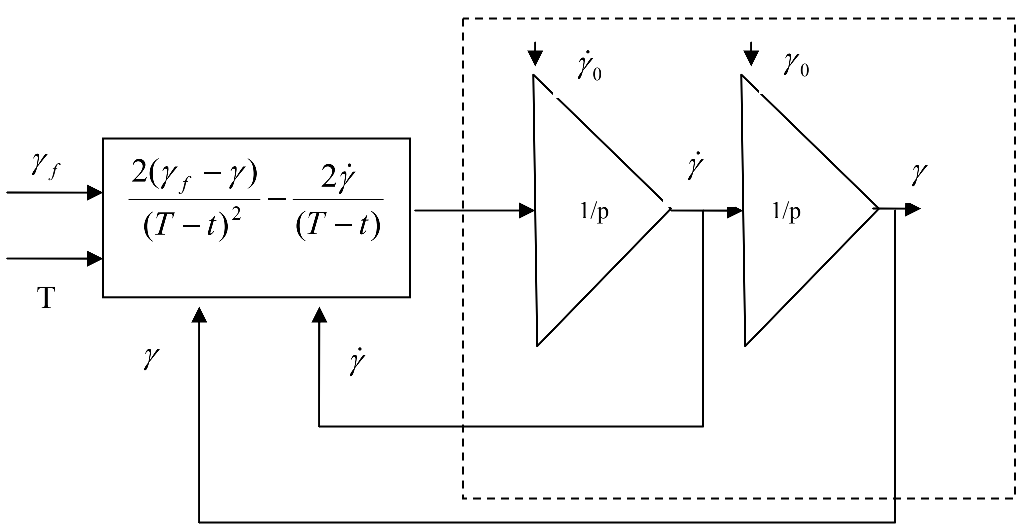


Fig. 3.1 The block-diagram of the ACU of the reduction problem.

3.2.2 Analysis of the Control Process Dynamics in the Reduction Problem

It is not difficult to see that the motion program (3.20) (open control) is accelerated motion with constant acceleration (3.22). As has already been said, the transition to the control with feedback (3.23) transforms it to motion with variable acceleration. However, in that case, for $t = T$ there arises one singularity – the denominator of the controlling function becomes equal to zero. This difficulty can be overcome by doing the following.

Assume that $T - t = \Delta T$, where ΔT is a constant time interval. From the physical standpoint this means that the target point of the reduction process is also moving, since it leaves the controlled object behind by the value ΔT. Denote its variable coordinate by γ_m. The controlling acceleration function on the time interval ΔT takes the form

$$\ddot{\gamma} = \frac{2(\gamma_m - \gamma)}{\Delta T^2} - \frac{2\dot{\gamma}}{\Delta T}, \qquad (3.24)$$

where γ *and* $\dot{\gamma}$ (the coordinate and velocity of the controlled object) are, as previously, the variable values which are functions of time.

Thus when using the left-hand side of expression (3.24) for the controlling acceleration, it is assumed that the object moves with a constant lag in time by the value ΔT from the target point γ_m and, after time T, its coordinate becomes equal to the given value $\gamma = \gamma_f$.

Now let us verify that this is really so.

We begin by noting that by analogy with (3.20) we have the following program for the control of the coordinate of the moving target point

$$\gamma_m = \left(\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T}\right)\frac{(t + \Delta T)^2}{2} + \dot{\gamma}_0(t + \Delta T) + \gamma_0 . \qquad (3.25)$$

This equation reflects the fact that the leading point γ_m leaves the controlled object behind by time ΔT.

Substituting (3.25) into (3.24) and performing some simple transformations, we obtain the following expression for the controlling acceleration

$$\ddot{\gamma} = k_0 + k_1 t + k_2 t^2 + k_\gamma^1 \gamma + k_\omega^1 \omega , \qquad (3.26)$$

where $\omega = \dot{\gamma}$ [4] is the velocity of the controlled object

$$k_0 = \frac{2\gamma_f}{T^2} + \frac{T - \Delta T}{\Delta T \cdot T}(2\gamma_0 \frac{T + \Delta T}{\Delta T \cdot T} + 2\omega_0);$$

$$k_1 = \frac{2\omega_0}{\Delta T}(\frac{1}{\Delta T} - \frac{2}{T}) + \frac{4(\gamma_k - \gamma_0)}{\Delta T \cdot T^2};$$

$$k_2 = \frac{2(\gamma_k - \gamma_0)}{\Delta T \cdot T^2} - \frac{2\omega_0}{\Delta T \cdot T};$$

$$k_\gamma^1 = -\frac{2}{\Delta T^2};$$

$$k_\omega^1 = -\frac{2}{\Delta T}.$$

Expression (3.26) is a linear non-homogeneous differential equation of second order with constant coefficients

$$\ddot{\gamma} + k_\omega \dot{\gamma} + k_\gamma \gamma = k_0 + k_1 t + k_2 t^2, \tag{3.27}$$

where $k_\gamma = -k_\gamma^1$ u $k_\omega = -k_\omega^1$.

As is known, its solution consists of two parts: a general solution of the corresponding homogeneous equation and a particular solution of the non-homogeneous equation. The first of these solutions is the so-called transitional component and the second solution is a stationary component [72÷78].

Let us first define a particular solution, i.e. a stationary component. It will be sought in the form of a polynomial of the same structure as the right-hand part

$$\dot{\gamma} = a_0 + a_1 t + a_2 t^2. \tag{3.28}$$

Substituting (3.28) into (3.27) and equating the right-hand parts, where powers *t* are assumed to be equal, we obtain the following equations for the coefficients a_i (i=0, 1, 2):

$$2a_2 + k_\omega a_1 + k_\gamma a_0 = k_0;$$

$$2k_\omega a_2 + k_\gamma a_1 = k_1;$$

$$k_\gamma a_2 = k_2. \tag{3.28}$$

(3.28) is easy to solve:

[4] We mean that the theory developed here will be used for controlling the rotation process of manipulating robots and therefore, along with the notation $\dot{\gamma}(t)$ we also use the traditional notation of angular velocity $\omega(t)$

$$a_2 = \frac{k_2}{k_\lambda};$$

$$a_1 = \frac{1}{k_\gamma}(k_1 - 2\frac{k_\omega k_2}{k_\gamma});$$

$$a_0 = \frac{1}{k_\gamma}(k_0 - \frac{2k_2}{k_\gamma} - \frac{k_\omega}{k_\gamma}(k_1 - 2\frac{k_\omega k_2}{k_\gamma})). \tag{3.29}$$

Substituting the coefficients k_i ($i = 0, 1, 2$) from (3.26) into expressions (3.29), we finally obtain

$$a_2 = \frac{(\gamma_f - \gamma_0)}{T^2} - \frac{\dot{\gamma}_0}{T};$$

$$a_1 = \dot{\gamma}_0;$$

$$a_0 = \gamma_0. \tag{3.30}$$

Hence a particular solution of the non-homogeneous equation (3.27) has the form

$$\gamma = (\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T})\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0, \tag{3.31}$$

which coincides with (3.20) and thus indeed satisfies the boundary conditions (3.13) and (3.14).

Now let define a solution of the homogeneous equation

$$\ddot{\gamma} + k_\omega \dot{\gamma} + k_\gamma \gamma = 0, \tag{3.32}$$

i.e. a transitional function of the reduction process. The characteristic equation (3.32) can be written in the form

$$\lambda^2 + k_\omega \lambda + k_\gamma = 0, \tag{3.33}$$

which, as is easily seen, has two complex-conjugate roots

$$\lambda_1 = -\frac{1}{\Delta T} + \frac{1}{\Delta T}i \text{ and } \lambda_1 = -\frac{1}{\Delta T} - \frac{1}{\Delta T}i, \tag{3.34}$$

where $i = \sqrt{-1}$.

By virtue of (3.34), a general solution of the homogeneous equation (3.32) can be written in the form

$$\gamma(t) = e^{-\frac{1}{\Delta T}t}(C_1 \cos\frac{t}{\Delta T} + C_2 \sin\frac{t}{\Delta T}), \tag{3.35}$$

which leads to a general solution of the non-homogeneous equation (3.27)

$$\gamma(t) = e^{-\frac{1}{\Delta T}t}(C_1 \cos\frac{t}{\Delta T} + C_2 \sin\frac{t}{\Delta T}) + (\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T})\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0 . \tag{3.36}$$

We also need a derivative (3.36)

$$\dot{\gamma}(t) = \omega(t) = -\frac{1}{\Delta T}e^{-\frac{1}{\Delta T}t}(C_1 \cos\frac{t}{\Delta T} + C_2 \sin\frac{t}{\Delta T}) + e^{-\frac{1}{\Delta T}t}(\frac{1}{\Delta T}C_2 \cos\frac{t}{\Delta T} - \frac{1}{\Delta T}C_1 \sin\frac{t}{\Delta T})$$
$$(\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T})t + \dot{\gamma}_0 . \tag{3.37}$$

The initial values of functions (3.36) and (3.37) defined according to the initial conditions

$$t = 0;\ \gamma = \gamma_{10};\ \ \dot{\gamma} = \dot{\gamma}_{10}, \tag{3.38}$$

allow us to define the constants C_1 and C_2

$$C_1 = \gamma_{10} - \gamma_0 \text{ и } C_2 = \Delta T(\dot{\gamma}_{10} - \gamma_0) - (\gamma_{10} - \gamma_0) \tag{3.39}$$

and thereby the final form of a solution of the differential equation (3.27)

$$\begin{aligned} \gamma(t) = e^{-\frac{1}{\Delta T}t}[(\gamma_{10} - \gamma_0)\cos\frac{t}{\Delta T} + (\Delta T(\dot{\gamma}_{10} - \gamma_0) - (\gamma_{10} - \gamma_0))\sin\frac{t}{\Delta T}] + \\ + [(\frac{2(\gamma_f - \gamma_0)}{T^2} - \frac{2\dot{\gamma}_0}{T})\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0] \end{aligned} . \tag{3.40}$$

Here we should make a remark concerning the initial conditions (3.38), since they differ from the first of the boundary conditions (3.13). The matter is that the reduction process can be started for any initial values of the coordinate and velocity of the controlled object. It is not obligatory that these values be equal to the calculated values of the coordinate and velocity of the controlled object which are given preliminarily in (3.13). If values (3.13) and (3.28) are not equal, then there occurs a transitional process defined by the exponential summand in (3.40). Otherwise, the transitional component is absent and the reduction process has to be content with the forced component, i.e. with the second summand in (3.40). It should also be noted that the transitional component has a damping character and the value ΔT plays the role of a time constant: the larger it is, the slower the damping process is, and vice versa. Thus the value ΔT can so-to-say serve as a measure of «strictness» of reduction process control.

3.3 The Acceleration Problem

3.3.1 Controlling Function Synthesis

In the acceleration problem the boundary condition (3.14) is replaced by t=T; $\dot{\gamma} = \dot{\gamma}_f$, (3.41)

which means that in this case it is required that at the given moment of time $t = T$ the velocity of the controlled object reach the given value $\dot{\gamma} = \dot{\gamma}_f$. The coordinate may have an arbitrary value [41,70].

This is again a variational problem with moving ends. The natural boundary condition (3.18) remains as before. Analogously to the reduction problem, we obtain the following values of the coefficients C_i (i = 0, 1, 2, 3) in the expression for the controlling program (3.10):

$$C_3 = 0; C_2 = \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T}; C_1 = \dot{\gamma}_0; C_0 = \gamma_0 . \tag{3.42}$$

Hence

$$\gamma = (\frac{\dot{\gamma}_f - \dot{\gamma}_0}{2T})t^2 + \dot{\gamma}_0 t + \gamma_0 , \tag{3.43}$$

$$\dot{\gamma} = (\frac{\dot{\gamma}_f - \dot{\gamma}_0}{T})t + \dot{\gamma}_0 . \tag{3.44}$$

$$\ddot{\gamma} = \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T} . \tag{3.45}$$

The last expression is the law of acceleration process control. It means that if on the time interval [0; T] the controlled object is subjected to control (3.45), then at the moment of time $t = T$ its velocity will satisfy the boundary condition (3.41), i.e. the acceleration problem will be thereby proved.

However this is again the program law of control and to make it self-correcting (adaptive) we proceed as in the case of the reduction problem, i.e. we replace the initial velocity and coordinate values by the respective current values, and the moment of time T by the difference $T - t$:

$$\ddot{\gamma} = \frac{\dot{\gamma}_f - \dot{\gamma}}{T - t} . \tag{3.46}$$

The block-diagram of the acceleration process control is shown in Fig. 3.2.

Control (3.46) has the same singularity as the law of adaptive control of the reduction process and therefore we should use an analogous method to eliminate it.

3.3.2 Analysis of the Control Process Dynamics in the Acceleration Problem

Assume that $T - t = \Delta T$, where ΔT is a constant time interval. From the physical standpoint this again means that the target point of the acceleration process is also the moving one, since it leaves behind the controlled object by the value ΔT. Denote its variable coordinate by γ_m.

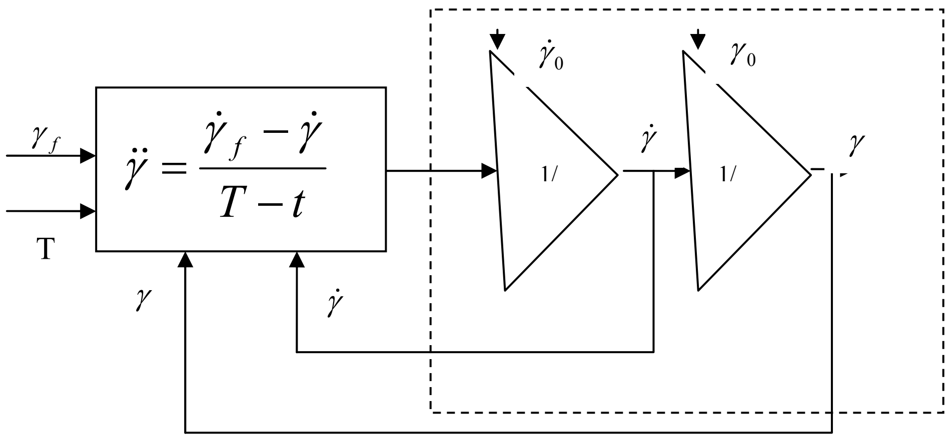


Fig. 3.2 The block-diagram of the acceleration process control

The function of controlling acceleration on the time interval ΔT will take the form

$$\ddot{\gamma} = \frac{\dot{\gamma}_m - \dot{\gamma}}{\Delta T}, \tag{3.47}$$

where $\dot{\gamma} = \omega$ is the velocity of the controlled object which is a function of time.

Thus the use of the left-hand part of expression (3.24) for the controlling acceleration means that the controlled object moves with a constant lag in time by the value ΔT from the target point γ_m and, after the time T, its velocity will be equal to the given value $\dot{\gamma} = \dot{\gamma}_f$.

Proceeding analogously to 3.2.2, we obtain the following program of velocity control of the moving target point

$$\dot{\gamma}_m = \dot{\gamma}_0 + \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T}(t + \Delta T). \tag{3.48}$$

Substituting (3.48) into (3.47) and performing some simple transformations, we obtain the following expression for the controlling acceleration

$$\ddot{\gamma} = k_0 + k_1 t + k_\omega^1 \omega, \tag{3.49}$$

where $\omega = \dot{\gamma}$ is the velocity of the controlled object;

$$k_0 = \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T} + \frac{\dot{\gamma}_0}{\Delta T};$$

$$k_1 = \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T\Delta T};$$

$$k_\omega^1 = -\frac{1}{\Delta T}.$$

Expression (3.49) is a linear non-homogeneous differential equation of second order with constant coefficients

$$\ddot{\gamma} + k_\omega \dot{\gamma} = k_0 + k_1 t, \qquad (3.50)$$

where $k_\omega = -k_\omega^1$.

Let us first define a particular solution, i.e. the stationary component (3.50). It will be sought in the form of a polynomial of the same structure as the right-hand part

$$\gamma = a_0 + a_1 t. \qquad (3.51)$$

Substituting (3.51) into (3.50) and equating the right-hand parts with the same powers *t*, we obtain the following expressions for the coefficients a_i ($i = 0, 1$):

$$2a_1 + k_\omega a_0 = k_0;$$

$$k_\omega a_1 = k_1. \qquad (3.52)$$

(3.52) is easy to prove

$$a_1 = \frac{k_1}{k_\omega};$$

$$a_0 = \frac{k_0}{k_\omega} - \frac{k_1}{k_\omega^2}. \qquad (3.53)$$

Substituting the values of the coefficients from (3.49) into the last expression, we obtain

$$a_1 = \frac{\dot{\gamma}_f - \dot{\gamma}_0}{T};$$

$$a_0 = \dot{\gamma}_0. \qquad (3.54)$$

Hence a particular solution of the non-homogeneous equation (3.49) will have the form

$$\gamma = \left(\frac{\dot{\gamma}_f - \dot{\gamma}_0}{T}\right)\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0, \qquad (3.55)$$

and its derivative will be

$$\dot{\gamma} = (\frac{\dot{\gamma}_f - \dot{\gamma}_0}{T})t + \dot{\gamma}_0, \tag{3.56}$$

which coincides with (3.44) and therefore indeed satisfies the boundary conditions (3.13) and (3.41).

Let us now define a solution of homogeneous equation

$$\ddot{\gamma} + k_\omega \dot{\gamma} = 0, \tag{3.57}$$

i.e. a transitional function of the acceleration process. The characteristic equation (3.57) can be rewritten as

$$\lambda^2 + k_\omega \lambda = 0, \tag{3.58}$$

which, as is easily seen, has two roots

$$\lambda_1 = 0 \text{ and } \lambda_1 = -k\omega = -\frac{1}{\Delta T}. \tag{3.59}$$

By virtue of (3.59) a general solution of the homogeneous equation (3.58) can be written in the form

$$\gamma(t) = C_1 + C_2 e^{-\frac{1}{\Delta T}t}, \tag{3.60}$$

which leads to a general solution of the non-homogeneous equation (3.50)

$$\gamma(t) = C_1 + e^{-\frac{1}{\Delta T}t} C_2 + (\frac{\dot{\gamma}_f - \dot{\gamma}_0}{T})\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0. \tag{3.61}$$

Derivative (3.61) is equal to

$$\dot{\gamma}(t) = -\frac{1}{\Delta T} e^{-\frac{1}{\Delta T}t} C_2 + (\frac{(\dot{\gamma}_f - \dot{\gamma}_0)}{T})t + \dot{\gamma}_0. \tag{3.62}$$

The initial values defined according to the initial conditions

$$t=0;\ \gamma = \gamma_{10};\ \ \dot{\gamma} = \dot{\gamma}_{10}, \tag{3.63}$$

allow us to define the constants C_1 and C_2

$$C_1 = \gamma_0 - \gamma_{10} \text{ and } C_2 = (\dot{\gamma}_{10} - \gamma_0) + (\gamma_{10} - \gamma_0), \tag{3.64}$$

and thereby we define a solution of the differential equation (3.50) and its derivative

$$\gamma(t) = (\gamma_0 - \gamma_{10}) + ((\dot{\gamma}_{10} - \gamma_0) + (\gamma_{10} - \gamma_0))e^{-\frac{1}{\Delta T}t} + (\frac{\dot{\gamma}_f - \dot{\gamma}_0}{T})\frac{t^2}{2} + \dot{\gamma}_0 t + \gamma_0. \tag{3.65}$$

Here we should make a remark concerning the initial conditions (3.38), since they differ from the first of the boundary conditions (3.13). The matter is that the reduction process can be started for any initial values of the coordinate and velocity of the controlled object. It is not obligatory that these values be equal to the calculated values of the coordinate and velocity of the controlled object which are given preliminarily in (3.13). If values (3.13) and (3.28) are not equal, then there occurs a transitional process defined by the exponential summand in (3.40). Otherwise, the transitional component is absent and the reduction process has to be content with the forced component, i.e. with the second summand in (3.40). It should also be noted that the transitional component has a damping character and the value ΔT plays the role of a time constant: the larger it is, the slower the damping process is, and vice versa. Thus the value ΔT can so-to-say serve as a measure of «strictness» of acceleration process control.

3.4 The Approach Problem

3.4.1 Controlling Function Synthesis

The approach problem employs four boundary conditions (3.11) and (3.12) which allow us to calculate immediately the coefficients C_i (i=0, 1, 2, 3) in the controlling function (3.10):

$C_0 = \gamma_0$;

$C_1 = \dot{\gamma}_0$;

$C_2 = \frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0)$;

$$C_3 = \frac{12}{T^3}(\gamma_0 - \gamma_f) - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0). \qquad (3.66)$$

Since for the acceleration (3.10) implies

$$\ddot{\gamma}(t) = C_2 + C_3 t, \qquad (3.67)$$

we obtain the synthesized control function

$$\ddot{\gamma}(t) = \left(\frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0)\right) + \left(\frac{12}{T^3}(\gamma_0 - \gamma_f) - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0)\right)t \text{ и} \qquad (3.68)$$

of the velocity and coordinate program

$$\dot{\gamma}(t) = \dot{\gamma}_0 + \left(\frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0)\right)t + \left(\frac{12}{T^3}(\gamma_0 - \gamma_f) - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0)\right)\frac{t^2}{2}; \qquad (3.69)$$

$$\gamma(t)=\gamma_0+\dot{\gamma}_0 t+\left(\frac{6}{T^2}(\gamma_f-\gamma_0)-\frac{2}{T}(2\dot{\gamma}_f+\dot{\gamma}_0)\right)\frac{t^2}{2}+\left(\frac{12}{T^3}(\gamma_0-\gamma_f)-\frac{6}{T^2}(\dot{\gamma}_f+\dot{\gamma}_0)\right)\frac{t^3}{6}. \quad (3.70)$$

The block-diagram of the approach problem is shown in Fig. 3.3.

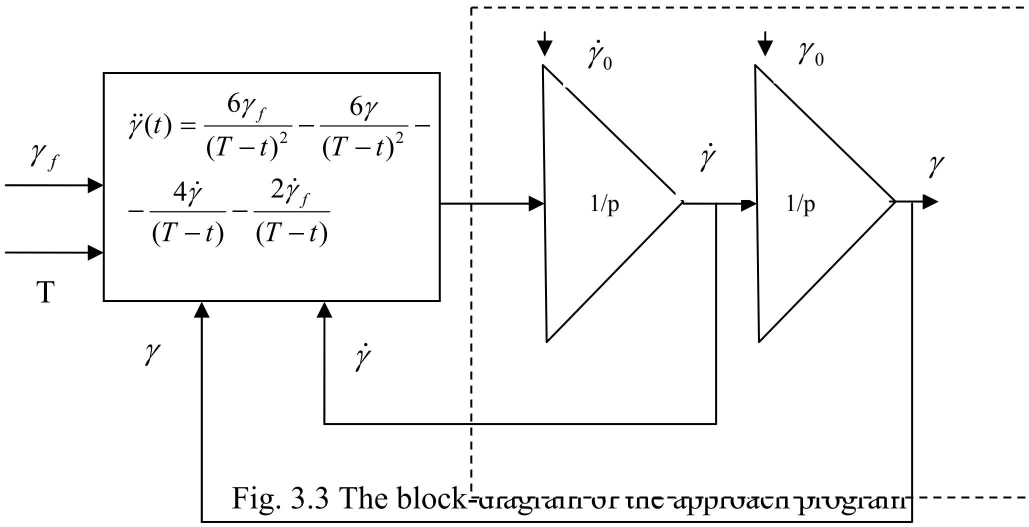


Fig. 3.3 The block-diagram of the approach program

In order to obtain an adaptive control algorithm we proceed as follows: since now the object is all the time at the initial point of time, it is assumed that t = 0 and the initial velocity and coordinate values are replaced by the respective current values, and the moment of time T is replaced by the difference $T-t$:

$$\ddot{\gamma}(t)=\frac{6\gamma_f}{(T-t)^2}-\frac{6\gamma}{(T-t)^2}-\frac{4\dot{\gamma}}{(T-t)}-\frac{2\dot{\gamma}_f}{(T-t)}. \quad (3.71)$$

Since control (3.46) again contains the same singularity as the law of adaptive control of the reduction process, we should use an analogous method of its elimination.

3.4.2 Analysis of the Control Process Dynamics in the Approach Problem

In (3.71), replace $T-t$ by ΔT, where ΔT is a constant time interval, i.e. it is again assumed that the target point of the approach process is mobile. Its variable coordinate denote by γ_m is obviously equal to

$$\begin{aligned}\gamma(t)&=\gamma_0+\dot{\gamma}_0(t+\Delta T)+\left(\frac{6}{T^2}(\gamma_f-\gamma_0)-\frac{2}{T}(2\dot{\gamma}_f+\dot{\gamma}_0)\right)\frac{(t+\Delta T)^2}{2}+\\&+\left(\frac{12}{T^3}(\gamma_0-\gamma_f)-\frac{6}{T^2}(\dot{\gamma}_f+\dot{\gamma}_0)\right)\frac{(t+\Delta T)^3}{6}\end{aligned}. \quad (3.72)$$

It is easy to see that the velocity of the mobile target point is equal to

$$\dot{\gamma}(t) = \dot{\gamma}_0 + (\frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0))(t + \Delta T) + (\frac{12}{T^3}(\gamma_0 - \gamma_f) - \\ - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0))\frac{(t+\Delta T)^2}{2}. \tag{3.73}$$

Substituting (3.72), (3.73) and $T - t = \Delta T$ into (3.71) and performing some transformations, we obtain the differential equation of second order

$$\ddot{\gamma} + K_v\dot{\gamma} + K_s\gamma = K_0 + K_1 t + K_2 t^2 + K_3 t^3, \tag{3.74}$$

where $K_0 = \frac{6\gamma_0}{\Delta T^2} + \frac{4\dot{\gamma}_0}{\Delta T} + C_2$

$K_1 = \frac{6\dot{\gamma}_0}{\Delta T^2} + \frac{4C_2}{\Delta T} + C_3$, $K_2 = \frac{3C_2}{\Delta T^2} + \frac{2C_3}{\Delta T}$,

$K_3 = \frac{C_3}{\Delta T^2}$, $K_s = \frac{6}{\Delta T^2}$, $K_v = \frac{4}{\Delta T}$,

C_2 and C_3 are defined from (3.66).

The forced component from the general solution (3.74) has the form

$$\gamma_{fr} = \frac{\Delta T^2}{6}[K_0 - \frac{2}{3}\Delta T K_1 + \frac{5}{9}\Delta T^2 K_2 - \frac{4}{9}\Delta T^3 K_3 + \left(K_1 - \frac{4}{3}\Delta T K_2 + \frac{5}{3}\Delta T^2 K_3\right)t + \\ + \left(K_2 - 2\Delta T K_3\right)t^2 + K_3 t^3] \tag{3.75}$$

The transitional component is written as follows:

$$\gamma_{tr} = e^{-\frac{2t}{\Delta T}}\left[\gamma_{10}\left(\cos\frac{\sqrt{2}}{\Delta T} + \sqrt{2}\sin\frac{\sqrt{2}}{\Delta T}t\right) + \dot{\gamma}_{10}\frac{\sqrt{2}}{2}\Delta T\sin\frac{\sqrt{2}}{\Delta T}t - \frac{\Delta T^2}{6}K_0\left(\cos\frac{\sqrt{2}}{\Delta T}t + \sqrt{2}\sin\frac{\sqrt{2}}{\Delta T}t\right) + \right. \\ + \frac{\Delta T^3}{9}K_1\left(\cos\frac{\sqrt{2}}{\Delta T}t + \frac{\sqrt{2}}{4}\sin\frac{\sqrt{2}}{\Delta T}t\right) - \frac{5}{54}\Delta T^4 K_2\left(\cos\frac{\sqrt{2}}{\Delta T}t - \frac{\sqrt{2}}{5}\sin\frac{\sqrt{2}}{\Delta T}\right) + \\ \left. + \frac{\sqrt{2}}{54}\Delta T^5 K_3\left(2\sqrt{2}\cos\frac{\sqrt{2}}{\Delta T}t - \frac{t}{2}\sin\frac{\sqrt{2}}{\Delta T}t\right)\right] = \\ = e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t + B\sin\frac{\sqrt{2}}{\Delta T}t\right), \tag{3.76}$$

where

$$A = \gamma_{10} - \frac{1}{6}\Delta T^2 K_0 + \frac{1}{9}\Delta T^3 K_1 - \frac{5}{54}\Delta T^4 K^2 + \frac{4}{54}\Delta T^5 K_3;$$

$$B = \sqrt{2}\,\gamma_{10} + \frac{\sqrt{2}}{2}\dot{\gamma}_{10}\Delta T - \frac{\sqrt{2}}{6}\Delta T^2 K_0 + \frac{\sqrt{2}}{36}\Delta T^3 K_1 + \frac{5\sqrt{2}}{270}\Delta T^4 K_2 - \frac{7\sqrt{2}}{108}\Delta T^5 K_3 .$$

It should be emphasized that in the above expressions the initial values γ_{10} *and* $\dot{\gamma}_{10}$ are not equal to the initial values given (3.11) and thus there arises the transitional process (3.76) which gets damped with time (in this case the time constant is equal to $\frac{\Delta T}{2}$), i.e. the object moves to the forced trajectory (3.75), which leads to a complete solution of the approach problem.

3.5 The Approach Problem with an Additional Condition Imposed on the Terminal Accelerations

Frequently, it is not enough to have four boundary conditions (3.11) and (3.12) of the approach problem to solve applied problems of terminal control. For example, in the case deceleration it is not enough to assume that the terminal velocity is equal to zero: for a complete stop it is necessary that the terminal acceleration, too, be equal to zero. Thus there arise an additional boundary condition (the fifth one) related to acceleration:

$$t = 0;\ \gamma = \gamma_0;\ \ \dot{\gamma} = \dot{\gamma}_0,$$
$$t = T;\ \gamma = \gamma_f;\ \ \dot{\gamma} = \dot{\gamma}_f;\ \ddot{\gamma} = \ddot{\gamma}_f . \tag{3.77}$$

It is clear that in this case the controlling function should be taken in the form of a polynomial of fourth order containing five coefficients, of which only three are to be defined, since it is obvious that the first two coefficients satisfy the first two (initial) conditions (3.77)

$$\gamma(t) = \gamma_0 + \dot{\gamma}_0 t + C_2 t + C_3 t^2 + C_4 t^3 + C_5 t^4 . \tag{3.88}$$

Calculating the first and second derivatives and substituting them into the last three equations (3.77), we obtain the values of the coefficients C_i (i = 2, 3, 4)

$$C_2 = \frac{12}{T^2}\left(\gamma_f - \gamma_0\right) - \frac{6}{T}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) + \ddot{\gamma}_f;$$
$$C_3 = \frac{48}{T^3}\left(\gamma_f - \gamma_0\right) + \frac{18}{T^2}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) - \frac{6}{T}\ddot{\gamma}_f;$$

$$C_4 = \frac{36}{T^4}\left(\gamma_f - \gamma_0\right) - \frac{12}{T^2}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) + \frac{6}{T^2}\ddot{\gamma}_f\,. \tag{3.89}$$

From (3.88) and (3.89) it follows that the controlling acceleration function has the form

$$\begin{aligned}\ddot{\gamma}(t) = {} & \frac{12}{T^2}\left(\gamma_f - \gamma_0\right) - \frac{6}{T}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) + \ddot{\gamma}_f + \left(\frac{48}{T^3}\left(\gamma_f - \gamma_0\right) + \frac{18}{T^2}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) - \frac{6}{T}\ddot{\gamma}_f\right)t + \\ & + \left(\frac{36}{T^4}\left(\gamma_f - \gamma_0\right) - \frac{12}{T^2}\left(\dot{\gamma}_f + \dot{\gamma}_0\right) + \frac{6}{T^2}\ddot{\gamma}_f\right)t^2.\end{aligned} \tag{3.90}$$

To pass to the control with feedback we proceed as in the preceding cases, i.e. we assume that in (3.90) t = 0, $\Delta T = T - t$, and replace the initial values of the phase coordinates by the respective current ones. As a result, we obtain $\ddot{\gamma}(t) = \frac{12}{(T-t)^2}\left(\gamma_f - \gamma\right) - \frac{6}{(T-t)}\left(\dot{\gamma}_f + \dot{\gamma}\right)$[5]. (3.91)

(3.90) is again the law of control with a singularity and to eliminate this singularity we proceed as before. We assume that T – t = ΔT = const and the terminal values of the phase trajectories are equal to the variable phase trajectories of the mobile target point

$$\begin{aligned}&\gamma_m(t) = \gamma_0 + \dot{\gamma}_{10}(t+\Delta T) + C_2\frac{(t+\Delta T)^2}{2} + C_3\frac{(t+\Delta T)^3}{6} + C_4\frac{(t+\Delta T)^4}{24};\\ &\dot{\gamma}_m(t) = \dot{\gamma}_{10} + C_2(t+\Delta T) + C_3\frac{(t+\Delta T)^2}{2} + C_4\frac{(t+\Delta T)^3}{6},\end{aligned} \tag{3.92}$$

where C_2, C_3 and C_4 are defined from (3.89).

Substituting functions (3.92) into (3.91) and performing simple but rather lengthy transformations, we obtain the differential equation of the approach problem which does not contain singularities

$$\ddot{\gamma} + K_\omega\dot{\gamma} + K_\gamma\gamma = \sum_{i=0}^{4} K_i t^i, \tag{3.93}$$

where

$$K_0 = \frac{12\gamma_0}{\Delta T^2} + \frac{6\omega_0}{\Delta T} + C_2;$$
$$K_1 = \frac{12\omega_0}{\Delta T^2} + \frac{6C_2}{\Delta T} + C_3$$

$$K_2 = \frac{6C_2}{\Delta T^2} + \frac{3C_3}{\Delta T}C_4;$$

[5] The terminal acceleration value is assumed to $\ddot{\gamma}_f = 0$, which is natural for the deceleration (stopping) problem.

$$K_2 = \frac{2C_3}{\Delta T^2} + \frac{2C_4}{\Delta T};$$

$$K_4 = \frac{C_4}{\Delta T^2};$$

$$K_\gamma = -\frac{12}{\Delta T^2};$$

$$K_\omega = -\frac{6}{\Delta T}.$$

Let us define the transitional and stationary components of equation (3.93). A particular solution of the non-homogeneous equation will be sought in the form $\gamma = \sum_{i=0}^{4} a_i t^i$, (3.94)

where a_i are the coefficients we want to define.

Differentiating (3.94) twice, substituting into (3.93) and equating the coefficients at equal powers t, we obtain a system of equations with respect to the desired coefficients a_i ($i = 0,\ldots,4$)

$$K_o = 2a_2 + K_\omega a_1 + K_\gamma a_0;$$

$$K_1 = 6a_3 + 2a_2\ K_\omega + a_1 K_\gamma;$$

$$K_2 = 12a_4 + 3a_3\ K_\omega + a_2 K_\gamma;$$

$$K_3 = 4a_4 K_\omega + a_3 K_\gamma;$$

$$K_4 = a_4\ K_\omega, \qquad (3.95)$$

from which they are defined quite easily:

$$a_4 = -\frac{K_4}{K_\gamma};$$

$$a_2 = \frac{K_2 + 3K_w a_3 - 12a_4}{K_\gamma};$$

$$a_1 = \frac{K_1 + 2K_w a_2 - 6a_3}{K_\gamma};$$

$$a_0 = \frac{K_w a_1 - 2a_2}{K_\gamma}. \qquad (3.96)$$

Expressions (3.96) define the stationary component of the approach process with the given terminal (zero) acceleration value.

The transitional component (a general solution of the non-homogeneous equation (3.93)) is likewise easy to write:

$$\gamma_{tr}(t)=e^{-\frac{K_w t}{2}}(s_1\cos\beta t+s_2 Sin\beta t), \qquad (3.97)$$

where $\beta=\sqrt{k_\gamma-\left(\frac{k_\omega}{2}\right)^2}$, s_1, s_2 are the constants we want to define.

A complete solution of the differential equation (3.93) can now be written as a sum of the transitional and the stationary process

$$\gamma(t)=\gamma_{tr}(t)+\gamma_{tr}(t)=e^{-\frac{K_\omega t}{2}}(s_1\cos(\beta t)+s_2\sin\beta t)+\sum_{i=0}^{4}a_i t^i \quad . \qquad (3.98)$$

To define the constants s_1 and s_2 we use the initial conditions (3.38) and derivative (3.98), which gives the following expressions for the sought constants

$$C_1=\gamma_{10}-a_0;\ C_2=\left(\dot{\gamma}_{10}-a_1 K_\omega\frac{C_1}{2}\right)\frac{1}{\beta} \qquad (3.99)$$

and eventually the final expression for a complete solution of (3.93).

$$\gamma(t)=e^{-\frac{K_\omega t}{2}}((\gamma_{10}-a_0)\cos\beta t+(\dot{\gamma}_{10}-a_1K_\omega\frac{(\gamma_{10}-a_0)}{2})\sin\beta t)+\sum_{i=0}^{4}a_i t^i \ . \qquad (3.100)$$

The transitional process (3.97) gets damped with time (the time constant is equal to $\frac{K_\omega}{2}$), i.e. the object moves to the forced trajectory (3.94).

The velocity of the controlled object is equal to

$$\begin{aligned}\dot{\gamma}(t)=e^{\frac{K_\omega}{2}}(-\tfrac{K_\omega}{2}(\gamma_{10}-a_0)\cos\beta t+\left(\omega_{10}-a_1K_\omega\frac{\gamma_{10-}a_0}{2}\right)\sin\beta t+\\+\left((\gamma_{10}-a_0)\beta sisn\beta t+\left(\omega_{10}-a_1K_\omega\left(\tfrac{\gamma_{10-}a_0}{2}\right)\right)\beta\cos\beta t\right)+a_1+2a_2t+3a_3t^2+4a_4t^3\end{aligned} \quad . \qquad (3.101)$$

Substituting the value $t = T$ into the stationary solution of equation (3.93) and into its derivative, it is not difficult to see that they indeed satisfy the boundary conditions (3.12) provided that the terminal acceleration is equal to zero, which solves the posed problem on the terminal state control in the approach problem.

4. Control of Terminal States of Spatial Rotations of Robot-Manipulators

The spinor model of the kinematics of spatial rotations developed on the basis of spinor representation of generalized spatial rotations (Chapter II) and the methods of the control theory of terminal states of motion of mechanical objects (Chapter III) made it possible to create simple methods of controlling terminal states of spatial rotations of robot-manipulators.

Here we would like to repeat what has already been stated previously (see subsection 2.3): the theory developed in Chapter II has enabled us to reduce the three-dimensional problem of spatial motion control to the one-dimensional problem because we have defined the coordinates of the rotating vector (2.17) as functions of one rotation angle lying in the rotation plane[6]. It is obvious that the trajectories corresponding to this kind of rotations consist of three natural stages [79,80,81]: acceleration, uniform rotation[7] and deceleration for the control of which we will use the results of Chapter III.

Let us consider the following problem. It is required to bring by means of rotation a mechanical object of control (for instance, a gripping device or a spherical link) with coordinates $x(x^1, x^2, x^3)$ to the point of a three-dimensional space with coordinates $y(y^1, y^2, y^3)$. As has been shown in Subsection 2.3, an intermediate rotating vector $\xi(\xi^1;\xi^2;\xi^3)$ performs rotation by an angle defined by the terminal and initial points of rotation - $\gamma_f = ar\cos(\frac{(x,y)}{|x|*|y|}) = ar\cos(\frac{(x,y)}{|x|^2})$.

In the same subsection we have obtained the kinematics expressions for the rotating vectors.

where $r = (x^2y^3 - x^3y^2; x^3y^1 - x^1y^3; x^1y^2 - x^2y^1)$, $\quad r_x = (r^2x^3 - r^3x^2; r^3x^1 - r^1x^3; r^1x^2 - r^2x^1)$;

$$r_y = (r^2y^3 - r^3y^2; r^3y^1 - r^1y^3; r^1y^2 - r^2y^1). \qquad (2.17')$$

[6] The rotation plane is defined by three principal points of each rotation: central, initial and terminal.

[7] The segment corresponding to uniform rotation may be zero.

It should be emphasized that, unlike (2.17), in expressions (2.17′) we have used the rotation angle function $\gamma(t)$ satisfying the following quite obvious condition: $0 \le \gamma(t) \le \gamma_f = ar\cos(\frac{(x,y)}{|x|^2})$.

It is obvious that this function defines in fact the motion dynamics of the object of control and in the theory of terminal control it is therefore the main sought for function. The preceding chapter was entirely dedicated to the solution of problems of this kind in the general case. Now it remains to use the obtained general results to solve the concrete problems of terminal control of spatial rotations of multimember mechanisms with spherical and rotational pairs.

Let us divide the interval $[0;\gamma_f]$ into three segments: $[0;\alpha_1\gamma_f]$ $[\alpha_1\gamma_f;\alpha_2\gamma_f]$ $[\alpha_2\gamma_f;\gamma_f]$, where $\alpha_2,\alpha_1 < 1\ u\ \alpha_2 \ge \alpha_1$. It is clear that the first segment corresponds to the beginning of the motion process, the second segment to uniform motion and, finally, the third subinterval to deceleration. If we give $\alpha_2 = \alpha_1$, then there will be no uniform motion (the length of the second segment is equal to zero), i.e. the initial stage of motion is immediately followed by the deceleration stage. This is evidently the most economical case, but in what follows we will all the same consider all three phases of spatial rotation control [82÷84].

4.1 Control in the Initial Rotation Stage

From the standpoint of dynamics, the initial process of rotation means that the object of control which is at rest must be accelerated to the desired velocity $\dot{\gamma}_f$. It might seem from this definition that in this case we should use the results of the solution of the acceleration control problem (3.3), but the matter is that if we want to finish the initial stage of motion in the right-hand end of the segment $[0;\alpha_1\gamma_f]$, then, certainly, we should use the methods of the approach problem which take into account all boundary conditions.

The boundary conditions of the initial rotation states are as follows:

$$t = 0;\ \gamma(0) = \gamma_0;\ \dot{\gamma}(0) = 0$$

$$t = T;\ \gamma(T) = \alpha_1\gamma_f;\ \dot{\gamma}(T) = \dot{\gamma}_f, \tag{4.1}$$

where T is the given duration time of the initial rotation stage.

Substituting sum (3.75) into (3.76) (the sum of the transient and stationary functions of the approach process control) we obtain the coordinates of the rotating vector as functions of time which represent the dynamics of the initial rotation stage process

$$\xi^1(t)=\frac{|x|^2}{|r|^2}(\cos(\gamma_f-e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)-\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+$$
$$+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])r_x^1-\cos(e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)+$$
$$+\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+$$
$$+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])\cdot r_y^1)$$

$$\xi^2(t)=\frac{|x|^2}{|r|^2}(\cos(\gamma_f-e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)-\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+$$
$$+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])r_x^2-\cos(e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)+$$
$$+\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+$$
$$+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])\cdot r_y^2)$$

$$\xi^3(t)=\frac{|x|^2}{|r|^2}(\cos(\gamma_f-e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)-\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+$$
$$+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])r_x^3-\cos(e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)+ \tag{4.2}$$
$$+\frac{\Delta T^2}{6}[K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3+\left(K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3\right)t+$$
$$+\left(K_2-2\Delta TK_3\right)t^2+K_3t^3])\cdot r_y^3)$$

where all parameters are defined in (3.75) and (3.76).

It is not difficult to define the functions of rotation velocity coordinates of the intermediate vector. For this we introduce the notation

$$a_0=K_0-\frac{2}{3}\Delta TK_1+\frac{5}{9}\Delta T^2K_2-\frac{4}{9}\Delta T^3K_3;\ a_1=K_1-\frac{4}{3}\Delta TK_2+\frac{5}{3}\Delta T^2K_3;$$
$$a_2=K_2-2\Delta TK_3;\ a_3=K_3.$$

Besides, taking into account that

$$e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t\right)'=-\frac{2}{\Delta T}e^{-\frac{2}{\Delta T}}(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t)+\frac{\sqrt{2}}{\Delta T}e^{-\frac{2t}{\Delta T}}\cdot$$
$$\cdot\left(B\cos\frac{\sqrt{2}}{\Delta T}t-A\sin\frac{\sqrt{2}}{\Delta T}t\right),$$

we obtain the expression for a derivative of the rotation angle function

$$\dot{\gamma}(t)=-\frac{2}{\Delta T}e^{-\frac{2}{\Delta T}}(A\cos\frac{\sqrt{2}}{\Delta T}t+B\sin\frac{\sqrt{2}}{\Delta T}t)+\frac{\sqrt{2}}{\Delta T}e^{-\frac{2t}{\Delta T}}\cdot$$
$$\cdot\left(B\cos\frac{\sqrt{2}}{\Delta T}t-A\sin\frac{\sqrt{2}}{\Delta T}t\right)+a_1+2a_2t+3a_3t^2, \quad (4.3)$$

and, finally, the velocity functions of the rotating vector $\xi(t)$

$$\dot{\xi}^1(t)=\frac{|x|^2}{|r|^2}\dot{\gamma}(t)(\sin(\gamma_f-\gamma(t))r_x^1-\sin\gamma(t)r_y^1);$$

$$\dot{\xi}^2(t)=\frac{|x|^2}{|r|^2}\dot{\gamma}(t)(\sin(\gamma_f-\gamma(t))r_x^2-\sin\gamma(t)r_y^2);$$

$$\dot{\xi}^2(t)=\frac{|x|^2}{|r|^2}\dot{\gamma}(t)(\sin(\gamma_f-\gamma(t))r_x^3-\sin\gamma(t)r_y^3) \quad (4.4)$$

The rotation process in the initial stage was modeled by means of the MathCAD software. As initial data we used we used the values from Subsection 2.4.

Numerical Example

As the initial and terminal vector we took $x(10, -45, 30)$ and $y(1, 20, 51.225)$. It is obvious that the angle between them is equal to $\gamma_f = ar\cos(\frac{(x,y)}{|x|^2}) = 77.65^0$. As shown in Table 2.1, the angle of rotation $\gamma_f = 77.65^0$ was divided into three equal angles $\gamma_f/3$ = 25.88^0; $2\gamma_f/3$ = 51.77^0 and γ_f = 77.65^0, i.e. in that case $\alpha_1=\frac{1}{3}$ *and* $\alpha_2=\frac{2}{3}$. In what follows we will use the angle values expressed in terms of radians; therefore $\gamma_f/3$ = 0.452; $2\gamma_f/3$ = 0.904 and γ_f =

1.355. Let us assume that the angular velocity is equal to $\omega = 1$ and the rotation time is also $T = 1$ sec. We also assume that all three rotation stages are of equal duration, i.e. $T_1 = \frac{T}{3} = 0.333$ sec.

Since now we are considering the initial rotation stage, the boundary conditions are written as follows:

t=0; $\gamma_0 = 0$; $\dot{\gamma}_0 = 0$,

t=T$_1$; $\gamma_f = 0.452$; $\dot{\gamma}_f = \omega_f = 1$. (4.5)

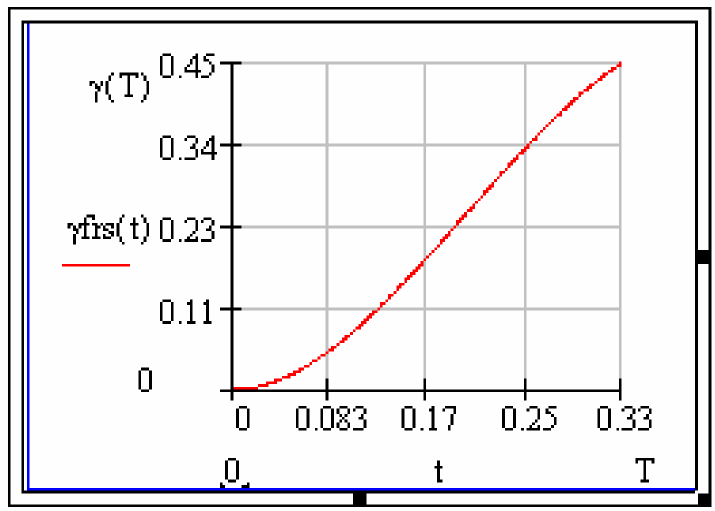
Using the results of Subsection 3.4 (transient and forced solutions of equation (3.74)) and also expressions (4.2) and (4.3), we obtain the dynamic characteristics of the control in the initial rotation stage which are given in Figs. 4.1 and 4.2.

From the analysis of the curves shown in these figures we see that the control completely satisfies the given boundary conditions. This conclusion can be easily verified by calculating the angle of rotation at the moment of time T_1 by the expression

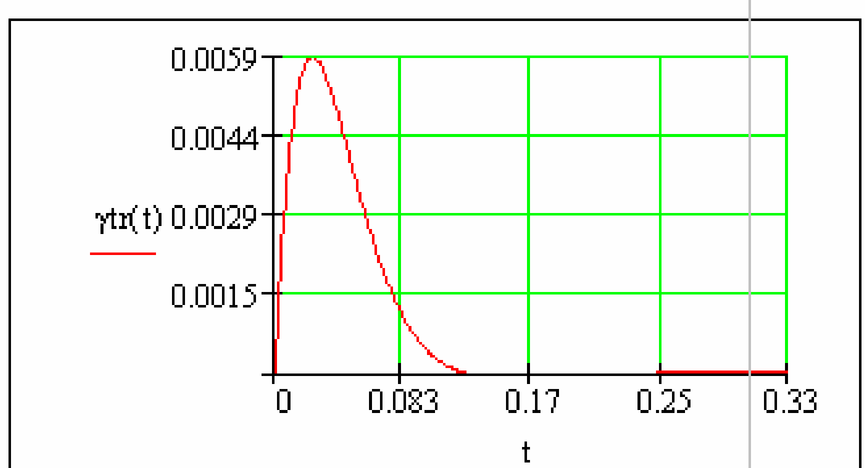
$$\cos\gamma(T_1) = \frac{\sum_{i=1}^{3} x_i \xi_i(T_1)}{|x||\xi(T_1)|}, \qquad (4.6)$$

from which we obtain $\gamma(T_1) = \gamma_f = 0.452$. The latter fact is important for yet another reason: it shows the correct consistency of the terminal control methods and the spinor method of representation of spatial solutions: expressions (4.2) containing the dynamic functions of the rotation angle. Note that by direct calculation it can be easily verified that $|x| = |\xi(0)| = |\xi(T_1)| = 55$, which tells us that the method really realizes rotation.

Finally, let us comment on the character of the curves. Fig. 4.1 b shows the presence of a transient process, but it is two orders weaker that the forced component (Fig. 4.1,a) and soon damps down. The transient component of the angular velocity also damps down soon (Fig. 4.2,b), but its order is comparable with the order of the forced component (Fig. 4.2,a). Weak deflections of the phase trajectory (Fig. 4.3) and the total velocity (the sum of the transient and forced components calculated by expression (4.3), (Fig.4.2,b)) is a result of the transient process.



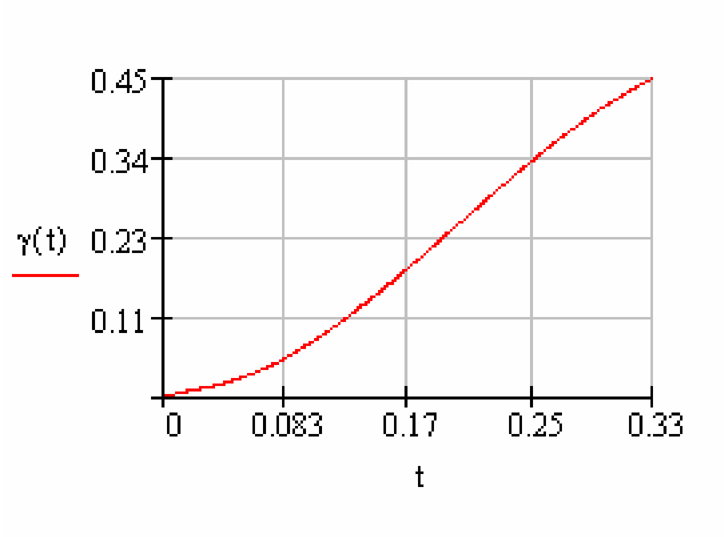
a)



b)

Fig. 4.1 The initial motion segment: rotation angle values as functions of time:

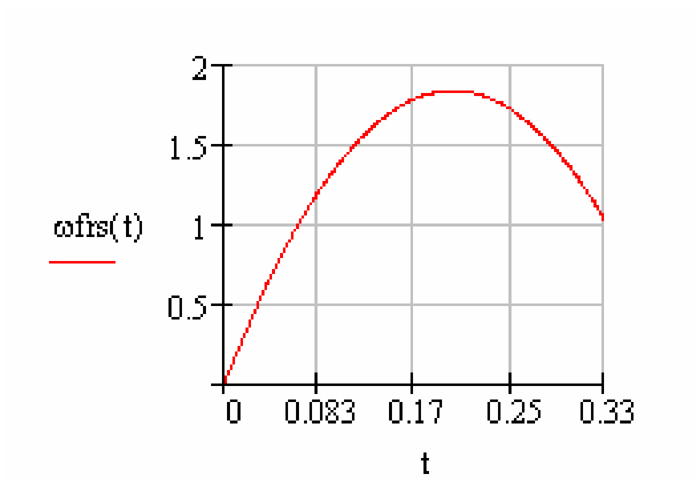
a) the forced component; b) the transient component.



c)

Fig. 4.1 The initial motion segment: rotation angle values as functions of time:

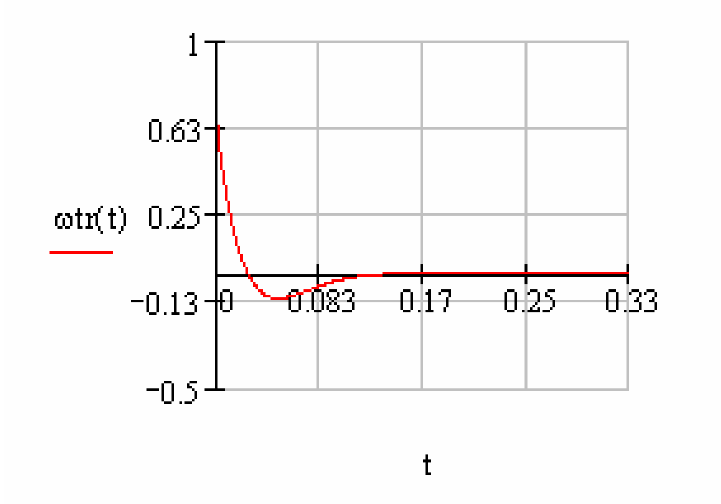
c) the complete solution: the sum of the forced and transient components.



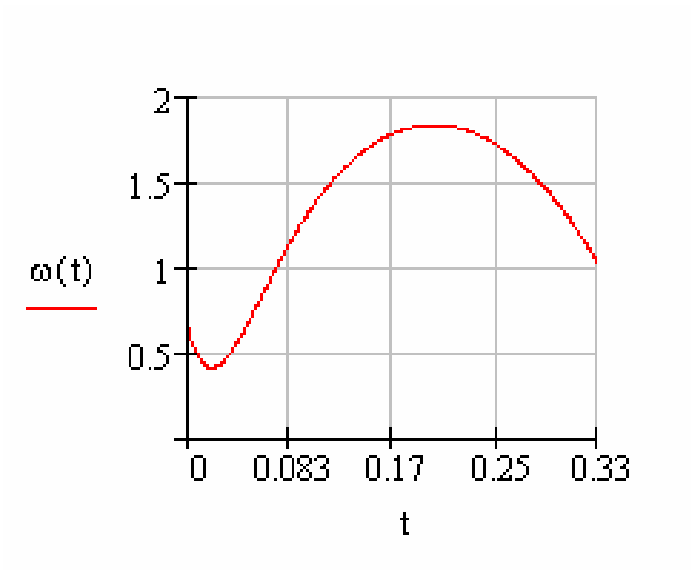
a)

Fig.. 4.2 The initial motion segment: the angular velocity value as a function of time:

a) the forced component



b)



c)

Fig. 4.2 The initial motion segment: the angular velocity value as a function of time:

b) the transient component;

c) the complete solution: the sum of the forced and transient components

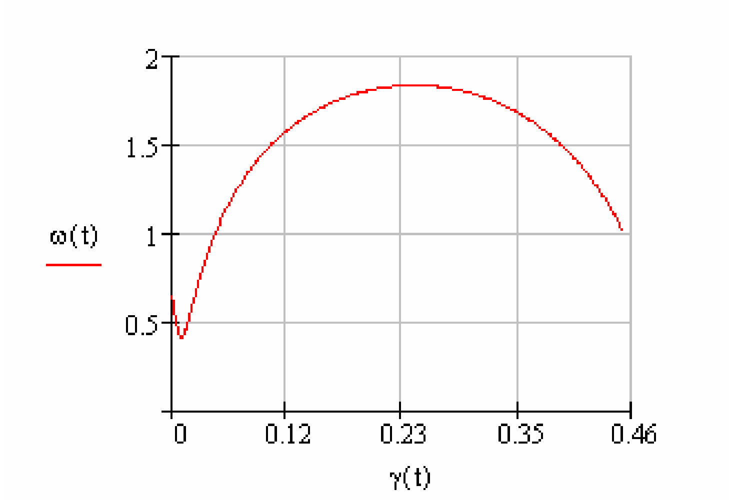


Fig. 4.3. The initial motion segment: the phase trajectory

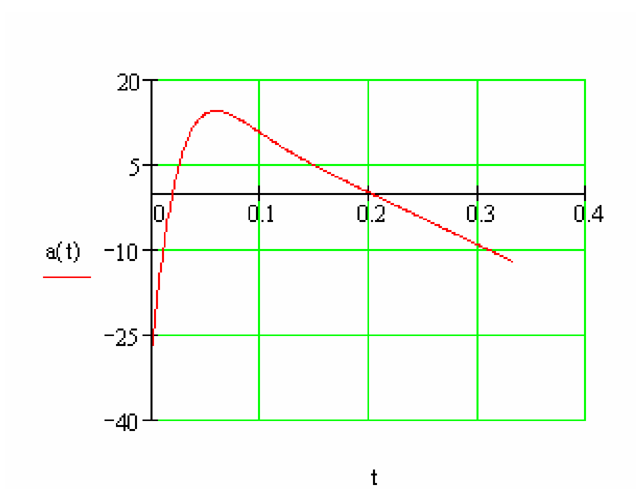


Fig. 4.4. The initial motion segment: acceleration as a function of time

4.2 Control in the Uniform Rotation Stage

In this case the control is not changed, i.e. the same equations and relations are used as in the initial motion stage.

We only change the boundary conditions

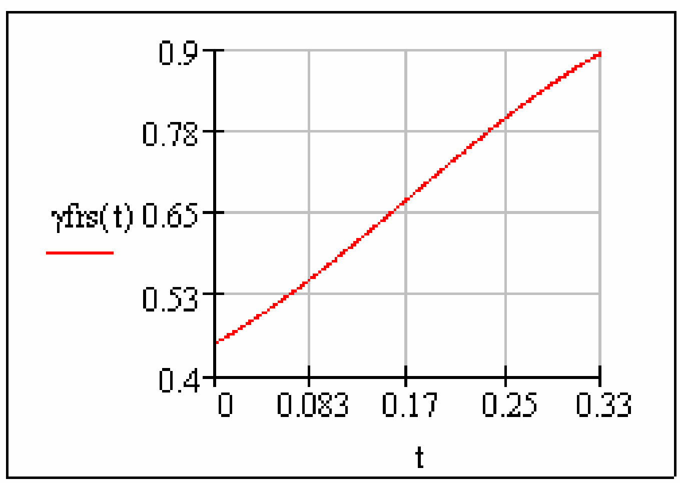
$$t = 0;\ \gamma_0 = 0.452;\ \ \dot{\gamma}_0 = 1,$$

$$t = T_1;\ \gamma_f = 0.904;\ \ \dot{\gamma}_f = \omega_f = 1. \qquad (4.7)$$

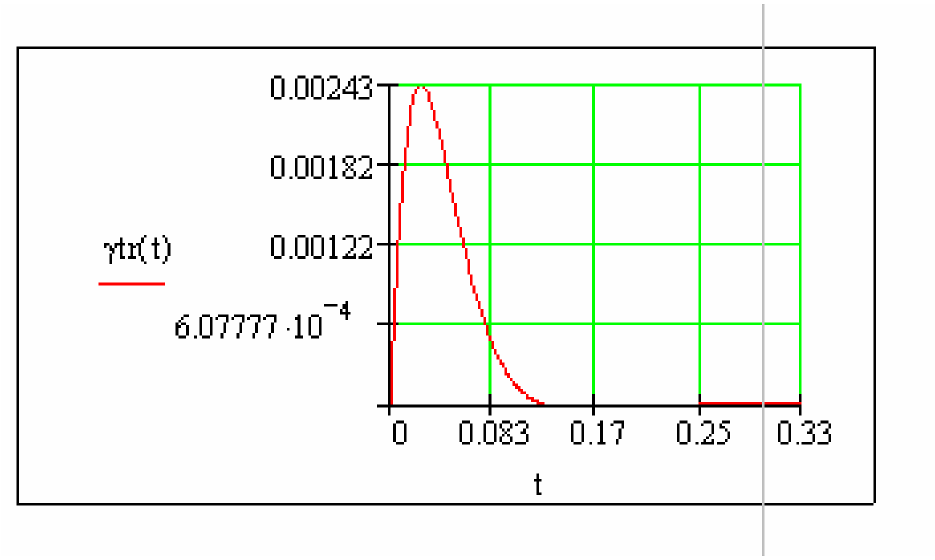
Figs. 4.5÷4.8 show the dynamic characteristics of the control process on the uniform rotation segment. Again we clearly see that the control satisfies the boundary conditions: at the end of the control period T = 0.33 sec. the controlled object really has the given angular coordinate $\gamma_f = 0.904$ and the velocity $\dot{\gamma}_f = 1$.

Though the transient process takes place, the transient component for the angular coordinate function is insignificant (Fig. 4.5,b), while the velocity function (Fig..4.6,b) is comparable with the forced (Fig. 4.6,a) and total (Fig. .4.6,c) components.

From expression (4.6) it follows that at the end of the segment $\gamma(T_1) = \gamma_f = 0.904$ and that $|x| = |\xi(0)| = |\xi(T_1)| = 55$.



a)

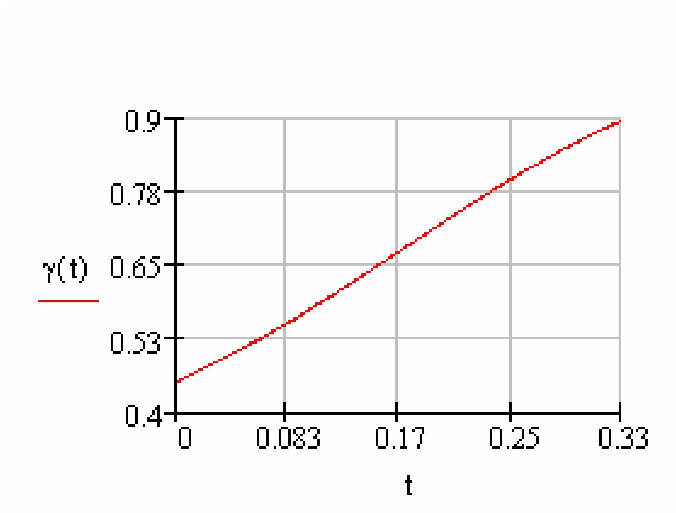


b)

Fig. 4.5 The uniform rotation segment: the rotation angle value as a function of time:

a) the forced component;

b) the transient component



c)

Fig. 4.5 The uniform rotation segment: the rotation angle value as a function of time:

c) the complete solution: the sum of the forced and transient components

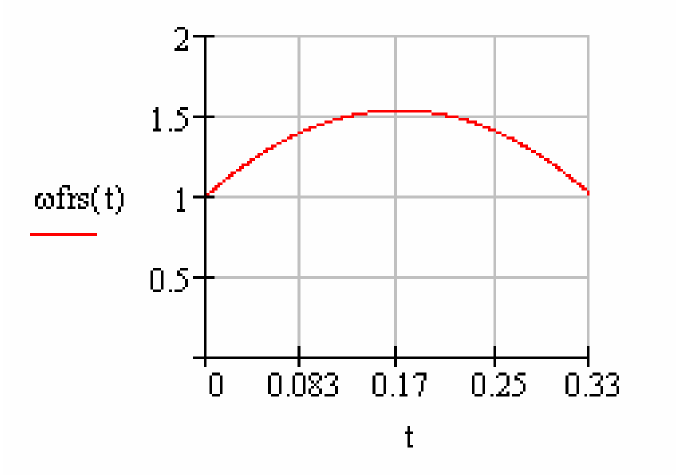
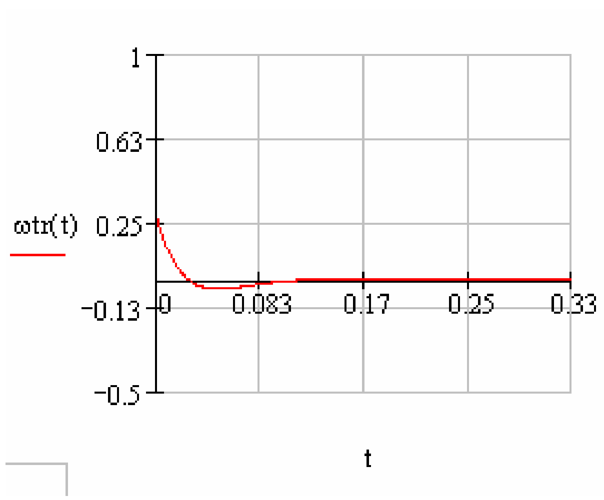
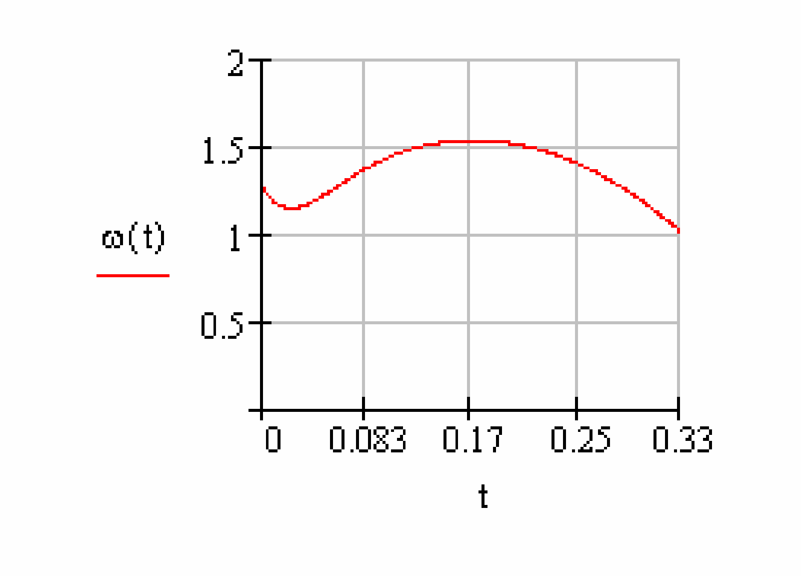


Fig. 4.6 The uniform rotation segment: the angular velocity value as a function of time: a) the forced component



b)



c)

Fig. 4.6 The uniform rotation segment: the angular velocity value as a function of time: b) the transient component;

c) the complete solution: the sum of the forced and transient components

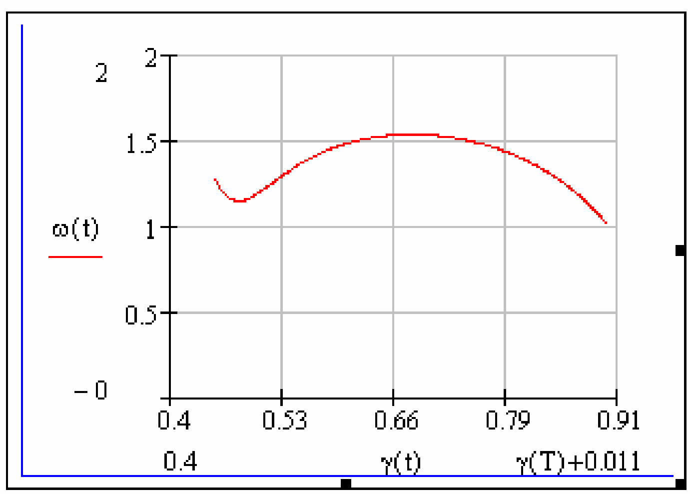


Fig. 4.7. The uniform rotation segment: the phase trajectory

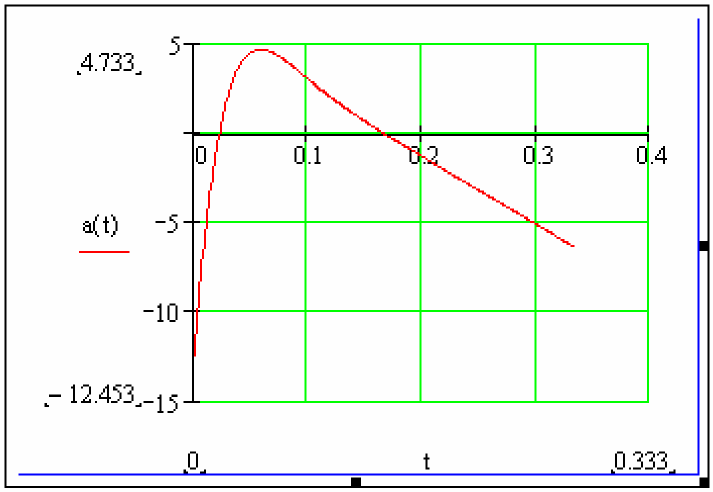


Fig. 4.8. The uniform rotation segment: acceleration as a function of time.

4.3 Deceleration

For the deceleration process ending in a complete stop we need to use the problem with five conditions since it is clear that at the end of the rotation process the acceleration must be equal to zero. Therefore the boundary conditions (3.77) take the following form:

$$t = 0;\ \gamma = 0.904;\ \ \dot{\gamma} = 1,$$
$$t = T;\ \gamma = 1.355;\ \ \dot{\gamma} = 0;\ \ddot{\gamma} = 0. \tag{4.8}$$

Substituting (3.100) into (2.17) we obtain the following expressions for the rotating vector coordinates as functions of time:

$$\xi^1(t) = \frac{|x|^2}{|r|^2}(\cos(\gamma_f - e^{-\frac{K_\omega t}{2}}((\gamma_{10} - a_0)\cos\beta t +$$
$$+(\dot{\gamma}_{10} - a_1 K_\omega\frac{(\gamma_{10} - a_0)}{2})\sin\beta t) + \sum_{i=0}^{4} a_i t^i)r_x^1 - \cos(e^{-\frac{K_\omega t}{2}}((\gamma_{10} - a_0)\cos\beta t + ;$$
$$+(\dot{\gamma}_{10} - a_1 K_\omega\frac{(\gamma_{10} - a_0)}{2})\sin\beta t) + \sum_{i=0}^{4} a_i t^i)r_y^1);$$

$$\xi^2(t)=\frac{|x|^2}{|r|^2}(\cos(\gamma_f-e^{-\frac{K_\omega t}{2}}((\gamma_{10}-a_0)\cos\beta t+$$

$$+(\dot{\gamma}_{10}-a_1K_\omega\frac{(\gamma_{10}-a_0)}{2})\sin\beta t)+\sum_{i=0}^{4}a_it^i)r_x^2-\cos(e^{-\frac{K_\omega t}{2}}((\gamma_{10}-a_0)\cos\beta t+$$

$$+(\dot{\gamma}_{10}-a_1K_\omega\frac{(\gamma_{10}-a_0)}{2})\sin\beta t)+\sum_{i=0}^{4}a_it^i)r_y^2);$$

$$\xi^3(t)=\frac{|x|^2}{|r|^2}(\cos(\gamma_f-e^{-\frac{K_\omega t}{2}}((\gamma_{10}-a_0)\cos\beta t+$$

$$+(\dot{\gamma}_{10}-a_1K_\omega\frac{(\gamma_{10}-a_0)}{2})\sin\beta t)+\sum_{i=0}^{4}a_it^i)r_x^3-\cos(e^{-\frac{K_\omega t}{2}}((\gamma_{10}-a_0)\cos\beta t+ \qquad (4.9)$$

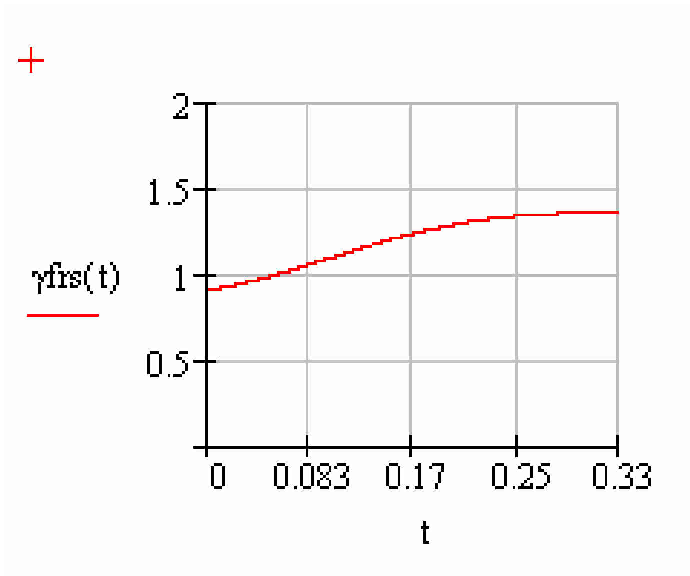
$$+(\dot{\gamma}_{10}-a_1K_\omega\frac{(\gamma_{10}-a_0)}{2})\sin\beta t)+\sum_{i=0}^{4}a_it^i)r_y^3),$$

where K_ω, a_i (i = 0, 1, 2, 3, 4) and β are defined from (3.95), (3.96) and (3.97), respectively. It is not difficult either to calculate the derivatives for (4.9), but we omit these calculations here because they are too long and tedious.

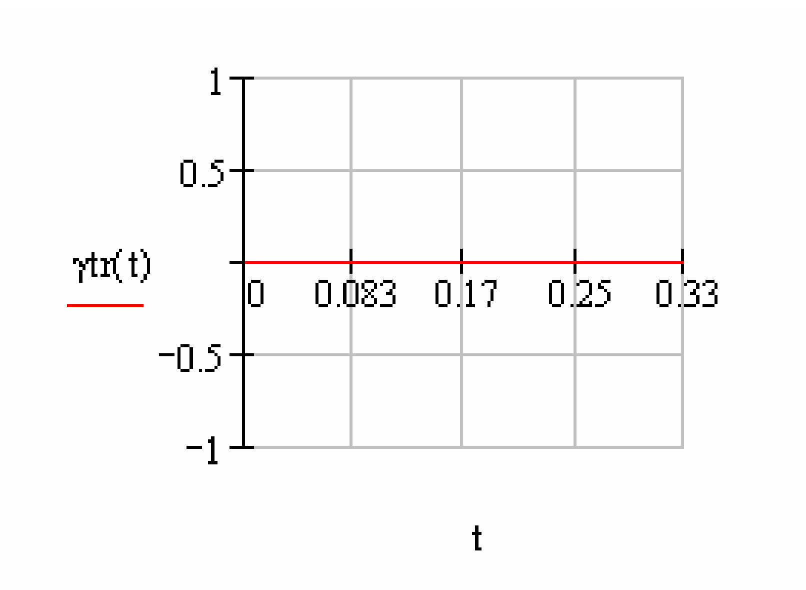
As stated in Chapter III, the values are equal to the initial deviations from the synthesized control trajectories and define the presence of a transient process. When they are equal to the boundary conditions (4.8) for $t = 0$, this means that there is no transient process at all.

Figs. 4.9÷4.12 show the dynamic characteristics of the control process on the deceleration segment when $\gamma_{10}=\gamma_0=0.905$ *and* $\dot{\gamma}_{10}=\dot{\gamma}_0=1$, i.e. when there is no transient process – this is clearly seen from Figs. 4.9,b and 4.10,b. Therefore the curves in Fig. 4.9,a and Fig. 4.10,a coincide, since the transient component is absent. Again we see that the control fully satisfies the boundary conditions and in this case the acceleration and the velocity become equal to zero at the end of the given time interval (Fig. 4.10,c and Fig. 4.12) , which results in a complete stop.

Figs. 4.13÷4.16 show the dynamic characteristics when $\gamma_{10}=\gamma_0=0$ *and* $\dot{\gamma}_{10}=\dot{\gamma}_0=0.$ In this case, as seen from Figs. 4.13,b and 4.114,b, there exists a transient process. As different from the preceding motion stages, in this case the intensity of transient processes is quite comparable with stationary functions though these transient processes damp down soon. It is obvious that the intensity of transient processes explains an essential difference between the stationary and complete functions of angular motion (Fig. 4.13,a and 4.13,b) and its velocity (Fig. 4.14,a and 4.14,b). Nevertheless the control again satisfies the boundary conditions – this fact also follows from (4.6), where we should substitute the values of functions (4.9) for $t = T_1$, which gives for the deceleration stage the values $\gamma(T_1) = \gamma_f = 1.355$ and $|x| = |\xi(0)| = |\xi(T_1)| = 55$.



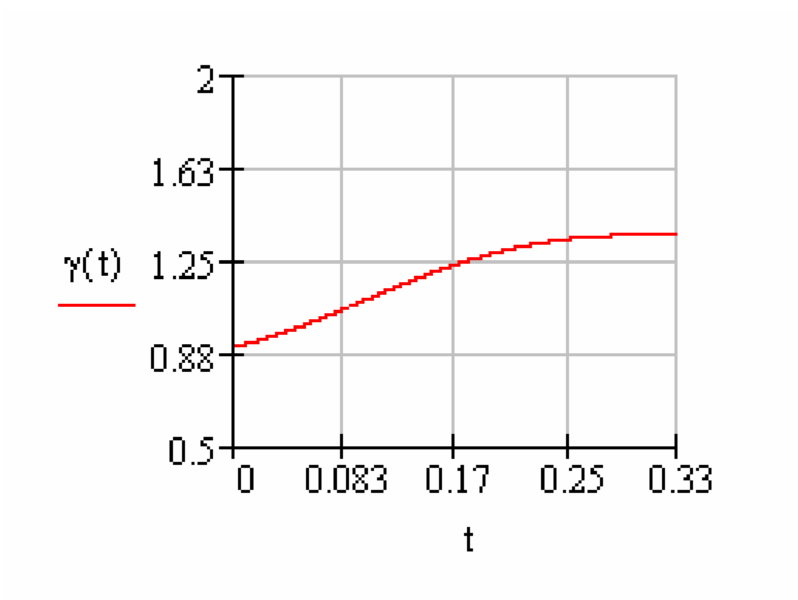
a)



b)

Fig. 4.9 The deceleration segment (there is no transient process): the rotation angle value as a function of time:

a) the forced component; b) the transient component



c)

Fig. 4.9 The deceleration segment (there is no transient process): the rotation angle value as a function of time:

c) the complete solution: the sum of the forced and transient components

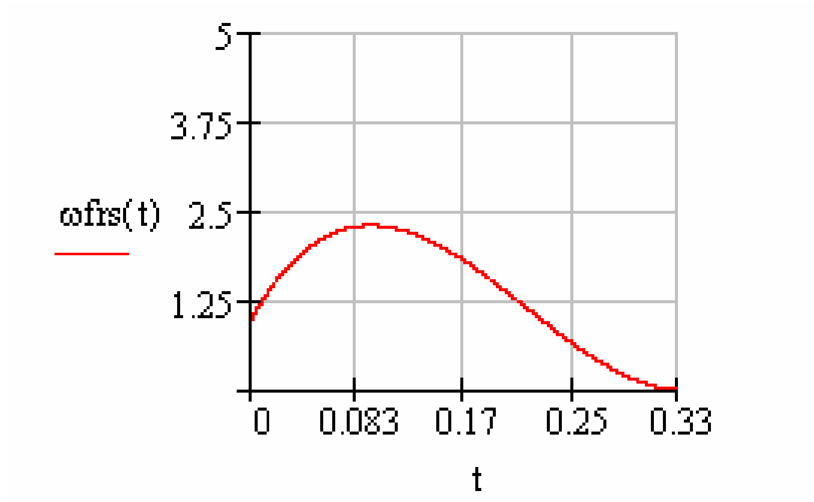
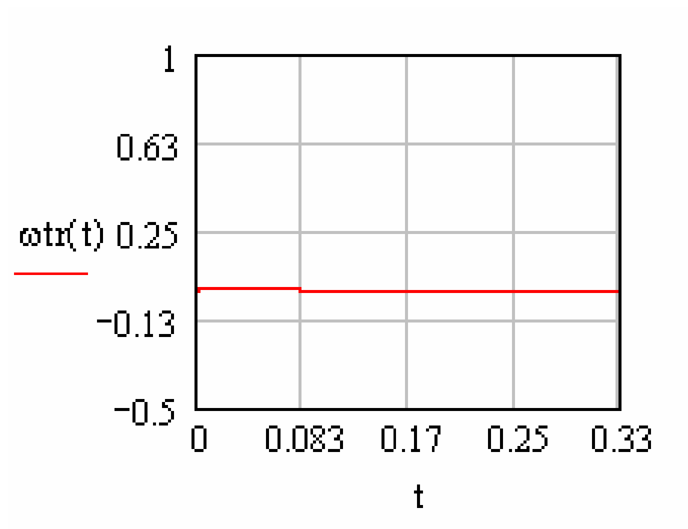
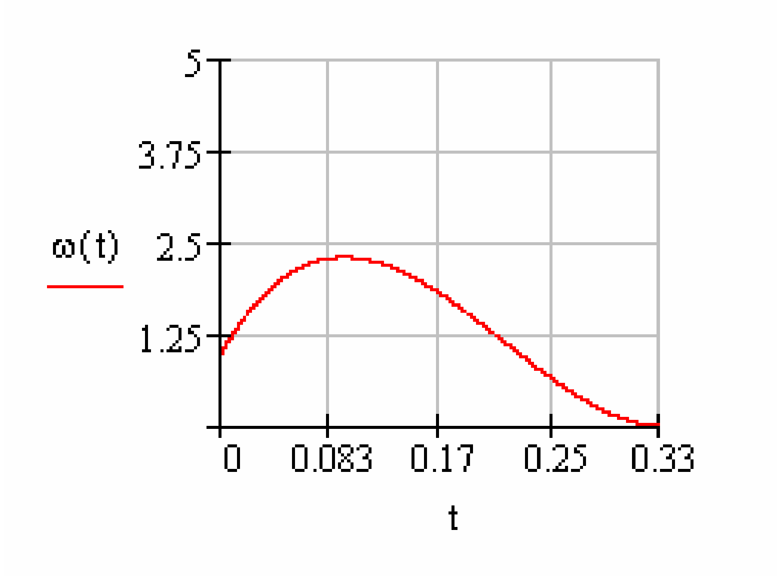


Fig. 4.10 The deceleration segment (there is no transient process): the rotation angle value as a function of time: a) the forced component



b)



c)

Рис. 4.10 The deceleration segment (there is no transient process): the angular velocity value as a function of time:

b) the transient component;

c) the complete solution: the sum of the forced and transient components

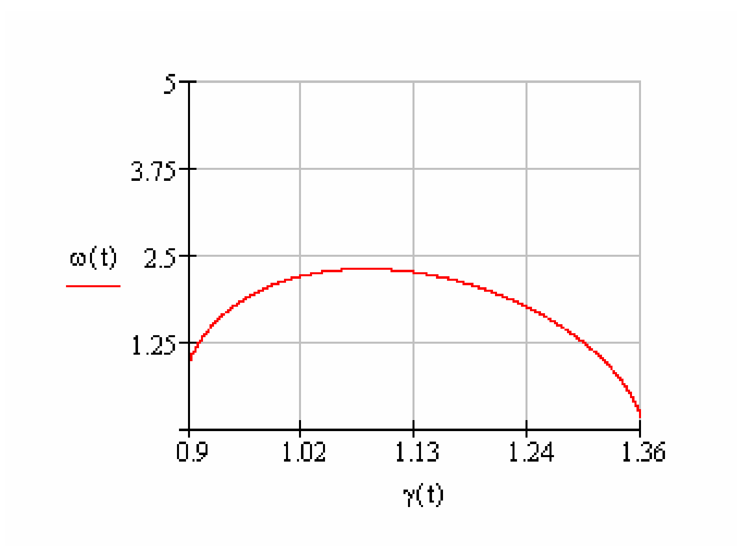


Fig. 4.11 The deceleration segment (there is no transient process): the phase trajectory

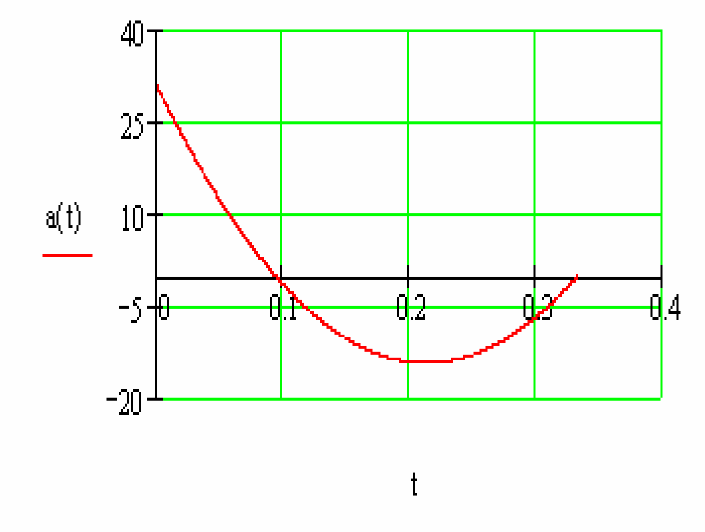
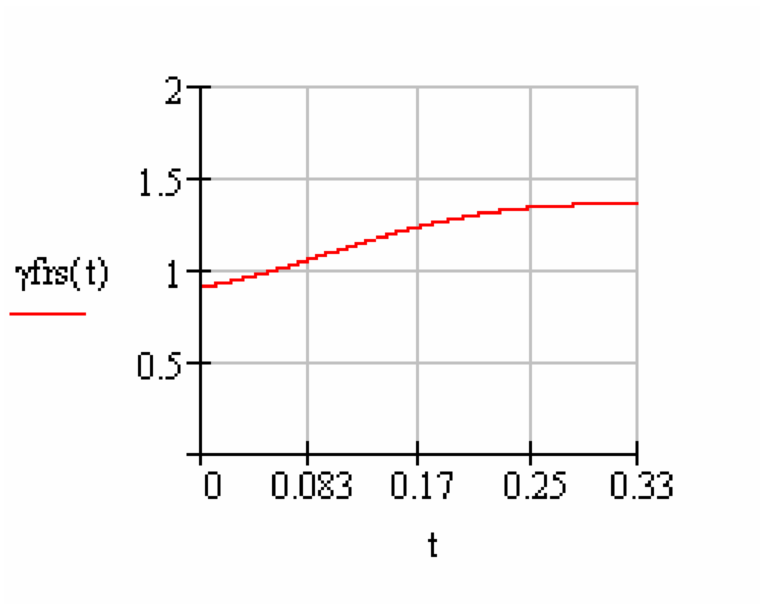
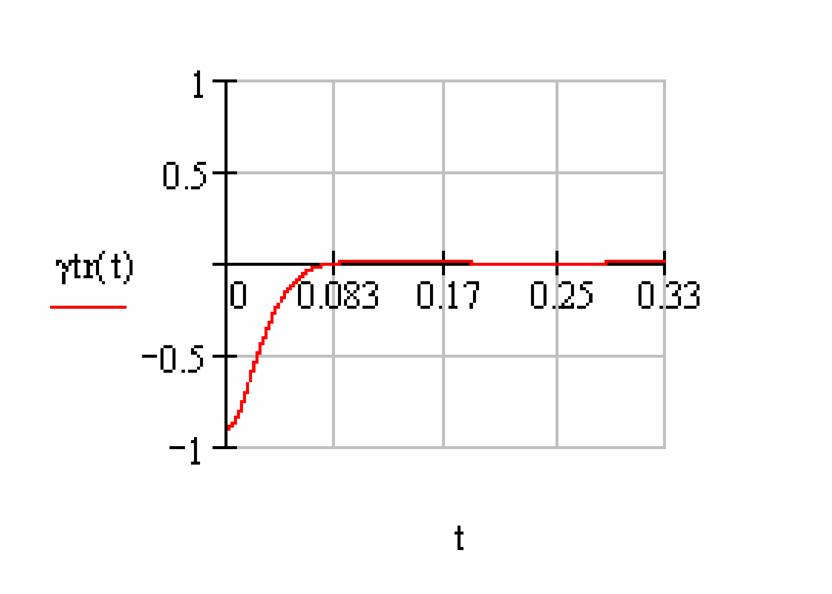


Fig. 4.12 The deceleration segment (there is no transient process): acceleration as a function time



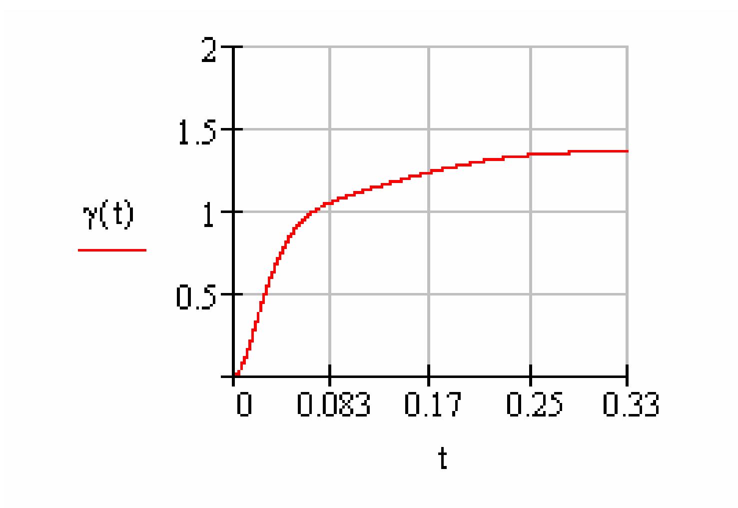
a)



b)

Fig. 4.13 The deceleration segment (there is a transient process): the rotation angle value as a function of time:

a) the forced component; b) the transient component



c)

Fig. 4.13 The deceleration segment (there is a transient process): the rotation angle value as a function of time:

c) the complete solution: the sum of the forced and transient components

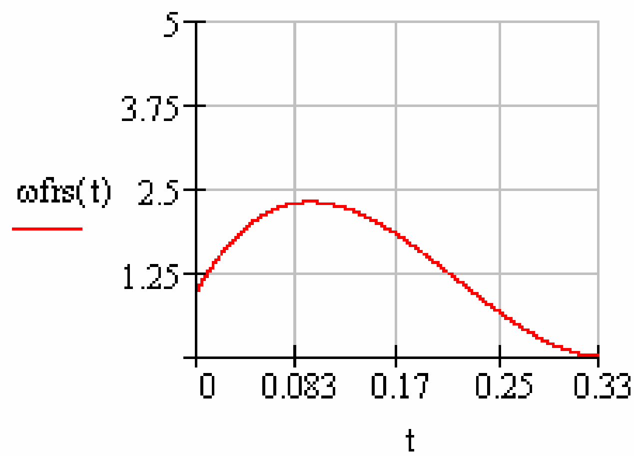
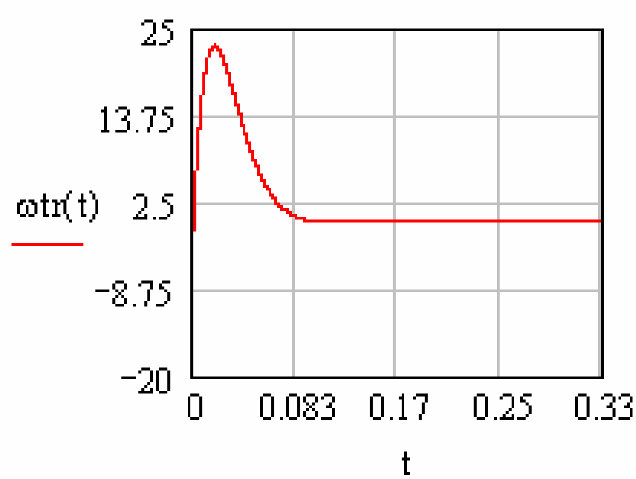
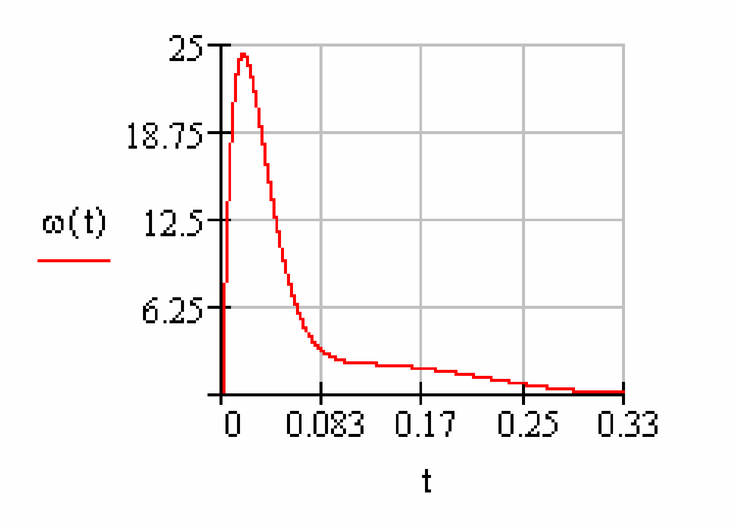


Fig. 4.14 The deceleration segment (there is a transient process): the angular velocity as a function of time

a) the forced component



b)



c)

Fig. 4.15 The deceleration segment (there is a transient process): the angular velocity as a function of time

b) the transient component;

c) the sum of the forced and transient components

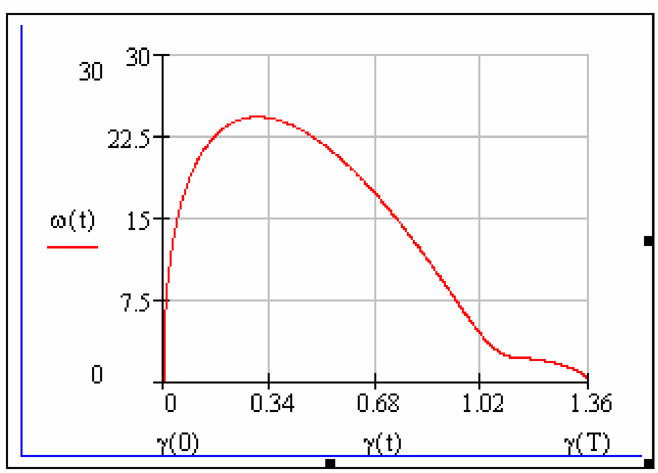


Fig. 4.15 The deceleration segment (there is a transient process): the phase trajectory:

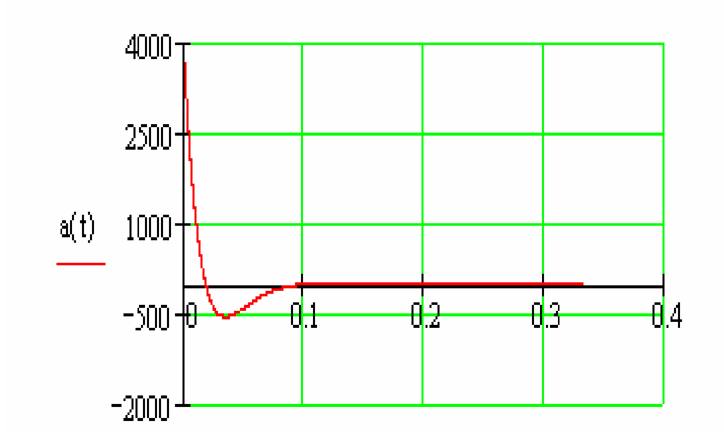


Fig. 4.16. The deceleration segment (there is a transient process): acceleration as a function of time

4.4 Development of an Optimal Control of the Electric Drive of Spatial Rotations of Manipulators

After we have obtained the algorithms of an adaptive terminal control of spatial rotations of robot-manipulators, there arises a problem on the development of an optimal control of the electric drive of these systems. In this case, too, we have used the variational methods connected with power losses. It should be said that these methods have found quite a wide application for the solution of problems of this kind [84÷88].

Let us derive an optimal diagram of the rotation velocity of the drive shaft. $\Omega_д$ and the armature current $i_я$ by considering as an example an electric motor working on direct current of separate excitation. The power dissipated in the armature winding during the transient process is assumed to have a minimal value.

We begin the consideration by assuming that the total moment of inertia in the drive system is $J_\sum$ = const and the static load moment is M_c — const. In the equation

$$c_Д i_Я \phi_Д - M_c = J_\Sigma \frac{d\Omega_Д}{dt} \tag{4.10}$$

we pass to relative units by denoting

$$M_c / M_{Д\,Н} = \mu_0; \quad J_\Sigma \Omega_Д / M_{ДН} = T_M; \quad \Omega_Д / \Omega_{ДН} = \upsilon; \; i_Я / i_{Я\,Н} = i$$

$$\Phi_Д / \Phi_Н = \Phi; \qquad t / T_M = \tau$$

Here $M_{Дн} = C_Д i_{Ян} \Phi^Н$ is the nominal moment of the motor; $i_{ян}$ and $\Phi_н$ are respectively the nominal armature current and the nominal excitation flow; $\Omega_{ДН}$ is the nominal velocity of the motor.

If $Ф_Д = Ф_Н$, then $\Phi = 1$ and the equation of drive motion is written in the form

$$i = \upsilon(1) + \mu_0. \tag{4.11}$$

It is required to find functions v(τ) and i(τ) that reduce the functional $W = \int_0^T i^2 d\tau$ to a minimum. The boundary conditions are given in the form $\upsilon(0) = 0$, $v(T) = 0$. In addition to this, the isoperimetric condition $\alpha = \int_0^T v d\tau$ is given, where α is the rotation angle. For the functional $W = \int_0^T \left(\upsilon^{(1)} + \mu_0\right)^2 d\tau$ we use the method of Lagrangian multipliers and obtain the integrand of an auxiliary functional

$$F^* = \left(\upsilon^{(1)} + \mu_0\right)^2 + \lambda\upsilon\,. \tag{4.12}$$

We define the partial derivatives

$$\partial F^* / \partial\upsilon = \lambda, \qquad \partial F^*/\partial\upsilon(1) = 2(\upsilon(1) + \mu 0). \tag{4.13}$$

After substituting then into the Euler equation

$$\partial F^*/\partial\upsilon - d/d\tau \quad \partial F^*/\partial\upsilon(1) = 0,$$

we obtain $\lambda - 2\upsilon(2) = 0$, i.e. $\upsilon(2) = \lambda/2$. Then

$$\upsilon^{(1)} = \frac{\lambda}{2}\tau + C_1; \qquad \upsilon = \frac{\lambda}{4}\tau^2 + C_1\tau + C_2\,. \tag{4.14}$$

From the boundary conditions we have

C2=0; $\frac{\lambda}{4}\tau^2 + C_1 T = 0$; $C_1 = \frac{-\lambda T}{4}$,

whence we have

$$\upsilon(\tau) = \frac{\lambda}{4}\tau(\tau - T), \qquad i(\tau) = \frac{\lambda}{4}(2\tau - T) + \mu_0. \tag{4.15}$$

The constant λ is defined from the isoperimetric condition

$$\alpha = \int_0^T \frac{\lambda}{4}\tau(\tau - T)d\tau = \frac{\lambda}{4}\left[\frac{T^3}{3} - \frac{T^3}{2}\right] = -\frac{\lambda T^3}{24},$$

i.e.

$$\lambda = -\frac{24\alpha}{T^3}.$$

Thus we obtain the optimal functions of velocity $\upsilon(\tau)$ and current $i\,(\tau)$ in the transient process

$$\begin{aligned} v_{opt}(\tau) &= \frac{6\alpha\tau}{T^3}(T - \tau), \\ i_{opt}(\tau) &= \frac{6\alpha}{T^3}(T - 2\tau) + \mu_0. \end{aligned} \tag{4.16}$$

In the above-considered problem we have illustrated the application of the variational method, i.e. the method which underlies the solution of the terminal problems in Chapter III, to the d.c. electric motor without taking into account the restrictions υ, i and control. Let us consider the synthesis of controls electric motors of the manipulator whose diagram is shown in Fig. 4.17:

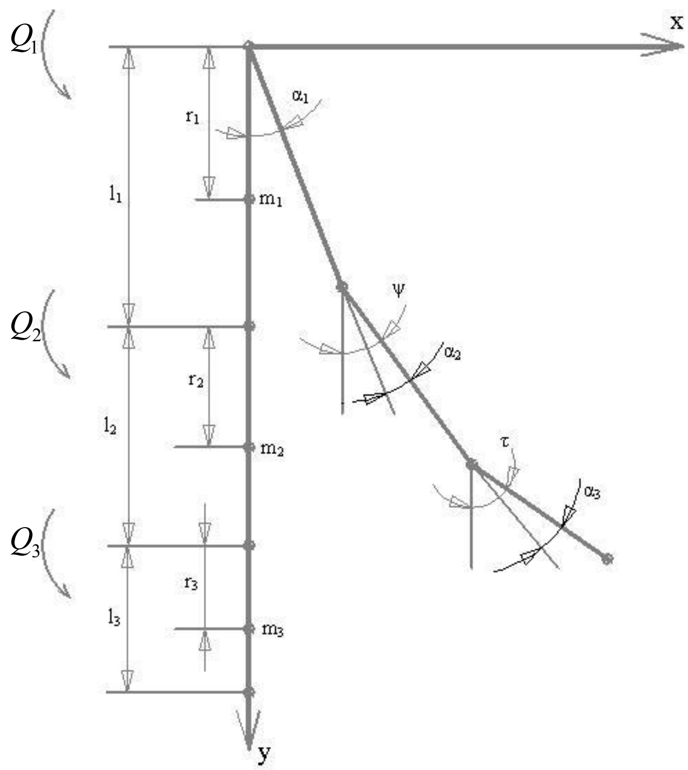


Fig. 4.17 The diagram of the manipulator with three joints

In order to find an optimal fast-acting positional control of the electric motors, we write the differential equations of the considered manipulator in form [87, 88]:

$$Q_1 = [\ \ddot{\alpha}_1 m_1 r_1^{\ 2} + m_2\left(l_1^{\ 2} + r_2^{\ 2} + 2l_1 r_2 \cos\alpha_2\right) + m_3(\ l_1^{\ 2} + l_2^{\ 2} + r_3^{\ 2} +$$
$$+ 2l_1 l_2 \cos\alpha_2 + 2l_2 r_3 \cos\alpha_3 + 2l_1 r_3 \cos(\alpha_2 + \alpha_3)\ )\ ] + q_1;$$
$$Q_2 = \ddot{\alpha}_2 m_2 r_2^{\ 2} + m_3\left(l_2^{\ 2} + r_3^{\ 2} + 2l_2 r_3 \cos\alpha_3\right)\ ] + q_2;$$
$$Q_3 = \ddot{\alpha}_3 m_3 r_3^{\ 2} + q_3. \tag{4.17}$$

Let us consider the rotation of the joints on a small time interval Δt, to which there corresponds the increments of the angles α_1, $\alpha 2$, α_3 equal to $\Delta\alpha_1$, $\Delta\alpha_2$, $\Delta\alpha_3$, and practically the constancy of the functions $q1$, $q2$, $q3$ which depend on the rotation angles, velocities and accelerations of the joints. Let us make the value $W = \int_0^{\Delta t} dt$ tend to a minimum (this means that we need a maximal quick action) by fulfilling the

conditions

$$\int_0^{\Delta t} M_{Дi}^2 dt = \Delta t M_{ДiH}^2; \quad \int_0^{\Delta t} \dot{\alpha}_3 dt = \Delta\alpha_3; \quad \alpha_i(0) = \alpha_{i0};$$

$$\dot{\alpha}_i(0) = \dot{\alpha}_{i0}; \qquad Q_i = M_{Дi}\dot{J}_i - J'_{Дi}\dot{\Omega}_{Пi},$$

where $M_{\text{дiн}}$ is the given moment of the electric motor; $\dot{\Omega}_{\text{П}i}$ is the acceleration of the drive shaft; the friction force is not taken into account.

For the third motor we minimize the functional

$$W_3 = \int_0^{\Delta t} \left[1 + \lambda \frac{1}{j_3^2} \left(\ddot{\alpha}_3 m_3 r_3^2 + q_3 + J'_{Д3}\Omega_{\text{П}3}^{(1)} \right)^2 \right] dt. \tag{4.18}$$

From the Euler equation $a_3^{(4)} = 0$ we obtain

$$\alpha_3(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3,$$

where $C_1 = \alpha_{30}$, $C_2 = \dot{\alpha}_{30}$.

We can show that [87, 88]

$$C_3 = \frac{1}{\Delta t^2} \left(\Delta\alpha_3 - \dot{\alpha}_{30}\Delta t - \frac{1}{6}\Delta\ddot{\alpha}_3 \Delta t^2 \right) \cong \frac{1}{\Delta t^2} \left(\Delta\alpha_3 - \dot{\alpha}_{30}\Delta t \right), \tag{4.19}$$

and therefore on the time interval Δt. the extremum α_3(t) is

$$\alpha_3(t) = \alpha_{30} + \dot{\alpha}_{30} t + \frac{\pm j_3 M_{Д3H} - q_3 - J'_{Д3}\Omega_{\text{П}3}^{(1)}}{2 m_3 r_3^2} t^2 + \frac{\Delta\ddot{\alpha}3}{6\Delta t} t^3. \tag{4.20}$$

At the moment of time

$$t = \Delta t = \frac{\dot{\alpha}_{30} m_3 r_3^2}{\pm j_3 M_{Д3H} - q_3 - J'_{Д3}\Omega_{\text{П}3}^{(1)}} \pm$$

$$\pm \sqrt{\left(\frac{\dot{\alpha}_{30} m_3 r_3^2}{\pm j_3 M_{Д3H} - q_3 - J'_{Д3}\Omega_{\text{П}3}^{(1)}} \right)^2 + \frac{2\Delta\alpha_3 m_3 r_3^2}{\pm j_3 M_{Д3H} - q_3 - J'_{Д3}\Omega_{\text{П}3}^{(1)}}}$$

the device, which controls the motor and the power-driven electronic converter, computes the new coefficients C_1, C_2, C_3, C_4 of the function $\alpha_3(t)$ for the next time interval Δt_1. Here of two different sign before the square root we should take the one which corresponds to the smallest positive value of Δt. The sign before $M_{\text{д3Н}}$ is taken positive or there take place the moments of force $M_{\text{д3}}$ = $M_{\text{д3н}}$ or $q_3 + J'_{Д3}\Omega_{\text{П}3}^{(1)}$.

For the second electric motor we minimize the functional

$$W_2 = \int_0^{\Delta t} \left\{ 1 + \lambda \frac{1}{j_2^{\,2}} \left[\ddot{\alpha}_2 \left(m_2 r_2^{\,2} + m_3 \left(l_2^{\,2} + r_3^{\,2} + 2 l_2 r^3 \cos\alpha_3 \right) \right) + q_2 + J'_{Д2} \Omega^{(1)}_{П2} \right]^2 \right\} dt \qquad (4.21)$$

by fulfilling the conditions

$$\int_0^{\Delta t} M^2_{Д2} dt = \Delta t M^2_{Д2H}; \qquad \alpha_2(0) = \alpha_{20}; \qquad \dot{\alpha}_2(0) = \dot{\alpha}_{20},$$

where $M_{Д2н}$ is the given moment for the second motor. The optimal trajectory of the second joint is

$$\alpha_2(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3,$$

where $C_1 = \alpha_{20}$, $C_2 = \dot{\alpha}_{20}$, $C_4 = \dfrac{\Delta\ddot{\alpha}_2}{6}$, the constant C_3 is defined from the equation

$$\frac{1}{j_2^{\,2}} \left\{ 2C_3 \left[m_2 r_2^{\,2} + m_3 \left(l_2^{\,2} + r_3^{\,2} + 2 l_2 r_3 \cos\alpha_3 \right) \right] + q_2 + J'_{Д2} \Omega^{(1)}_{П2} \right\}^2 = M^2_{Д2H}$$

whence we have

$$C_3 = \frac{\pm j_2 M_{Д2H} - q_2 - J'_{Д2} \Omega^{(1)}_{П2}}{m_2 r_2^{\,2} + m_3 \left(l_2^{\,2} + r_3^{\,2} + 2 l_2 r_3 \cos\alpha_3 \right)}, \qquad (4.22)$$

where the sign before $M_{Д2н}$ depends on whether the direction of the force $M_{д2н}$ coincide or does not coincide with that of $q_2 + J'_{Д2}\Omega^{(1)}_{П2}$).

For the first electric motor we find the optimal trajectory $\alpha_1(t)$ in an analogous manner as for the second motor. Thus we obtain

$$\alpha_1(t) = C_1 + C_2 t + C_3 t^2 + C_4 t^3, \qquad (4.23)$$

where $C_1 = \alpha_{10}$; $\quad C_2 = \dot{\alpha}_{10}$; $\quad C_4 = \dfrac{\Delta\ddot{\alpha}_1}{6\Delta t}$;

$$C_3 = \frac{\pm j_1 M_{Д1H} - q_1 - J'_{Д1} \Omega^{(1)}_{П1}}{\left[m_1 r_1^{\,2} + m_2 \left(l_1^{\,2} + r_2^{\,2} + 2 l_1 r_2 \cos\alpha_2 \right) + m_3 \begin{pmatrix} l_1^{\,2} + l_2^{\,2} + r_3^{\,2} + 2 l_1 r_2 \cos\alpha_2 + \\ 2 l_1 r_3 \cos\alpha_3 + 2 l_2 r_3 + 2 l_1 r_2 \cos(\alpha_2 + \alpha_3) \end{pmatrix} \right]}$$

Displacements of the first and second joints performed on the time interval Δt can be calculated by the formulas

$$\Delta\alpha_1 = \int_0^{\Delta t} \dot{\alpha}_1(t) dt; \qquad \Delta\alpha_2 = \int_0^{\Delta t} \dot{\alpha}_2(t) dt. \qquad (4.24)$$

For the known ε_i, Ω_i the electric motor can be correctly chosen after constructing the load characteristics, i.e. the dependence of velocities on moments of force for each motor, in the transient process. These load characteristics are obtained by means of the laws of motion of manipulator joints. These laws are constructed by the given manipulator capacity and the results of the synthesis of optimal motions. The constructed load characteristics are used to define a maximal moment and a maximal velocity of the drive. If the drive has these values, then the given law of motion is realizable. Next, after choosing a reduction gear, we estimate the nominal power value by the motor heating condition. In Fig. 4.18 we give the examples of the simplified diagrams of accelerations ε_1, decelerations ε_2 and velocities $\Omega_Д$ of the motor of the robot with remote control. Using these values, we can preliminarily estimate $M_Д$ max, $\Omega_Д$ max for the diagrams shown in Fig. 4.18,a while the load characteristic has the form of an ellipse

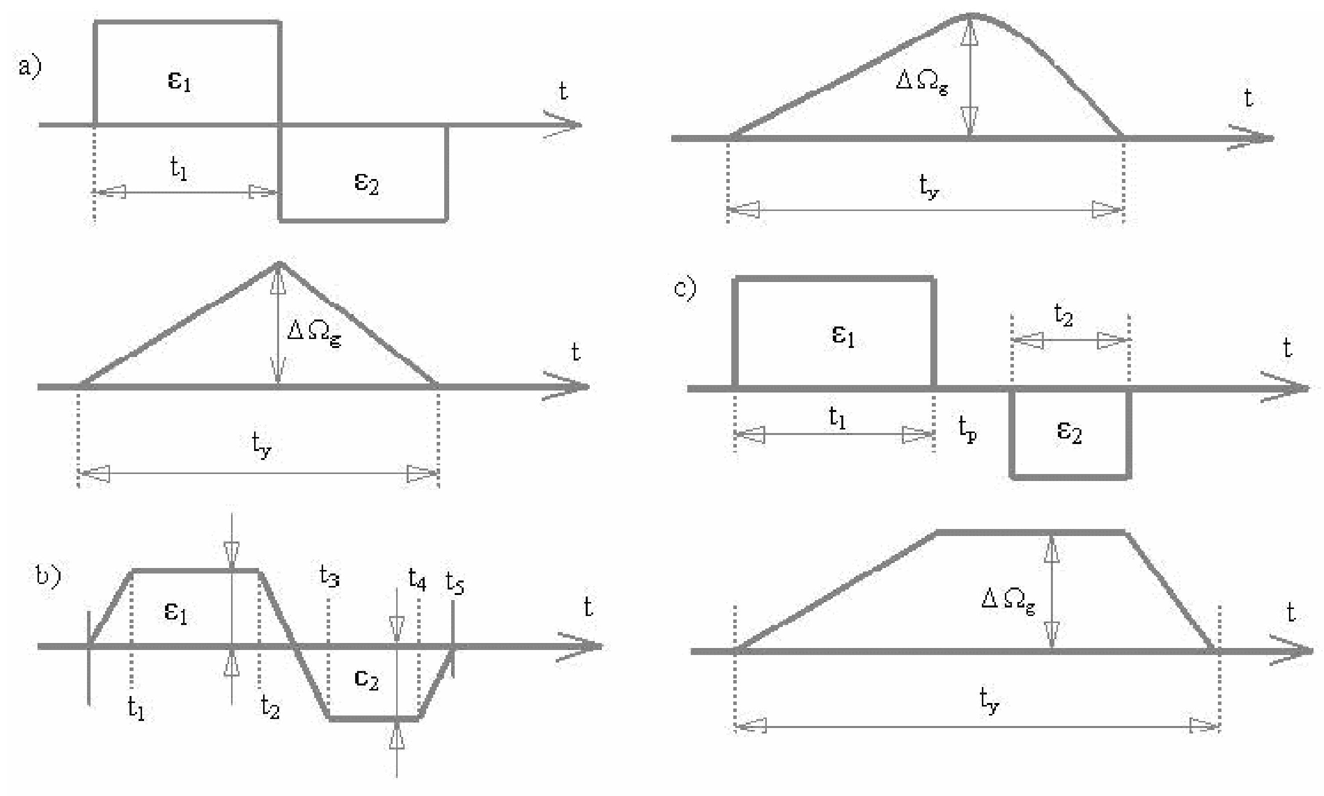


Рис. 4.18 Diagrams of accelerations ε_1, decelerations ε_2 and velocities $\Omega_Д$ of the motor of a robot with positional control.

$$\Omega_Д{}^2 / \Omega^2_{Д\max} + Q^2 / j^2 M^2_{Д\max} = 1, \qquad (4.25)$$

where

$$\Omega^2_{Д\max} = \frac{\Delta\alpha_Д M_{Д_{\max}}}{J_Д} = \Delta\Omega^2_Д; \;\; M_{Д\max} = \frac{4\Delta\alpha_Д J_Д}{t_y^{\,2}\left(1-k_M^{\,2}\right)} 1;$$

$$k_M = \frac{Q}{jM_Д \max}; \qquad Q = const;$$

is the control time $t_у = 2\Delta\alpha_Д/\Delta\Omega_Д$; $\Delta\alpha_Д$ is an angular displacement of the motor in the transient process. shaft угловое перемещение вала двигателя в переходном процессе. For the diagrams in Fig. 4.18,b the load characteristic is

$$\Omega_Д / \Omega_{Д_{\max}} + Q^2 / j^2 M^2_{Д\max} = 1, \tag{4.26}$$

where $\Omega_{Д\max} = \Delta\Omega_Д = \dfrac{M_{Д_{\max}}}{2J_Д}\left(t_y - \dfrac{2M_{Д\max}}{J_Д\gamma}\right)\left(1-k_M^{\,2}\right);$

$\gamma = \left|\dot{\varepsilon}_{1,2}\right| = \text{const}$ on the intervals from zero to t_1, from t_2 to t_3 and from t_4 до t_5;

$$M_Д \max = \frac{J_Д\gamma t_y}{4}\left[1+\sqrt{1-\frac{32\Delta\alpha_Д}{t_{y^3}\left(1-k_{M^2}\right)\gamma}}\right];$$

$$t_y = \frac{M_Д \max}{J_Д\gamma} + \sqrt{\frac{4\Delta\alpha_Д J_Д}{M_Д \max\left(1-k_M^{\,2}\right)} + \frac{M^2_{Д\max}}{\gamma^2 J_Д^{\,2}}}$$

For the diagrams in Fig. 4.18,c the load characteristic has the form of an ellipse and

$$\Omega^2_{Д\max} = \frac{\Delta\alpha_Д M_Д \max(1-\rho)}{J_Д(1+\rho)}; \qquad \rho = t_p / t_y;$$

$$M_Д \max = \frac{4\Delta\alpha_Д J_Д}{t_y^{\,2}\left(1-k_M^{\,2}\right)\left(1-\rho^2\right)}; \tag{4.27}$$

$$t_y = \frac{2\Delta\alpha_Д}{\Delta\Omega_Д(1+\rho\;)}$$

In the formulas for $\Omega_{Д\max}$ and $M_{Д\max}$ we do not take into account the feedback in the drive with respect to $\Omega_Д$ and $M_Д$. If the initial and terminal values of joint trajectories $\alpha_{iН}$ and $\alpha_{iк}$ are arbitrary (for example, for tracking systems), i.e. probable in the working zone of the manipulator, then the motor is chosen by a trajectory which is the most difficult one from the standpoint of obtaining $M_{д\max}$, $\Omega_{д\max}$ and motor heating. Note that the condition of motor heating used above in the motion synthesis does not take into account heat exchange and is regarded as a repeated short-lived phenomenon when the heating of the armature winding has no time to reach an admissible heat value during the motion cycle.

Fig. 4.19 shows the circuit realizing the control of the manipulator motors. The circuit inputs receive information on the velocities $\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3$, accelerations $\dot{\alpha}_1, \dot{\alpha}_2, \dot{\alpha}_3$, moments of force Q_1 Q_2, Q_3 calculated or measured on the preceding time interval and recorded in the memory within the given time interval Δt. This information is used to calculate q_1, q_2, q_3 and, after that, the control for the next time interval which begins when new memorized potentials are delivered to the circuit inputs; in that case, the signal values at the integrator outputs are set to zero by closing the discharge loops of the capacitors.

The control realized by the circuit shown in Fig. 4.19 is cancelled by the programming device in the initial and terminal parts of the arm trajectory when the inequalities $\beta_{1\,i} < \alpha_{3i} - \alpha_i < \beta_{2i}$ are fulfilled. This ensures the smoothness of the start and stop. Here α_{3i} is the given stepwise displacement of the i-th joint set in time by the programming device.

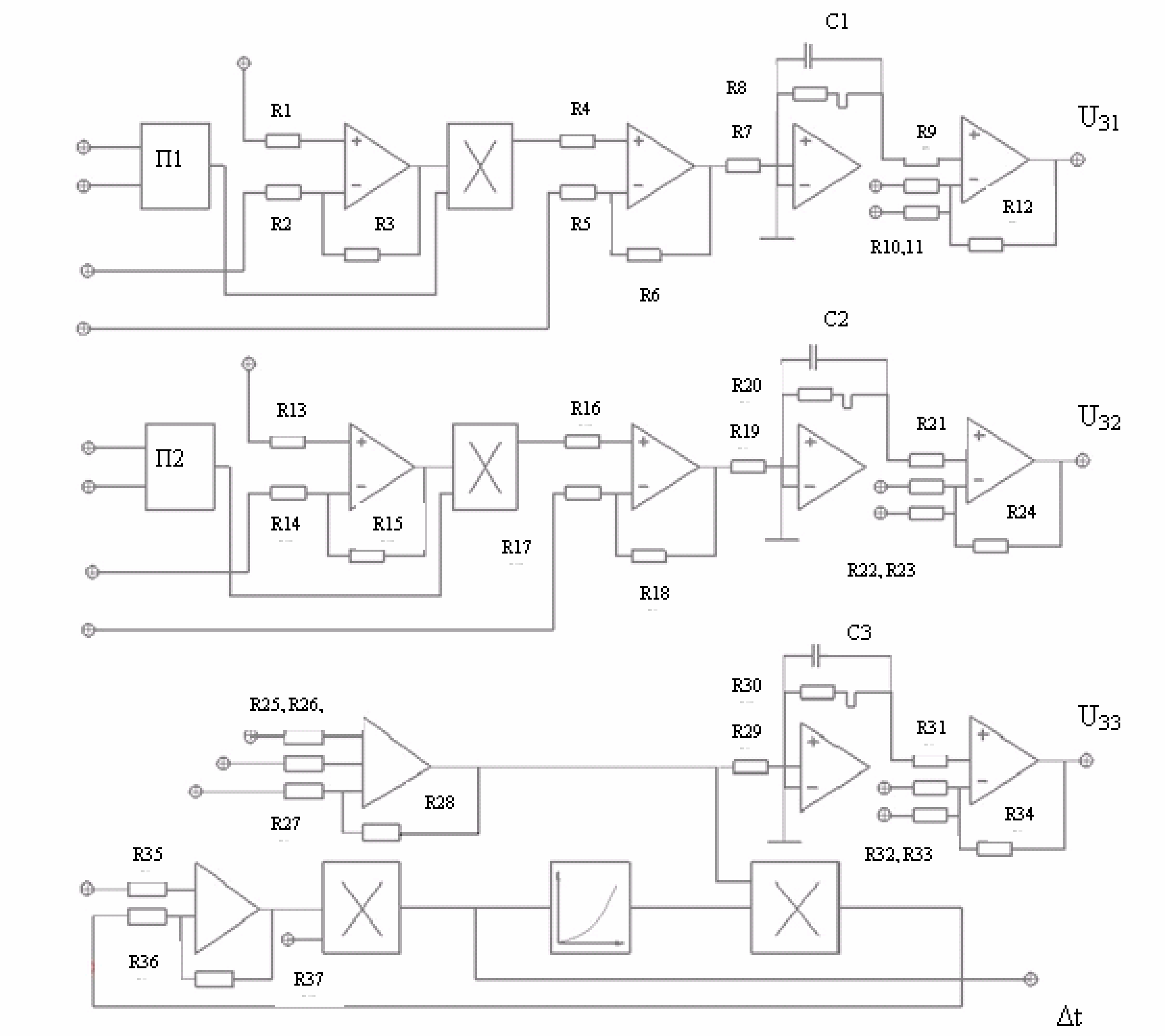


Fig. 4.19 The Circuit Realizing the Control of the Manipulator Motors

Conclusions

1. Spatial rotations are for the first time described by their spinor representation, which made it possible to obtain simple relations for describing by means of an element of the controlling orthogonal matrix of the basic representation by the known coordinates of three defining rotation points: central, initial and terminal.

2. Simple formulas are obtained for calculation of controlling Euler angles;

3. The obtained results have enabled us to reduce the actually three-dimensional problem of spatial motion control to the one-dimensional problem;

4. A general variational method is obtained to solve problems of terminal control of spatial rotations;

5. Simple adaptive algorithms are obtained, by means of which various partial problems on the terminal control of acceleration, transfer of the object to a given point, and approach are solved under various terminal conditions.

6. New algorithms of control of spatial rotations of manipulating robots are studied;

7. An optimal control circuit is developed for the work of the electric drive realizing the algorithms of control of spatial rotations of manipulating robots.

References

1. S.Sakai, K.Osuka, H.Fukushima, M.Iida: Watermelon Harvesting Experiment of a Heavy Material Handling Robot for Agriculture Using LQ Control, Proc.of IROS,pp.769-774,2002
2. Asada, H. 1984. Dynamic analysis and design of robot manipulators using inertia ellipsoids.
3. Corke, P. I. 1996. A robotics toolbox for MATLAB. *IEEE Robotics and Automation Magazine* 3, 24-32.
4. Craig, J. J. 2003. *Introduction to robotics – mechanics and contro*l. Pearson Education, Delhi, India.
5. Y.Tokunaga, T.Hakukawa, T.Inoue, ”Algorithm and Design of an Robot ”,J.of Robotics and Mechatronics,Vol.11,No.3,pp.173-182,1999.
6. Reid JF, Searcey SW, An Algorithm for separating guidance information from row crop images Trans ASEA 1988 pp 1624-1632
7. S.Sakai, K.Osuka, M.Iida, M.Umeda: Control of a Heavy Material Handling Agricultural Manipulator Using Robust Gain-scheduling and μ-Synthesis, Proc.of IEEE ICRA pp.96-102,2003

8. Kondo, N., M. Monta, T. Fujiura. 1996a. Fruit harvesting robots in Japan. *Advances in Space Research* 18 (1/2), 181-184.
9. Kondo, N., M. Monta, and T. Fujiura. 1996. Basic constitution of a robot for agricultural use. *Advanced Robotics* 10 (4), 339-353.
10. Davies, E. R. (1997). Machine Vision Theory Algorithms, Practicalities. London, Academic Press.
11. Illingworth, J. and J. Kittler (1988). "A survey of the Hough Transform." Computer Vision Graphics and Image Processing 44(1): 87-116.
12. S.Sakai, K.Osuka, M.Umeda: Global Performance of Agricultural Robots, Proc.of IROS,to appear.2004
13. Van Straten, G. 2004. Field Robot Event, Wageningen, 5-6 June 2003. *Computers and Electronics in Agriculture* 42(2004) 51-58.
14. Reed, J.N., S.J. Miles, J. Butler, M. Baldwin, and R. Noble. 2001. Automatic mushroom harvester development. J. of Agricultural Engineering Research, 78(1) 15-23.
15. Torii, T.: 2000. Research in autonomous agriculture vehicles in Japan. *Computers and Electronics in Agriculture* 25 (12), 133–153.
16. Agricultural Automation in the new Millennium. *Computers and Electronicsin Agriculture* 30 (2001), 237–240.
17. Baerveldt, A., J., B., Astrand. 1998. Vision-guided mechanical weed-control. Halmstad University.
18. N.Kawamura, N.Namikawa, T.Fujiura, M.Ura, ”Studyon Agricultural Robot (Part1)”,J.of Japanese Society of Agricultural Machinery,vol.46,no.3,pp.353-358,1984 (in Japanese)
19. Reid, J.F., Q., Zhang, N., Noguchi, M., Dickson: 2000. Agricultural automatic guidance research in North America. *Computers and Electronics in Agriculture* 25 (12), 155–167.
20. Van Henten, E.J., J. Hemming, B.A.J. Van Tuyl, J.G. Kornet, J. Meuleman, J., Bontsema, E.A. van Os. 2002. An autonomous robot for harvesting cucumbers in greenhouses. *Autonomous Robots* 13, 241-258.
21. Hannan, M. W., and T.F. Burks. 2004. Current developments in automated citrus harvesting. Paper No. 04-3087. St. Joseph, Mich.: ASAE. Kassler, M. 2001.
22. Pellenc, R., Montoya, J. L., D’Esnon, A. G., and Rombaut, M. 1990. Automated machine for detection and grasping of objects. U.S. Patent No. 4975016.

23. Reid JF, Searcy SW, Automatic tractor guidance with computer vision. Proc Int Off-Highway & Powerplant Congress, Milwaukee Sept 14-17 1987.
24. Harrell, R. C., P. D. Adsit, T. A. Pool, and R. Hoffman. 1988. The Florida robotic grove lab. ASAE Paper No. 88-1578. St. Joseph, Mich.
25. Keicher, R., H., Seufert: 2000. Automatic guidance for agricultural vehicles in Europe. *Computers and Electronics in Agriculture* 25 (12), 169–194.
26. Kondo, N., Y. Nishitsuji, P.P. Ling and K. C. Ting, 1996b. Visual feedback guided robotic cherry tomato harvesting. *Transactions of the ASAE* 39-6, 2331-2338
27. G.Grand, R.Rabatel, A.Pellenc, A.Journeau, M.J.Aldon, "Magali:A self-propelled robot to pick apples ",ASAE Paper,vol.46, no.3,pp.353-358,1987
28. Setiawan, A.I.Furukawa, T.Preston, A."A Low-Cost Gripper for an Apple Picking Robot ",ICRA2004,pp.4448-4453,2004.
29. Kassay, L., and D. C. Slaughter. 1993. Electrical, mechanical and electronic problems with the Hungarian apple harvester robot during the 1992 fall field tests. ASAE Paper No. 933088. St. Joseph, Mich.: ASAE.
30. D'Esnon, A. G. 1985. Robotic harvesting of apples. In: *Proc. Agri-Mation* 1, 210-214. Chicago, IL., 25-28 February.
31. Bulanon, D.M., T. Kataoka, Y. Ota, and T. Hiroma. 2001. A machine vision system for the apple harvesting robot. *Agricultural Engineering International: the CIGR Journal of Scientific Research and Development.* Manuscript PM 01 006. Vol. III
32. Ehsani, M. R., S. K. Upadhyaya, D. Slaughter, S. Shafii, M. Pelletier. 1999. A NIR Technique for Rapid Determination of Soil Mineral Nitrogen. *Precision Agriculture* 1(2): 217-234.
33. Bak, T., H., Jakobsen. 2004. Agricultural robotic platform with four wheel steering for weed detection. *Biosystems Engineering* 87(2): 125-136.
34. Rossing, W. and P.H. Hogewerf. 1997. State of the art of automatic milking systems. *Computers and Electronics in Agriculture* 17 (1), 1–17.
35. N.Kondo, K.C.Ting, "Robotics for Bioproduction Systems ", American Society of Agricultural Engineers,1998.
36. Михлин С.Г. Вариационные методы в математической физике. М.: Гостехиздат, 1957, 375с.

37. Курант Р., Гилберт Д. Методы математической физики, т.1. М.: Гостехтеоретиздат, 1951, 525с.
38. Михлин С.Г. Численная реализация вариационных методов. М.: Наука, 1966, 432с.
39. Блисс Дж.А. Лекции по вариационному исчислению. М.: Иностранная литература, 1950, 348с.
40. Янг Л. Лекции по вариацинному исчислению и оптимальному управлению. М.: Мир, 1974, 488с.
41. Гельфанд И.М., Фомин С.В. Вариационное исчисление.М.: Физматгиз, 1961, 228с.
42. Понтрягин Л.С. и др. Математическая теория оптимальных процессов.М.: Физматгиз, 1961,
43. Коллатц Л. Функциональный анализ и вычислительная математика. М.: Мир, 1968, 447с.
44. Беллман Р. Динамическое программирование. М.: Иностранная Литература, 1960
45. Егоров А.И. Об одной вариационной задаче в теории уравнений элиптического типа. // Сибирский математический журнал 1964, 4, №3(5)
46. Ли Э.Б., Маркус Л., Основы теории оптимального управления. М.: Наука, 1972
47. Брайсон А. Хо Ю-ши. Приклпднпя теория оптимального управления. М.: Мир, 1972
48. Красовский Н.Н. Теолрия управления движением. М: Наука, 1971
49. Моисеев Н.Н. Численные методы в теории оптимальных систем. М:.Наука, 1971
50. Летов А.М. Динамика полета и управление. М.: Наука, 1969
51. Milnikov A.A., Prangishvili A.I., Rodonaia I.D. A Spinor Model of 3-Dimensional Generalized Rotations// // Automation and Remote Control, Moscow, 2005, № 6, p. 30-37
52. Milnikov A.A., Onal H., Erguven J., Rodonaia I. A spinor Method of Solution of Manipulators Inverse Kinematic Problem// Problems of Mechanics, Tbilisi, 2006 №1, p.41-48
53. Milnikov A.A., Onal H., Erguven J., Rodonaia I. A Kinematics of Spatial Rotations and Euler’s Angles// Problems of Mechanics, Tbilisi, 2006 №1, p.102-108
54. Гельфанд И.М., Минлос Р.А., Шапиро З.Я. Представления группы вращений и группы Лоренца. М.: Физматгиз, 1958, 751 с.
55. Наймарк М.А. Теория представления групп. М.: “Наука“,, 1976, 453 с.
56. Хаммермеш М. Теория групп и её приложение к физическим проблемам. М.: Мир, 1966, 540с.

57. Вейль. Г. Теория групп и квантовая механика. М.: “Наука“, 1986б 321 с.
58. Вигинер Е. Теория групп и её приложения к квантовомеханической теории атомных спектров. М.: “Наука“, 1961, 278 с.
59. Виленкин Н.Я. Специальные функции и теория представления групп. М.: “Наука“, 1965, 375с.
60. Картан Э. Теория спиноров. М.: “Физматгиз“, 1939, 163с.
61. Годунов С.К. Теория спиноров и представления группы вращений. Новосибирск., Издательство новосибирского университета, 1978, 90 с.
62. Гельфанд И.Н. Лекции по линейной алгебры. М.: «Наука», 1966, 271с.
63. Ланкастер П. Теория матриц. М.: «Наука», 1978, 280с.
64. Гатнмахер Ф.Р. Теория матриц. М.: «Наука», 1967, 575с.
65. Постников М.М. Лекции по геометрии. Линейная алгебра. М.: Наука, 1986, 400с.
66. Ефимов. Н.В. Линейная алгебра и многомерная геометрия. М.: Наука, 1970, 527с.
67. Мальцев А.И. Основы линейной алгебры М.: «Наука», 1970, 400с.
68. Батенко А.П. Управление конечным состоянием движущихся объектов. М.: Советское Радио, 1977, 256 с.
69. Erguven J., Milnikov A.A.,. Rodonaia I.D., Suladze A.S. Moving Mechanical Objects Terminal Adaptive Control Problems // Problems of Applied Mechanics, Tbilisi, 2004 №2(15), p.54-58
70. Смирнов В.И. Курс высшей математики. М.: Гостехтеоретиздат,1951, 804с.
71. Колмогоров А.Н., Фомин С.В. Элементы теории функций и функционального анализа. М.:Наука, 1972, 496с.
72. Мандельштам Л.И. Лекции по теории колебаний. М.: Наука, 1972, 470с.
73. Стрелков С.П. введение тв теорию колебаний. М.: Гостехтеоретиздат, 1950, 344с.
74. Мигулин В.В. и др. Основы теории колебаний. М.: Наука, 1978, 391с.
75. Бидерман В.Л. Прикладная теория механических колебаний. М.: высшая школа, 1972, 413с.
76. Андронов А.А. и др. Теория колебаний.М.: Наука, 1981, 568с.
77. Вибрации в технике т.1. М.: машиностроение, 1978, 352с.
78. Вибрации в технике т.2. М.: машиностроение, 1978, 351с.
79. Fu.K.S., Gonzales R.G., Lee C.S.G. Robotics: Control, Sensing, Vision and Intelligence. New York, McRaw – Hill Book Company, 1984, 621 p.

80. Lee C.S.G., Ziegler M.A. A geometric approach in Solving the Inverse Kinematics of PUMA Robots, IEEE Trans. Aerospace and Electronic Systems, AES-20, No.6, pp. 695-706, 1984
81. Артоболевский И.И. Теория механизмов и машин. М.: “Наука“, 1975, 576 с.
82. Левитский Н.И. Теория мехагизмов и машин. М.: “Наука“, 1979, 574 с.
83. Юдин В.А., Петрокас Л.В. Теория мехагизмов и машин. М.: “Высшая школа”, 1977, 527 с.
84. Юдин В.А. Працтирование кинематических схем механизмов. М.: “Искусство“. 1963, 426 с.
85. Глазенко Т.А., Иришков В.И. Тиристорные широтно-импульсгые преобразователи с дросселями насыщения для систем электропровода постоянного тока.Л., ЛДНТП, 1972, 35 с.
86. Гшрузов В.Л., Саьинин Ю.А. Асихронные маломощные проводы ссо статистическими преобразователями.Л., Энергия, 1974, с 34-158
87. Бродовский В.Н., Иванов Е.С. Приводы с частотно-токовым управлением.М., Энергия. 1974
88. Петров Ю.П. Оптимальное управление электроприводом. Л., Энергия. 1971